GENERAL ARITHMETIC
FOR SCHOOLS

is issued in the following styles:

ORDINARY EDITION
COMPLETE, with and without answers.
PARTS I, II, III, separately, with answers perforated.
PARTS I and II, bound together, without answers.
PARTS II and III, bound together, with answers.

EDITION WITH APPENDIX
COMPLETE, with and without answers.
PARTS I, II, III, separately, with and without answers.
PARTS I and II, bound together, without answers.
PARTS II and III, bound together, with whole of appendix
to PARTS I-III, with and without answers.

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GENERAL ARITHMETIC
FOR SCHOOLS

BY

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PREFACE

This character of this book has been determined by the belief that the primary object in the teaching of elementary arithmetic is to secure accuracy. In pure computation, the less a pupil has to think, the more likely is it that mistakes will be avoided, but this state can only be reached by ample practice in fundamental processes. For this reason, numerous straightforward exercises have been included.

The book is designed for pupils from the age of 11 plus up to School Certificate standard, and is divided into three parts. In the early stages where explanations of processes are always given by the teacher, the bookwork is confined to simple statements of definitions and rules and illustrative examples; in the later chapters the treatment is sufficiently detailed for the pupil to be able to read ahead by himself.

A number of exercises are headed "oral" or "for class discussion"; these are constructed so that the working can be done mentally and are intended to accustom the pupil to the routine of the argument: it is suggested that these exercises should be taken viva-voce, but that all pupils should be required to write down the answers.

The examples in the other exercises are classified under three heads:

(A) Minimum course: plain numbers.
These examples cover all essential points of the work and have been graded carefully. They may be done entire by all pupils.

(B) Extra practice: numbers enclosed in brackets.
These examples provide extra practice, if needed; they are parallel to those in A and do not extend the ground covered.

(C) Advanced: asterisked numbers.
These examples bring out the finer points of the work under consideration and are intended for those pupils who run ahead of the class.

For the convenience of teachers who prefer to make their own selection, these groups are not printed separately, the examples being arranged in order of difficulty.

A new feature which it is believed will be found useful is the inclusion of 3-minute and 4-minute oral-practice exercises at the
PREFACE

ends of Parts I, II. Additional drill practice is provided by the comprehensive Tests in Computation and more miscellaneous work by the Revision Papers at the end of each Part.

Although accuracy is of primary importance, the subject naturally offers considerable scope for developing power of expression and thought; the miscellaneous examples at the end of most chapters are intended to serve this purpose.

An Appendix to each Part has been compiled, containing (i) drill exercises, (ii) additional revision papers, (iii) parallel exercises of all important exercises in the main book, and for convenience of reference numbered in the same way except for a distinguishing (a). The main purpose is to provide material for a revision course in a systematic form, but it will also be of use to those who like to have a very large supply of examples at their disposal. The book may be obtained with or without this appendix.

The author has followed closely the recommendations of the recent report on the teaching of Arithmetic issued by the Mathematical Association, and some quotations from that Report will be found in the text. In cases where there is considerable divergence of opinion (e.g. in multiplication of decimals), each of the methods in general use has been explained. Use has also been made of various practical suggestions in the Board of Education pamphlet (No. 101, 1954) on Senior School Mathematics.

CONTENTS

NOTE. This Arithmetic is issued complete and also in parts and combinations of parts. There are two editions: (i) Ordinary Edition; (ii) Edition with Appendix of additional examples. A full list of styles will be found on page ii. The following is the Table of Contents of the whole book.

TABLES AND FORMULE

PART I

[Pages 1-128]

I. USE OF PRIME FACTORS
   (Tests of divisibility, p. 1; prime factors, index notation, p. 3; \( x \) and \( y \) by factors, p. 4; laws of indices, p. 5; powers, roots, p. 6; H.C.F. by factors and rule, p. 8; L.C.M., p. 11; miscellaneous examples, p. 12.)

II. SIMPLE AREAS, VOLUMES
   (Area of rectangle, p. 15; land measurement, p. 17; further area, p. 18; volume of cuboid, p. 21; material used for a box, p. 23; solids of uniform cross-section, p. 24.)

III. EASY UNITARY METHOD
   (Direct variation, p. 26; inverse variation, p. 29; miscellaneous examples, p. 31.)

IV. FRACTIONS: ADDITION AND SUBTRACTION
   (Notation, p. 33; lowest terms, p. 35; common denominators, p. 36; fractions and division, p. 37; addition, p. 38; subtraction, p. 40; problems, p. 41.)

V. FRACTIONS: MULTIPLICATION AND DIVISION
   (Multiplication, p. 44; division, p. 46; brackets, p. 47; one quantity as fraction of another, p. 48; problems, p. 49.)

VI. DECIMALS: NUMERATION, ADDITION AND SUBTRACTION
   (Notation, p. 52; metric units of length, p. 53; multiplication and division by powers of 10, p. 56; fractions as decimals, p. 58; addition and subtraction, p. 60; miscellaneous examples, p. 62.)
CONTENTS

X.

VII. DECIMALS: MULTIPLICATION AND DIVISION
(Short multiplication, p. 64; long multiplication, p. 65; short division, p. 68; fractions as decimals, p. 69; approximations, p. 70; significant figures, p. 72; division by factors, p. 74; long division, p. 74; decimalization of money, p. 77; decimalization of money by rule, p. 80; decimalization of compound quantities, p. 81; averages, p. 82; miscellaneous examples, p. 84.)

VIII. METRIC SYSTEM
(Metric tables, p. 86; addition and subtraction, p. 89; multiplication and division, p. 90; practice method, p. 90; miscellaneous examples, p. 92.)

IX. GRAPHS
(Graphs of statistics, p. 93; axes and scales, p. 94; loci graphs, p. 97; straight line graphs, p. 102.)

THREE-MINUTE AND FOUR-MINUTE ORAL PRACTICE

O R A L P R A C T I C E
1-8 (Chaps. I-III).
9-16 (Chaps. IV-VIII).

TESTS IN COMPUTATION

Tests, 1-8 (Chaps. IV-VIII).

REVISION PAPERS
Papers 1-8, Elementary work.
Papers 9-16 (Chaps. I-III).

PART II

[Pages 129-272]

X.

FRACTIONAL PARTS AND PRACTICE
(One quantity as fraction of another, p. 129; simple practice, p. 130; compound practice, p. 134; miscellaneous examples, p. 135.)

XI.

RATI E, RATIO AND PROPORTION
(Rate and ratio, p. 137; taxation, bankruptcy, p. 139; scale of map, p. 143; increase and decrease in given ratio, p. 144; variation, direct and inverse, p. 147; compound units, p. 150; proportional parts, p. 153; ratio in problems, p. 156.)

XII.

PERCENTAGE
(Meaning of percentage, p. 158; fractions as percentages, p. 160; insurance, p. 162; percentage changes, p. 165; gain and loss per cent, p. 170; miscellaneous examples, p. 174.)

XIII.

FURTHER AREAS AND VOLUMES
(Rectangle, p. 176; papering, carpeting, p. 180; borders, subtraction method, p. 182; cuboid, p. 184; problems on volume, p. 186; material for making a box, p. 188; volume of solid of uniform cross-section, p. 190; miscellaneous examples, p. 192.)

XIV.

CIRCLES AND CYLINDERS
(Circumference, p. 194; area of circle and annulus, p. 196; surface of cylinder, p. 200; volume of cylinder, p. 203; miscellaneous examples, p. 206.)

XV.

SIMPLE INTEREST AND DISCOUNT
(Meaning of simple and compound interest, p. 207; calculation of simple interest, p. 209; formula, p. 210; inverse interest problems, p. 212; true present worth, discount, p. 216; practical discount, p. 217; banker's discount, p. 218; miscellaneous examples, p. 222.)

XVI.

APPROXIMATIONS
(Contracted addition and subtraction, p. 224; contracted multiplication, p. 227; contracted division, p. 228; errors, p. 229; addition, p. 231; subtraction, p. 232; multiplication and division, p. 233; miscellaneous examples, p. 236.)

XVII.

MISCELLANEOUS EXTENSIONS
(Averages, p. 238; inverse problems on mixtures, p. 240; rates of working, p. 242; harder fractions, p. 244; problems involving fractions, p. 245; miscellaneous examples, p. 247.)

THREE-MINUTE AND FOUR-MINUTE ORAL PRACTICE

O R A L P R A C T I C E
17-24 (Chaps. IV-VIII).

TESTS IN COMPUTATION

Tests, 9-16 (Chaps. IV-XII).
Tests, 17-24 (Chaps. IV-XV).

REVISION PAPERS
Papers, 25-32 (Chaps. I-XII).
Papers, 33-40 (Chaps. I-X IV).
Papers, 41-48 (Chaps. I-XVII).

PART II

[Pages 273-412]

XVIII.

SQUARE ROOT AND USE OF TABLES
(Graph of x², p. 273; square root rule, p. 274; table of squares, p. 278; table of square roots, p. 280; Pythagoras, p. 281; table of reciprocals, p. 283; miscellaneous examples, p. 285.)

XIX.

INDICES AND LOGARITHMS
(Integral indices, p. 286; fractional and negative indices, p. 287; graph of log₁₀ x, p. 291; logarithm tables, numbers between 1 and 10, p. 292; multiplication, division, p. 294; numbers greater than 10, p. 295; numbers between 0 and 1, p. 301; index problems, p. 306; miscellaneous examples, p. 306.)
CONTENTS

XX. HARDER MEASUREMENT
(Parallelogram, triangle, trapezium, p. 309; volume of
prism, p. 312; volume of pyramid, p. 315; circles and
cylinders, p. 317; cone, p. 320; sphere, p. 321; similar
figures and solids, p. 324; miscellaneous examples, p. 327.)

XXI. SPECIFIC GRAVITY
(Density and specific gravity, p. 329; principle of
Archimedes, p. 333.)

XXII. COMPOUND INTEREST
(Calculation of compound interest, p. 334; formula,
p. 339; tables, p. 342.)

XXIII. HARDER PERCENTAGE
(Percentage factors, p. 344; mixtures, p. 348.)

XXIV. SHARES AND STOCKS
(Shares, terminology, p. 351; share transactions, p. 355;
stocks, terminology, p. 359; miscellaneous transactions,
p. 364.)

XXV. MISCELLANEOUS PROBLEMS
(Change of units, p. 367; relative velocity, p. 370; clocks
and races, p. 373; miscellaneous examples, p. 375.)

TESTS IN COMPUTATION

REVISION PAPERS
Papers, 49–56 (Chaps. I–XX).
Papers, 57–64 (Chaps. I–XXV).

TABLES
402

ANSWERS
at end

APPENDIX
[Pages 413–572]

PARALLEL EXERCISES: Part I, p. 413; Part II, p. 461; Part III, p. 525

DRILL EXERCISES: Part I, p. 439; Part II, p. 502; Part III, p. 556

REVISION PAPERS: Part I, p. 447; Part II, p. 509; Part III, p. 562

ANSWERS
at end

TABLES

LENGTH

British

12 inches (in.) = 1 foot (ft.),
3 feet = 1 yard (yd.),
22 yards = 1 chain (ch.),
10 chains = 1 furlong (fur.),
8 furlongs = 1 mile (mi.).

220 yards = 1 furlong.
1760 yards = 1 mile.

100 links = 1 chain.
54 yards = 1 pole (p.), rod,
perch.
40 poles = 1 furlong.
4 inches = 1 hand.
6 feet = 1 fathom. (p.)
600 feet = 1 cable.
1 knot = 1 nautical mile per hour.

= about 6080 feet per hour.

Metric

10 millimetres (mm.) = 1 centimetre
(cm.).
10 centimetres = 1 decimetre
(dm.).
10 decimetres = 1 metre (m.).
10 metres = 1 decametre
(Dm.).
10 decametres = 1 hectometre
(Hm.).
10 hectometres = 1 kilometre
(Km.).

Approximate equivalents

1 inch = 2.540 cm.
1 yard = 0.9144 m.
1 mile = 1.6093 Km.
1 metre = 39.37 in.
1 K.m. = 0.6214 mi.

= about 1.6 mi.

100 (or 10¹) sq. mm. = 1 sq. cm.
100 (or 10³) sq. cm. = 1 sq. dm.
100 (or 10⁶) sq. dm. = 1 sq. m.

and so on.

1 acre (a.) = 1 sq. kilometre.
100 acres = 1 hectare (Ha.).

Approximate equivalents

1 sq. in. = 6.4516 sq. cm.
1 sq. cm. = 0.15500 sq. in.
1 acre = 0.4047 Ha.
1 are = 0.1196 sq. yd.
1 hectare = 2.471 ac. (about 2½ ac.).
### TABLES

#### VOLUME

<table>
<thead>
<tr>
<th>1728 (or 12^3) cu. inches = 1 cu. foot.</th>
<th>1000 (or 10^3) cu. mm. = 1 cu. cm.</th>
</tr>
</thead>
<tbody>
<tr>
<td>27 (or 13^3) cu. feet = 1 cu. yard.</td>
<td>(cc.)</td>
</tr>
<tr>
<td>1 cu. in. = 16.387 cu. cm.</td>
<td>and so on.</td>
</tr>
<tr>
<td>1 cu. cm. = 0.061024 cu. in.</td>
<td></td>
</tr>
</tbody>
</table>

#### CAPACITY

| 4 gills = 1 pint (pt.) | 10 centilitres (cl.) = 1 decilitre (dl.) |
| 2 pints = 1 quart (qt.) | 10 decilitres = 1 litre (l.) |
| 4 quarts = 1 gallon (gall.) | and so on. |
| 2 gallons = 1 peck (pk.) | 1 litre = 1 cu. dm. = 1000 c.c. |
| 4 pecks = 1 bushel (bush.) | Galls are used only for liquids; pecks, bushels, quarters are used only for dry measures. |
| 8 bushels = 1 quarter (qr.) | |

#### Approximate equivalents

| 1 gallon = 277.3 cu. in. = 4.546 cu. dm. = 4.546 litres. | 12 units = 1 dozen. |
| 1 cu. ft. = 6.23 gall. = 28.32 cu. dm. = 28.326 litres. | 12 dozen = 1 gross. |
| 1 litre = 1.699 cu. in. = 0.9081 pints (about 1.1 pints). | 20 units = 1 score. |

#### WEIGHT

| 16 ounces (oz.) = 1 pound (lb.). | 10 milligrams (mg.) = 1 centigram (cg.) |
| 28 lb. = 1 quarter (qr.). | 10 centigrams = 1 decigram (dg.) |
| 4 qr. = 1 hundredweight (cwt.). | 10 decigrams = 1 gram (gm.). |
| 20 cwt. = 1 ton (t.). | 10 grams = 1 dekagram (Dg.). |
| 14 lb. = 1 stone (st.). | 10 dekagrams = 1 hectogram (Hg.). |
| 112 lb. = 1 cwt. | 10 hectograms = 1 kilogram (Kg.). |
| 2240 lb. = 1 ton. | 1000 kilograms = 1 metric ton or tonne. |

#### Approximate equivalents

| 1 c.c. of water (at 4°C) weighs 1 gram. |
| 1 litre of water (1 cu. dm.) weighs 1 kilogram. |
| 1 cu. ft. of water weighs 62.3 lb. approx. (about 1000 oz.). |
| 1 gallon of water weighs 10 lb. (a pint of water weighs a pound and a quarter). |

### TABLES

#### Apothecaries' Weight (drugs)

| 24 grains (gr.) = 1 pennyweight (p.wt.) | 1 oz. apothecaries = 417 grams; 1 ounce troy = 480 grams. |
| 20 drt. = 1 ounce. | |
| 3 scruples = 1 dram. | |
| 8 drachms = 1 ounce. | |

#### Troy Weight (precious metals)

<table>
<thead>
<tr>
<th>1000 (or 10^3) cu. mm. = 1 cu. cm.</th>
</tr>
</thead>
<tbody>
<tr>
<td>and so on.</td>
</tr>
</tbody>
</table>

#### NUMBER

| 12 units = 1 dozen. | 365 days = 1 common year. |
| 12 dozen = 1 gross. | 366 days = 1 leap year. |
| 20 units = 1 score. | 24 hours = 1 day. |
| 10 years = 1 decade. | 12 months = 1 year. |
| 100 years = 1 century. | 14 days = 1 fortnight. |

#### TIME

<table>
<thead>
<tr>
<th>365 days = 1 common year.</th>
</tr>
</thead>
<tbody>
<tr>
<td>366 days = 1 leap year.</td>
</tr>
<tr>
<td>24 hours = 1 day.</td>
</tr>
<tr>
<td>12 months = 1 year.</td>
</tr>
<tr>
<td>10 years = 1 decade.</td>
</tr>
<tr>
<td>100 years = 1 century.</td>
</tr>
</tbody>
</table>

"Thirty days hath September, April, June, and November, all the rest have thirty-one, excepting February alone, which has but twenty-nine days in each leap year."

#### MONEY

| 4 farthings = 1 penny (d.). | 1 florin = 2s. |
| 12 pence = 1 shilling (s.). | 1 half-crown = 5s. 6d. |
| 20 shillings = 1 pound (l.) or sovereign. | 1 crown = 5s. |
| 100 centimes = 1 peseta (pta.). | 1 guinea = 21s. |

#### French

<table>
<thead>
<tr>
<th>100 centimes (c.) = 1 franc (fr.).</th>
</tr>
</thead>
</table>

#### American

| 100 cents = 1 dollar ($). |

#### Spanish

| 100 centimos = 1 peseta (pta.). |

#### Italian

| 100 centesimi = 1 lira (pl. lire). |
FORMULAÉ

Percentage

(i) \( \frac{a}{b} \times 100 \) per cent.; \( \frac{a}{b} \) per cent. of \( b \).

(ii) If \( P \) is increased by \( r \) per cent., it becomes \( P \left(1 + \frac{r}{100}\right) \).

(iii) For a gain of \( r \) per cent., C.P. : S.P. = 100 : (100 + r).

(iv) If the simple interest on \( LP \) for \( T \) years at \( r \) per cent. p.a. is \( IL \),

\[
I = \frac{PRT}{100} \quad \text{and} \quad P = \frac{100 \times I}{RT} \quad R = \frac{100 \times I}{PT} \quad T = \frac{100 \times I}{PR}.
\]

(v) If the amount at compound interest of \( LP \) for \( n \) years at \( r \) per cent. p.a. is \( LA \),

\[
A = P \left(1 + \frac{r}{100}\right)^n \quad \text{where} \quad R = 1 + \frac{r}{100};
\]

\[
\log P + n \log R = \log A.
\]

Mensuration

(i) Area of Parallelogram = base x height.

(ii) Area of Triangle = \( \frac{1}{2} \) base x height.

\[
= \sqrt{(a-c)(a-b)(b-c)} \quad \text{where} \quad a, b, c \text{ are the lengths of the sides and} \quad s = \frac{1}{2}(a+b+c).
\]

(iii) Volume of Prism = base-area x height.

(iv) Volume of Pyramid = \( \frac{1}{3} \) base-area x height.

(v) For a circle, radius \( r \),

\[
\text{circumference} = 2\pi r; \quad \text{area} = \pi r^2.
\]

(vi) For a circular cylinder, radius \( r \), height \( h \),

\[
\text{area of curved surface} = 2\pi rh; \quad \text{volume} = \pi r^2 h.
\]

(vii) For a circular cone, radius \( r \), height \( h \), slant height \( l \),

\[
\text{area of curved surface} = \pi rl; \quad \text{volume} = \frac{1}{3} \pi r^2 h.
\]

(viii) For a sphere, radius \( r \),

\[
\text{area of surface} = 4\pi r^2; \quad \text{volume} = \frac{4}{3} \pi r^3.
\]

PART I

CHAPTER I

USE OF PRIME FACTORS

Factors. Since 24 = \( 8 \times 3 \), 8 and 3 are each called factors of 24, and 24 is called a multiple of 8, also a multiple of 3.

Any number, of which 2 is a factor, is called even; numbers which are not even are called odd. Thus 2, 4, 6, 8, . . . are even, and 1, 3, 5, 7, . . . are odd.

EXERCISE 1 (Oral)

Express as the product of two factors in as many ways as possible:

1. 12 2. 18 3. 28 4. 30 5. 36 6. 60 7. 96 8. 108

Complete the following:

\begin{align*}
9. & \quad 63 = 7 \times \ldots \\
10. & \quad 72 = 8 \times \ldots \\
11. & \quad 75 = 5 \times \ldots \\
12. & \quad 132 = 11 \times \ldots \\
13. & \quad 144 = 9 \times \ldots \\
14. & \quad 112 = 7 \times \ldots \\
15. & \quad 42 = 2 \times 3 \times \ldots \\
16. & \quad 84 = 4 \times 3 \times \ldots \\
17. & \quad 180 = 5 \times 9 \times \ldots \\
18. & \quad 120 = 8 \times 3 \times \ldots \\
19. & \quad 154 = 11 \times 2 \times \ldots \\
20. & \quad 140 = 7 \times 4 \times \ldots \\
21. & \quad \text{Find the multiples of 7, which are less than 50}. \\
22. & \quad \text{Find the multiples of 12, which are less than 70}. \\
23. & \quad \text{Find the even multiples of 13, which are less than 100}. \\
24. & \quad \text{Find the odd multiples of 17, which are less than 100}.
\end{align*}

Tests of Divisibility

(i) A number is divisible by 2 if its last digit is even.

It is divisible by 4 if the number formed by the last two digits is divisible by 4.

It is divisible by 8 if the number formed by the last three digits is divisible by 8.
USE OF PRIME FACTORS

(2) A number is divisible by 5 if the last digit is 5 or 0.
It is divisible by 10 if the last digit is 0.
(3) A number is divisible by 3 if the sum of its digits is divisible
by 3; it is divisible by 9 if the sum of its digits is divisible
by 9.
(4) A number is divisible by 11 if the sum of the digits in the
odd places and the sum of the digits in the even places
are equal or differ by a multiple of 11.

Also it should be noted that a number is divisible by 6 if it is
divisible both by 2 and by 3, and is divisible by 12 if it is divisible
both by 4 and by 3; and so on. There is no simple test for
divisibility by 7.
Further, a number is divisible by 25 if the number formed by
the last two digits is divisible by 25; and it is divisible by 125 if
the number formed by the last three digits is divisible by 125.

EXERCISE 2 (Oral)
Which of the following numbers are divisible by the first number:

1. 4; 712, 822, 14780, 59376, 716834, 26308.
2. 8; 67344, 89020, 95752, 4567128.
3. 5; 852, 1435, 687930, 4182806, 2793485.
4. 3; 261, 705, 1433, 2655, 101001.
5. 9; 432, 873, 606, 7857, 8427, 30708, 5469.
6. 11; 3267, 814, 6425, 7282, 1001, 33333, 909183.
7. 6; 744, 543, 7134, 9258, 8060, 51234.
8. 12; 4128, 4242, 9456, 3147, 71562.
9. 25; 71820, 36885, 107075, 418225.
10. 33; 28512, 47058, 52492, 82632, 73282.

Find all the possible values of the missing digits in Nos. 11-15:
11. 7*3, 1*43, 60*18 are divisible by 3.
12. 83*, 95*2, 70*4 are divisible by 4.
13. 8*7, 4*75, 7123*5 are divisible by 9.
14. 6*2, 7*81, 919*8 are divisible by 11.
15. 396*5, 4875*, 2948** are divisible by 25.

INDEX NOTATION

Prime Numbers. A number is called prime if it is divisible
only by itself and 1.
Thus 2, 3, 5, 7, 11, 13, 17,... are prime numbers. Any number,
which is not prime, can be expressed as the product of two or more
prime numbers, that is in prime factors.

For example, 600 = 6 \times 10 \times 10 = 2 \times 3 \times 2 \times 5 \times 2 \times 5
= 2 \times 2 \times 3 \times 5 \times 5.

Index Notation. For brevity,
2 \times 2 is written 2^2, called "2 squared" or "the square of 2";
2 \times 2 \times 2 is written 2^3, called "2 cubed" or "the cube of 2";
2 \times 2 \times 2 \times 2 is written 2^4, called "2 to the power 4" or "the 4th
power of 2";
and so on.
Similarly for other numbers, 3 \times 3 \times 3 \times 3 \times 3 is written 3^5, called
3 to the power 5.
In the symbol 3^5, 5 is called the index; it denotes the number
of factors in the product 3 \times 3 \times 3 \times 3.
When expressing a number in prime factors, it is best at first to
proceed systematically. Take the prime numbers in ascending
order 2, 3, 5, 7, 11, ... and divide as often as possible by anyone
before passing on to the next.

Example 1. Express 5148 in prime factors:
5148 = 2 \times 2574 = 2 \times 2 \times 1287
= 2 \times 2 \times 3 \times 429 = 2 \times 2 \times 3 \times 3 \times 143
= 2 \times 2 \times 3 \times 11 \times 13
= 2^2 \times 3^2 \times 11 \times 13.

Alternatively the working may be set out as follows:

2 \times 2 \times 2 \times 3 \times 3 \times 11 \times 13

10 11 143

15
USE OF PRIME FACTORS

EXERCISE 3

State which of the following numbers are prime, and express the others in prime factors, using the index notation:


Factors and Brackets

The expression \((2 \times 3) \times 5\) equals 30 and may be written in any of the forms: \(2 \times (3 \times 5), 6 \times 5, 2 \times 15, 10 \times 3\); it is not equal to the expression \((2 \times 5) \times (3 \times 5)\). This statement must not be confused with the fact that \((2 \times 3) \times 5 = (2 \times 5) + (3 \times 5)\).

The expression \((20 \times 30) \div 10\) equals 60 and may be written in either of the forms: \(2 \times 30, 20 \times 3\); it is not equal to the expression \((20 \div 10) \times (30 \div 10)\). This statement must not be confused with the fact that \((20 + 30) \div 10 = (20 \div 10) + (30 \div 10)\).

The value of an expression like \((12 \times 25) \div 15\) is found by successive short division:

\[
(12 \times 25) \div 3 = 4 \times 25
\]

\[
\therefore (12 \times 25) \div 15 = (4 \times 25) \div 5 = 4 \times 5.
\]

\((12 \times 25) \div 15\) is written in the form, \(12 \times 25 \div 15\), and the working is arranged as shown in Exercise 2.

Example 2 (i) Example 2 (ii)

\[
\begin{array}{c|c}
12 \times 25 & 135 \times 119 \\
15 & 27 \times 119 \\
\hline
5 & 21 \\
4 \times 5 & 9 \times 119 \\
=20 & \div 3 \\
\end{array}
\]

\[
\begin{array}{c|c}
5 & 20 \\
\hline
=9 \times 17 = 153.
\end{array}
\]

EXERCISE 4 (Nos. 1-12, Oral)

State the values of:

1. \((4 + 3) \times 2\); \((4 \times 3) \times 2\).

2. \((2 + 5) \times 3\); \((2 \times 5) \times 3\).

3. \((8 + 12) \div 4\); \((8 \times 12) \div 4\).

4. \((10 + 15) \div 5\); \((10 \times 15) \div 5\).

5. One-third of \((6 \times 9)\).

6. One-tenth of \((40 \times 50)\).

LAWS OF INDICES

Express as the product of two factors in three ways:

7. \((4 \times 6) \times 2\).

8. \((5 \times 7) \times 3\).

9. \((7 \times 11) \times 5\).

10. 10 times \((3 \times 7)\).

11. 6 times \((7 \times 10)\).

12. 12 times \((3 \times 7)\).

Find the values of:

13. \((4 \times 6) \div 2\).

14. \((20 \times 25) \div 5\).

15. \((6 \times 24) \div 8\).

16. \((8 \times 6) \div 4 \times 3\).

17. \((100 \times 80) \div (10 \times 10)\).

18. \((20 \times 30 \times 40) \div 10\).

19. \((10 \times 20 \times 30) \div (10 \times 10)\).

20. \((21 \times 15 \times 2) \div (5 \times 7)\).

21. \((12 \times 8 \times 6) \div (6 \times 4 \times 3)\).

22. \((8 \div 2) \times 5\).

23. \((8 \times 2) \times 8\).

24. \((8 \times 2) \times 8\).

25. \((8 \div 2) \times 8\).

26. \((8 \times 2) \times 8\).

27. \((8 \times 2) \times 8\).

28. \((8 \div 2) \times 8\).

29. \((8 \times 2) \times 8\).

30. \((8 \div 2) \times 8\).

Laws of Indices

Example 3. Express in the index notation, \(5^{2} \times 5^{4}\).

\[5^{2} \times 5^{4} = (5 \times 5) \times (5 \times 5 \times 5) = 5 \times 5 \times 5 \times 5 \times 5 \times 5 \text{, six factors,} = 5^{6}.
\]

Similarly, if we write out in full the expression \(7^{4} \times 7^{9}\), we obtain \(7 \times 7 \times 7 \times \ldots \times 7\), where there are \((4 + 5)\) factors, and this is written \(7^{5}\); \(: = 7^{4} \times 7^{9} = 7^{14}\).

These results are special cases of the general rule that, if \(x, p\) and \(q\) are any numbers,

\[x^{p} \times x^{q} = x^{p+q}.
\]

This formula is sometimes expressed shortly by the words: "in multiplication, add the indices"; but it must be remembered that this applies only to the product of powers of the same number; it cannot be applied to the product \(2^{3} \times 3^{3}\).

Example 4. Express in the index notation, \(5^{6} \div 5^{2}\).

\[5^{6} \div 5^{2} = \frac{5^{6}}{5^{2}} = 5^{6-2} = 5^{4}.
\]

This result is a special case of the general rule that if \(p\) and \(q\) are any numbers such that \(p\) is greater than \(q\),

\[x^{p} \div x^{q} = x^{p-q}.
\]
USE OF PRIME FACTORS

This formula is sometimes expressed shortly by the words: “in division, subtract the indices”; but it must be remembered that this only applies to powers of the same number.

Example 5. Express in the index notation, \(8 \times 10 \times 12 \times 15\).

Reduce each number to its prime factors,

\[8 \times 10 \times 12 \times 15 = 2^3 \times (2^3) \times (2^2 \times 5) \times (2 \times 3) \times (3 \times 5) = 2^5 \times 2^3 \times 3 \times 5 = 2^8 \times 3^2 \times 5^2.\]

EXERCISE 5

Express in prime factors \((a, b\) being prime) using the index notation:

1. \(3^1 \times 3^2 \times 3^3 \times 3^4\),  2. \(5^2 \times 5^3 \times 5^4\),  3. \(a^3 \times a^2 \times a^3 \times a^4\).
4. \(2^2 \times 2^3\),  [5] \(3^3 \times 3^4\),  6. \(5^3 \times 5^5\),  [7] \(6^4 \times 6^4\).
8. \(2^6 \times 2^9\),  [9] \(3^6 \times 3^9\),  10. \(5^6 \div 5^3\),  [11] \(7^6 \div 7^4\).
12. \(2^3 \times 2^9 \div 2\),  13. \(3^5 \times 3^9 \div 3^4\),  14. \(5^4 \times 5^9 \div 5^2\),
*15. \(10^4 \times 10^5 \div 1000\),  *16. \(7^8 \div (7^2 \times 7^3)\),  *17. \(2^4 \times 2^9 \div 2\),
18. \(3 \times 3 \times 3 \times 5 \times 5\),  [19] \(5 \times 7 \times 5 \times 7 \times 7\),
*20. \(a \times b \times a \times b \times a\),  21. \(2 \times 12\),  [22] \(9 \times 15\),  23. \(2^8 \div 18\).
24. \(3^3 \times 6^2\),  25. \(4 \times 6 \times 9\),  *26. \(12 \times 16 \times 18\),  [27] \(10 \times 20 \times 25\).
*28. \(2^3 \times 4 \times 18\),  *29. \(2^5 \times 3^4 \times 6^3\),  *30. \(4 \times 18 \times 21^2\).

POWERS AND ROOTS

The square root of a given number is that number whose square is equal to the given number.

Thus 7 is the square root of 49 because \(7^2 = 49\). The symbol for “square root” is \(\sqrt{\phantom{x}}\) (derived from \(r\)), we therefore write

\[\sqrt{(49)} = 7.\]

The cube root of a given number is that number whose cube is equal to the given number.

Thus 2 is the cube root of 8 because \(2^3 = 8\).

The symbol for “cube root” is \(\sqrt[3]{\phantom{x}}\), we therefore write

\[\sqrt[3]{8} = 2.\]

The fourth root, fifth root, etc., of a number are defined in a similar way and are denoted by \(\sqrt[4]{\phantom{x}}, \sqrt[5]{\phantom{x}}, \) etc.

Example 6. Express in powers of prime factors,

(i) the square of \(2^4 \times 3^5\); (ii) the cube of \(3^2 \times 5^4\).

\[\begin{align*}
(2^4 \times 3^5)^2 &= 2^8 \times 3^{10} \times 5^4; \\
(3^2 \times 5^4)^3 &= 3^6 \times 5^{12}.
\end{align*}\]

Example 7. Express in powers of prime factors,

(i) the square root of \(3^6 \times 5^2 \times 7^4\); (ii) the cube root of \(2^6 \times 3^4 \times 5^2\).

\[\begin{align*}
\sqrt{(3^6 \times 5^2 \times 7^4)} &= 3^3 \times 5 \times 7^2; \\
\sqrt[3]{(2^6 \times 3^4 \times 5^2)} &= 2^2 \times 3 \times 5^\frac{2}{3}.
\end{align*}\]

The argument used in Example 7 shows that if a number is expressed in powers of prime factors, its square root is obtained by halving each index; if any index is odd, there is no exact square root. Similarly, its cube root is obtained by dividing each index by 3; if any index is not a multiple of 3, there is no exact cube root.

Example 8. Find the square root of 7056.

\[\begin{align*}
7056 &= 2 \times 3528 = 2^2 \times 1764 = 2^3 \times 882 = 2^4 \times 220; \\
&= 2^4 \times 3 \times 147 = 2^4 \times 3^2 \times 49 = 2^4 \times 3^2 \times 7^2; \\
&=: \sqrt{(7056)} = 2^2 \times 3 \times 7 = 84.
\end{align*}\]

Note. As soon as the process is understood, the working may be abbreviated by dividing in succession by factors which are perfect squares:

\[\begin{align*}
7056 &= 4 \times 1764 = 4^2 \times 441 = 4^3 \times 9 \times 49; \\
&=: \sqrt[3]{7056} = 4 \times 3 \times 7 = 84.
\end{align*}\]

EXERCISE 6

Write down in index form:

1. The squares of (i) \(2^4\); (ii) \(3^3 \times 5^3\); (i)ii) \(5^4 \times 7^9\).
2. The cubes of (i) \(2^3\); (ii) \(3^8 \times 7^7\); (iii) \(5^9 \times 7^11\).
3. The square roots of (i) \(2^8\); (ii) \(4^2\); (iii) \(3^4\).
4. The square roots of (i) \(2^3 \times 5^2\); (ii) \(3^9 \times 7^4\); (iii) \(2^8 \times 5^4 \times 7^8\).
5. The cube roots of (i) \(2^8\); (ii) \(3^2\); (iii) \(3^3 \times 5^2 \times 7^8\).

Write down the values of:

6. \(\sqrt{(4 \times 9)}\); (7) \(\sqrt{(25 \times 64)}\); 8. \(\sqrt{(49 \times 36 \times 121)}\); 9. \(\sqrt[3]{(27 \times 64)}\).
USE OF PRIME FACTORS

Find the square roots of:

Find the cube roots of:

*23. Find the length of the side of a square whose area is equal to that of a rectangle 80 yd. long, 45 yd. wide.

*24. Find the length of the side of a square field 10 acres in area. [1 acre = 4840 sq. yd.]

*25. Find the length of the edge of a cube whose volume is equal to that of a rectangular block, 12 in. by 9 in. by 2 in.

Find the least integers by which the following numbers must be multiplied to give perfect squares, and find the square roots of the products obtained:
26. $3^8 \times 5^9$.  [27] $2^8 \times 7^3 \times 11$.  28. 294.  *29. 5544.

Find the least integers by which the following numbers must be multiplied to give perfect cubes, and find the cube roots of the products obtained:
30. $2^4 \times 3^3$.  [31] $2^7 \times 3 \times 5^8$.  32. 360.  *33. 2352.

It is proved in algebra that $a^2 - b^2 = (a + b)(a - b)$; verify this if $a = 5$, $b = 3$, and if $a = 9$, $b = 4$, and use this fact to express in prime factors:
*34. $23^2 - 17^2$.  *35. $38^2 - 9^2$.  *36. $147^2 - 93^2$.  *37. $28^2 - 16$.

HIGHEST COMMON FACTOR

A number which is a factor of two or more given numbers is called a common factor of these numbers.

The greatest number which is a common factor of two or more given numbers is called their highest common factor or for short their H.C.F.

HIGHEST COMMON FACTOR

EXERCISE 7 (Oral)

Write down the H.C.F. of:
1. 10, 15.  2. 12, 30.  3. 12, 30, 40.  4. 6, 10, 14.
5. 21, 35.  6. 27, 45, 63.  7. 6, 18.  8. 10, 21.

Write down in prime factors the H.C.F. of:
13. $2^3 \times 3^2$, $2^3 \times 3^4$.  14. $2 \times 3^3 \times 5$, $2^2 \times 3^3 \times 7$.
15. $2^4 \times 3 \times 5^3$, $2^3 \times 3^3 \times 5 \times 7$.  16. $2^4 \times 3^3 \times 5^3$, $2^3 \times 3^4 \times 7^2$.
17. $2^2 \times 3^4 \times 5^3 \times 11$, $2^4 \times 3^3 \times 5^2 \times 7$, $2^3 \times 3^3 \times 5 \times 7 \times 11$.
18. $2^4 \times 3^3 \times 5^3$, $3^4 \times 5^3 \times 7^2$, $5^4 \times 7^3 \times 11^2$.

The H.C.F. can be found by expressing each number as the product of powers of prime factors.


84 = $4 \times 21 = 2^2 \times 3 \times 7$;
180 = $9 \times 20 = 2^2 \times 3^2 \times 5$;
264 = $8 \times 33 = 2^3 \times 3 \times 11$.

The highest power of 2 which is a factor of each number is $2^2$; the highest power of 3 which is a factor of each number is 3; there is no other common factor. \[ \therefore \] the H.C.F. is $2^2 \times 3$, that is 12.

Any common factor of two numbers is also a factor of their difference.

For example, since 5 is a factor of 70 and of 55, it is also a factor of 70 - 55, because

$70 - 55 = (14 \text{ fives}) - (11 \text{ fives}) = 3 \text{ fives}$.

Hence in looking for common factors of two numbers it is only necessary to consider factors of their difference.

For example, any common factors of 406 and 371 must be factors of 406 - 371, that is 35, and can therefore only be 5 and 7; 5 is obviously not a factor; and by trial we find that 7 is a common factor.

This fact is also used in the following process for finding the H.C.F. of two numbers which cannot be expressed easily in prime factors.
USE OF PRIME FACTORS

Example 10. Find the H.C.F. of 9271 and 19783.

First divide 19783 by 9271,

\[
\begin{array}{c|c|c}
9271 & 19783 \\
& 18542 \\
& 1241 \\
& 7 \\
& 2 \\
& 8687 \\
& 584 \\
& 1168 \\
& 73 \\
& 8 \\
& 73 \\
& 584 \\
& 584 \\
& 1168 \\
& 73 \\

This completes the process because the division is exact. The last divisor 73 is the H.C.F.

The working may be arranged concisely as follows:

\[
\begin{array}{c|c|c}
9271 & 19783 & 2 \\
& 18542 & 73 \\
& 1241 & 8687 \\
& 7 & 584 \\
& 2 & 1168 \\
& 8 & 73 \\
& 584 & 584 \\
& 1168 & 1168 \\
& 73 & 73 \\
\end{array}
\]

EXERCISE 8

Find by prime factors the H.C.F. of:

1. 60, 126. 8. 24, 42, 78. 9. 54, 88, 924. 10. 189, 882, 1071. 11. 84, 144, 264, 360, 420.

Find by the division method the H.C.F. of:


LEAST COMMON MULTIPLE

Least Common Multiple

A number which is a multiple of two or more given numbers is called a common multiple of these numbers.

The least number which is a common multiple of two or more given numbers is called their least common multiple or for short their L.C.M.

EXERCISE 9 (Oral)

1. Name 5 multiples of 6, and name 4 multiples of 8. What is the L.C.M. of 6 and 8?
2. Name 3 multiples of 15 which are also multiples of 10. What is the L.C.M. of 15 and 10?

Write down the L.C.M. of:

3. 4, 10. 4. 8, 10. 5. 4, 12. 6. 6, 9.

7. 7, 14. 8. 9, 12. 9. 10, 12. 10. 12, 24.


Write down in index form the L.C.M. of:

15. \(2^2 \times 3, 2^4 \times 5\). 16. \(2^3 \times 3^2, 2^2 \times 3^2\). 17. \(2^4 \times 5, 3^2\).

18. \(2^2 \times 3^2, 2^2 \times 3^2, 2^4 \times 3\). 19. \(2^2 \times 3^2, 2^3 \times 3^2, 2^4 \times 3^2\).

The L.C.M. can be found by expressing each number as the product of powers of prime factors.

Example 11. Find the L.C.M. of 48, 56, 105, 225.

\[
\begin{align*}
48 &= 3 \times 16 = 2^2 \times 3; \\
56 &= 8 \times 7 = 2^3 \times 7; \\
105 &= 5 \times 21 = 3 \times 5 \times 7; \\
225 &= 9 \times 25 = 3^2 \times 5^2.
\end{align*}
\]

Every common multiple of these numbers must have 2, 3, 5, 7 as factors, and each of these factors must occur to as high a power as it occurs in any of the numbers. The highest power of 2 is \(2^3\), the highest power of 3 is \(3^2\), the highest power of 5 is \(5^2\), and the highest power of 7 is \(7^2\).

\[
\therefore \text{the L.C.M. is } 2^3 \times 3^2 \times 5^2 \times 7, \text{ that is } 25200.
\]

Note. It is usually more convenient to keep the L.C.M. in factors than to multiply it out.
USE OF PRIME FACTORS

EXERCISE 10

Express in prime factors the L.C.M. of:

1. \(2^2 \times 3^5, 2^2 \times 5^3\)

2. \(5^2, 2 \times 3 \times 5^2\)

3. \(3^2 \times 5 \times 7, 3^3 \times 5^2 \times 7, 3 \times 7 \times 11\)

4. \(6, 10, 15\)

5. \(7, 9, 21, 35\)

6. \(12, 18, 24, 36, 42\)

**EXERCISE 11**

1. Find the largest number which is a factor of each of the numbers 594, 792, 1080.

2. Find the smallest number which is a multiple of each of the numbers 54, 63, 105.

3. What is the least sum of money which is an exact number of half-crowns and an exact number of florins?

4. What is the least length of tape which can be cut up exactly both into equal strips 1 ft. 3 in. and into equal strips 1 ft. 6 in. long?

5. Either by striding 30 in. or by striding 32 in., I take an exact number of steps to walk across a road. Find the least length of the road.

6. Divide the L.C.M. of 20, 24, 36 by their H.C.F.

7. Lines are drawn so as to divide a rectangle, 1 ft. 8 in. long, 1 ft. wide, into equal squares. Find the least possible number of squares.

8. Three bells toll at intervals of 4, 6, 9 seconds respectively. They start together; after what time will they next toll together?

9. Find the smallest sum of money which can be divided into an exact number of shares either of 6d. each or of 8d. each or of 1s. 3d. each or of 1s. 8d. each.

10. The rims of the wheels of a tractor and van are 7 ft. and 5 ft. 3 in. long respectively. What is the least distance in which both make an exact number of turns?

MISCELLANEOUS EXAMPLES

11. Three bells toll at intervals of 36 sec., 40 sec., 48 sec. respectively. They start together; after how many minutes will they next toll together?

12. Telegraph poles occur at intervals of 84 yd. along a road, and heaps of stones are deposited at intervals of 60 yd. The first heap is at the foot of the first pole. How far along the road is the next heap which lies at the foot of a pole?

13. Three people take steps of lengths 30 in., 36 in., 40 in. respectively. They start off together each with the right foot; how far have they gone before they are next in step, each with the right foot?

14. Four bells toll at intervals of 40 sec., 45 sec., 60 sec., 75 sec. respectively. They start together; after how many minutes will they next toll together?

16. A rectangular block, 6 in. by 12 in. by 15 in., is cut up into an exact number of equal cubes. Find the least possible number of cubes.

17. Find the least number such that the remainder is 5 when it is divided either by 10 or by 15 or by 18.

18. What is the least number of steps in a staircase if when I go up 2 steps at a time, 3 steps at a time, or 4 steps at a time, there is always 1 odd step at the top?

19. The total weight of a certain number of equal packets is 24 lb. 12 oz.; and when some more of the same weight are added the total weight is 33 lb. 12 oz. Find the greatest possible weight of a packet.

20. What pair of numbers between 100 and 130 have 14 as their H.C.F.?

21. If 90, 146, and 230 are each divided by the same number, the remainder is 6. What is the greatest possible value of the divisor?

22. The H.C.F. of 3240, 3600, and a third number is 36, and their L.C.M. is \(2^4 \times 3^3 \times 5^2 \times 7^2\). Find the third number.

23. Three men A, B, C go round a circular track one mile in circumference, in the same direction and starting together from the same place. A bicycles at 352 yd. per minute, B runs at 220 yd. per minute, C walks at 132 yd. per minute. After how many minutes will they first be together again at the starting-point?
CHAPTER II

SIMPLE AREAS AND VOLUMES

Area. A rectangle 4 in. long, 3 in. wide can be divided into \((4 \times 3)\) squares, each of side 1 in.; we therefore say that its area \(ABCD\) is \((4 \times 3)\) sq. in.; the shape of the rectangle is described by saying that it measures "4 inches by 3 inches."

If each side of a rectangle is 3 in. long, the rectangle is said to be 3 inches square. The APQD, 3 in. square, can be divided into \((3 \times 3)\) squares each of side 1 in.; its area is therefore 9 sq. in.

EXERCISE 12 (Oral)

1. Draw freehand diagrams to show how many squares, each of side 1 in., are contained in (i) a rectangle 3 in. long, 2 in. broad; (ii) a square of side 4 in.

How many squares each of side 1 in. are contained in the following rectangles?

2. 5 in. high, 3 in. wide.
3. 4 in. by 6 in.
4. A square of side 6 in.
5. A square of side 1 ft.
6. How many square inches are there in 1 sq. ft.?

How many squares each of side 1 ft. are contained in the following rectangles?

7. 4 ft. broad, 7 ft. long.
8. 8 ft. high, 12 ft. wide.
9. A square of side 5 ft.
10. A square of side 1 yd.
11. How many square feet are there in 1 sq. yd.?

Find the areas of the following rectangles:

12. 5 in. long, 2 in. broad.
13. 6 in. high, 10 in. wide.
14. 7 in. by 3 in.
15. A square of side 4 in.
16. 8 in. square.
17. 10 in. square.

18. 5 ft. long, 4 ft. wide.
19. 10 ft. long, 8 ft. high.
20. A square of side 7 ft.
21. 9 ft. square.
22. 1 yd. long, 2 ft. broad.
23. 1 ft. long, 7 in. wide.
25. A square of side 10 mi.
26. How many square chains are there in 1 sq. mi.?
27. The area of a rectangle, 1 fur. by 1 ch., is called an acre. How many square chains are there in 1 acre?

The examples in Exercise 12 illustrate the following statement:

The number of units in the area of a rectangle is equal to the product of the number of units in its length and the number of units in its breadth.

This rule is usually abbreviated as follows:

\[
\text{Area of Rectangle} = \text{length} \times \text{breadth}
\]

Using the notation of Algebra, we say that if a rectangle is \(l\) ft. long, \(b\) ft. broad, its area \(A\) sq. ft. is given by the formula,

\[
A = l \times b
\]

This formula can also be written in either of the following ways:

\[
I = A \div b = \frac{A}{b}; \quad b = A \div I = \frac{A}{I}
\]

where \(A\), \(l\), \(b\) are numbers, not quantities.

These facts are often stated in words in the abbreviated forms,

- Breadth of Rectangle = Area \div length;
- Length of Rectangle = Area \div breadth.

The Perimeter of a figure bounded by straight lines is the sum of the lengths of the sides.

For example, the perimeter of a field is the total length of the fence which encloses it.

Since the opposite sides of a rectangle are equal, the perimeter of a rectangle is twice the sum of its length and breadth; if a rectangle is \(l\) ft. long, \(b\) ft. broad, its perimeter \(P\) ft. is given by the formula, \(P = 2(l + b)\).

Example 1. A strip of carpet is 4 ft. wide, 5 yd. long. Find its area.

The carpet is 4 ft. wide, 15 ft. long:

\[
\therefore \text{area of carpet} = (4 \times 15) \text{ sq. ft.} = 60 \text{ sq. ft.}
\]
Example 2. The area of a rectangular courtyard, 30 yd. long, is 450 sq. yd. Find its breadth and perimeter.

Breadth of courtyard = \(\frac{450}{30}\) yd.
\[=15 \text{ yd.}\]
\[\therefore \text{perimeter of courtyard} = 2(30 + 15) \text{ yd.} = (45 \times 2) \text{ yd.} = 90 \text{ yd.}\]

Exercise 13

Find the areas of the rectangles in Nos. 1–10:

1. 8 in. long, 15 in. broad.  
2. 100 yd. long, 60 yd. wide.  
3. 5 yd. wide, 10 ft. high.  
4. 15 yd. square.  
5. 17 ft. by 13 ft.  
6. 1 ch. by 5 yd., in square yards.  
7. 2 fur. by 5 ch., in square chains.  
8. 2 ft. by 1 ft. 3 in., in square inches.  
9. 1 ft. 6 in. by 1 ft. 8 in., in square inches.  
10. 8 yd. 1 ft. by 16 yd. 2 ft., in square feet.

Find the area of a window 4 ft. wide, 6 ft. high.

Find the area of the floor of a room 24 ft. long, 15 ft. wide.

Find the area in square inches of a plank 5 in. wide, 4 ft. long.

Find the area of a courtyard 45 yd. long, 50 ft. wide.

How many square feet are there in 2 sq. yd.? Illustrate your answer by a sketch.

1 acre = 10 sq. ch.; how many square yards are there in 1 acre?

Find the breadths and perimeters of the rectangles in Nos. 17–22:

17. Area 48 sq. in., length 6 in.  
18. Area 108 sq. yd., length 12 yd.  
19. Area 10 sq. yd., length 6 ft.  
20. Area 1 sq. ft., length 16 in.  
21. Area 6 sq. ft., height 27 in.  
22. Area 10 sq. yd., height 10 ft.

Find the perimeter of a rectangle is 30 in., and its breadth is 6 in.; find its length and its area.

The perimeter of a football ground 120 yd. long is 400 yd.; find the area in square yards.

The perimeter of a rectangle is 2 ft., and its length is 7 in.; find its breadth and area.

The area of a rectangular field is 4840 sq. yd.; it is 55 yd. wide. Find its perimeter.

Land Measurement

If each side of a square is 221 ch. long, its area is 1 sq. ch.; but 1 ch. = 22 yd., therefore the area of this square is \(22 	imes 22\) sq. yd., or \(484\) sq. yd.

\[\therefore 1 \text{ sq. ch.} = 484 \text{ sq. yd.}\]

If a rectangle measures 1 fur. (10 ch.) by 1 ch., its area is 10 sq. ch., or 4840 sq. yd., and this area is called 1 acre (ac.).

\[\therefore 1 \text{ acre} = 10 \text{ sq. ch.} = 4840 \text{ sq. yd.}\]

Areas of fields and estates are usually expressed in acres. A full-sized Association football ground measures 120 yd. by 80 yd., therefore its area is 9600 sq. yd., which is a little less than 2 acres. This fact is useful when estimating by eye an area in acres.

Similarly, since 1 mi. = 90 ch., the area of 1 sq. mi. is 6400 sq. ch.; but 10 sq. ch. = 1 acre.

\[\therefore 1 \text{ sq. mi.} = 640 \text{ acres.}\]

Example 3. The area of a rectangular field is 10 ac.; its breadth is 200 yd. Find its length.

The area of the field = \(4840 \times 10\) sq. yd.; and the breadth = 200 yd.; therefore the length = \(48400 \div 200\) yd. = 242 yd.

Example 4. A carpet 15 ft. long, 12 ft. wide is laid in a room 20 ft. square. Find the area of the floor left uncovered.

Area of floor = \((20 \times 20)\) sq. ft. = 400 sq. ft.

Area of carpet = \((15 \times 12)\) sq. ft. = 180 sq. ft.

\[\therefore \text{area of part uncovered} = (400 - 180) \text{ sq. ft.} = 220 \text{ sq. ft.}\]

Example 5. Find the area of the given figure, where all corners are right-angled and dimensions are given in inches.

Divide the figure into rectangles A, B, C.

The width of B is \((8 - 6)\) in., i.e. 2 in.

Area of rect. A = \((8 \times 3)\) sq. in. = 24 sq. in.;
area of rect. B = \((2 \times 5)\) sq. in. = 10 sq. in.;
area of rect. C = \((10 \times 4)\) sq. in. = 40 sq. in.;

\[\therefore \text{total area} = 74 \text{ sq. in.}\]
SIMPLE AREAS AND VOLUMES

Example 6. A lawn 30 yd. long, 16 yd. wide is surrounded by a path 2 yd. wide. Find the area of the path.

\[ PQRS \text{ represents the rectangular lawn, and} \quad ABCD \text{ represents the outer edge of the path.} \]

Since \( PQ = 30 \text{ yd.} \), and since the path is 2 yd. wide,

\[ AB = (30 + 2 + 2) \text{ yd.} = 34 \text{ yd.} \]

\[ CD = (30 + 2 + 2) \text{ yd.} = 34 \text{ yd.} \]

\[ AD = (16 + 2 + 2) \text{ yd.} = 20 \text{ yd.} \]

\[ \therefore \text{area } ABCD = (34 \times 20) \text{ sq. yd.} = 680 \text{ sq. yd.} \]

and area PQRS \( = (30 \times 16) \text{ sq. yd.} = 480 \text{ sq. yd.} \)

\[ \therefore \text{area of path} = \text{area } ABCD - \text{area } PQRS \]

\( = (680 - 480) \text{ sq. yd.} = 200 \text{ sq. yd.} \)

**Note.** It is better to find the area of the path by the subtraction method than by dividing it up into separate rectangles.

Example 7.

An open box (no lid) is 5 in. long, 4 in. wide, 3 in. high. Find the area of the total external surface.

The external surface is made up as follows:

Front or back

<table>
<thead>
<tr>
<th>3&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>5&quot;</td>
</tr>
</tbody>
</table>

Each side face

<table>
<thead>
<tr>
<th>2&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>4&quot;</td>
</tr>
</tbody>
</table>

Base

<table>
<thead>
<tr>
<th>3&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>5&quot;</td>
</tr>
</tbody>
</table>

The total area of the front and back is \( (2 \times 3 \times 5) \text{ sq. in.} = 30 \text{ sq. in.} \);

the total area of the two side faces is \( (2 \times 3 \times 4) \text{ sq. in.} = 24 \text{ sq. in.} \);

the area of the base is \( (4 \times 5) \text{ sq. in.} = 20 \text{ sq. in.} \);

\[ \therefore \text{area of total external surface } = (30 + 24 + 20) \text{ sq. in.} = 74 \text{ sq. in.} \]

EXERCISE 14

Find in acres the areas of the rectangular fields, Nos. 1–3:

1. 5 ch. by 4 ch. 2. 220 yd. by 44 yd. 3. 4 fur. square.

4. How many square chains are there in 2 ac., 10 ac.?

5. How many square yards are there in 5 ac., 100 ac.?

6. How many acres are there in 2 sq. mi., half a square mile?

7. A rectangular 5-acre field is 100 yd. wide; find its length and perimeter.

FURTHER AREAS

Find the areas of the figures, Nos. 8–11, where all corners are right-angled and dimensions are given in inches.

Divide the figures into rectangles; No. 8 two rectangles, Nos. 9–11 three rectangles. Set out the working clearly as in Example 5.

8. 4

9. 8

10. 2

11. 2

Find the areas of the shaded parts of the figures, Nos. 12–15, by subtracting one or more areas from another area. All corners are right-angled and dimensions are given in feet.

12. 5

13. 2

14. 3

15. 2

Make freehand sketches showing the data of Nos. 16–22, and find the required areas.

16. A carpet 12 ft. by 10 ft. is laid in a room 15 ft. square. Find the area of the part of the floor left uncovered.

17. A carpet 16 ft. by 9 ft. is laid in a room 18 ft. long, 12 ft. wide. Find the area of the part of the floor left uncovered.
18. A picture 18 in. by 15 in. is mounted on a card 2 ft. long, 1 ft. 6 in. wide; what area of the card is uncovered?

19. A lawn 10 yd. by 8 yd. is surrounded by a path 1 yd. wide. Find the area of the path.

20. A picture 30 in. by 24 in. is mounted on a card so that there is a margin 3 in. wide all the way round. Find the area of the margin.

21. A courtyard is 40 yd. long, 24 yd. wide, and a paved path 3 yd. wide runs round the edge of it, inside it. Find the area of the path.

22. There are four flower beds, each 10 ft. by 10 ft., at the corners of a square with grass paths 4 ft. wide between them (compare the figure of No. 13). Find the total area of the grass. If the whole is surrounded by a gravel path 6 ft. wide, find the area of the gravel path.

23. The diagram represents the four walls of a rectangular room, folded out flat, dimensions in feet. Find the total area of the four walls.

<table>
<thead>
<tr>
<th>Long wall</th>
<th>Short wall</th>
<th>Long wall</th>
<th>Short wall</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>14</td>
<td>18</td>
<td>14</td>
</tr>
</tbody>
</table>

24. Find the total area of the four walls of a room, 15 ft. long, 10 ft. wide, 7 ft. high. [Make a sketch as in No. 23.]

25. Find the total area of the four walls of a room, 21 ft. long, 16 ft. broad, 10 ft. high.

26. Find the area of the total surface of a rectangular block, 8 in. long, 5 in. wide, 4 in. high.

27. Find the area of the total surface of a rectangular block, 9 in. square, 6 in. high.

28. Find the area of the total external surface of a closed box 12 in. long, 7 in. wide, 6 in. high.

29. The diagram represents the base and sides of an open tank, dimensions in feet. Find the area of the total external surface.

<table>
<thead>
<tr>
<th>Base</th>
<th>Side</th>
<th>Side</th>
<th>End</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>4</td>
<td>4</td>
<td>8</td>
</tr>
</tbody>
</table>

30. Make hand sketches, as in No. 29, to show the faces of a closed rectangular box, 12 in. long, 9 in. wide, 6 in. high, and find the total area of the surface of the box.

31. Find the area of the total external surface of an open box (i.e. without a lid), 15 in. long, 8 in. wide, 10 in. high.

32. Find the area of thin sheeting required for making an open cistern (i.e. without a lid), 5 ft. long, 3 ft. wide, 4 ft. deep.

33. Repeat No. 24 for a room 1 ft. long, 2 ft. wide, 6 ft. high.

34. Repeat No. 32 for an open cistern 1 ft. long, 2 ft. wide, 6 ft. deep.

Volume. A solid like an ordinary brick, each face of which is a rectangle, is called a rectangular solid or cuboid; and if each face is a square, the solid is called a cube.

A cube, each edge of which is 1 in. long, is called a cubic inch block or, for short, a cubic inch.

By using cubic inch blocks, we can build up cuboids of various sizes. The diagram represents a cuboid 5 in. long, 4 in. broad, 3 in. high. This cuboid contains $(5 \times 4 \times 3)$ cubic inch blocks; so we say its volume is $(5 \times 4 \times 3)$ cubic inches (cu. in.). The shape of the cuboid is described by saying that it measures "5 in. by 4 in. by 3 in."

This form of argument can be used to show that the number of units of volume of any cuboid is obtained by multiplying together the numbers of units in the length, breadth, and height.

This rule is abbreviated as follows:

**Volume of a cuboid** = length \times breadth \times height.

Further, by using the formula for the area of a rectangle, this rule may be stated in the abbreviated form:

**Volume of a cuboid** = (area of base) \times height.

Using the notation of Algebra, we say that if a cuboid is $l$ in. long, $b$ in. broad, $h$ in. high, its volume $V$ cu. in. is given by the formula,

\[ V = l \times b \times h. \]

Hence also

\[ h = V \div (l \times b), \]

and this fact may be stated in the abbreviated form,

**Height of a cuboid** = **Volume** \div (area of base)**
Example 8. The internal measurements of a tank are 10 ft. by 8 ft. by 5 ft.; find how much water the tank can hold.

The volume of the tank is \((10 \times 8 \times 5)\) cu. ft.;

\[\therefore\text{ the tank can hold } 400\text{ cu. ft. of water.}\]

Example 9. The volume of a rectangular solid is 240 cu. in.; it is 8 in. long and 5 in. broad, find its height.

The area of the base \(= (8 \times 5)\) sq. in. \(= 40\) sq. in.

\[\because \text{ height of solid } = \text{ volume} \div (\text{area of base})\]

\[= (240 \div 40)\text{ in. } = 6\text{ in.}\]

EXERCISE 15

[No. 1–11 are intended for oral work]

How many cubic inch blocks are required to build up rectangular solids having the following dimensions?

1. 5 in. by 8 in. by 7 in.  
2. 4 in. by 6 in. by 1 ft.  
*4. A 2-ft. cube.

How many cubic inch blocks can be packed into rectangular boxes of the following internal dimensions?

5. 4 in. by 7 in. by 9 in.  
6. 5 in. by 1 ft. by 1 ft. 6 in.  
[8] 5 in. by 1 ft. by 1 ft. 6 in.

Find the volumes of the following cuboids:

7. 4 ft. by 8 ft. by 3 ft.  
8. 1 yd. by 2 ft. by 4 ft.  
[9] 1 yd. by 2 ft. by 1 ft.  
10. 1 ft. 3 in. by 1 ft. 4 in. by 1 in.

*11. How many (i) 2-in. cubes, (ii) 6-in. cubes can be packed into a box 1 ft. by 1 ft. by 1 ft., internal measurements?

12. The volume of a rectangular block is 720 cu. in.

(i) Find its height if its length is 15 in. and breadth is 6 in.
(ii) Find its length if its breadth is 9 in. and height is 4 in.
(iii) Find its height if the area of the base is 45 sq. in.

13. The volume of a rectangular tank is 432 cu. ft.

(i) Find its height if the base is 4 yd. square.
(ii) Find its breadth if its length is 3 yd. and height is 8 ft.
(iii) Find its length if the area of the end face is 16 sq. ft.

14. A rectangular tank 8 ft. long, 6 ft. wide, 10 ft. high contains water to a depth of 7 ft. How much more water will it hold?

VOLUME OF MATERIAL FOR A BOX

*15. A rectangular block of lead, 1 ft. by 10 in. by 9 in., is melted and recast into a cuboid 8 in. high, 5 in. wide. Find its length.

*16. How many cubic feet of water must be pumped into a swimming-bath 80 ft. long, 15 yd. wide, to raise the water-level 2 ft.?

*17. A rectangular tank 9 ft. by 8 ft., and 5 ft. deep (internal measurements), contains water to a depth of 3 ft. How many concrete blocks measuring 6 in. by 6 in. by 4 in. can be put into the tank before the water overflows?

*18. The water in the tank in No. 17 is run off into an empty tank 18 ft. long, 6 ft. wide; what will be the depth of the water in the second tank?

Volume of Material used in making a Box

Suppose the external dimensions of a closed wooden box are 10 in. by 9 in. by 6 in., and that the wood is 1 in. thick. The volume of the material is found by subtracting the internal volume (i.e. the volume of space inside the box) from the external volume.

The diagrams show that

\[\text{internal length } = (10 - 1 - 1) \text{ in. } = 8 \text{ in.}\]
\[\text{internal breadth } = (9 - 1 - 1) \text{ in. } = 7 \text{ in.}\]
\[\text{internal height } = (6 - 1 - 1) \text{ in. } = 4 \text{ in.}\]

\[\therefore \text{ internal volume } = (8 \times 7 \times 4) \text{ cu. in. } = 224 \text{ cu. in.}\]

but total external volume \(= (10 \times 9 \times 6) \text{ cu. in. } = 540 \text{ cu. in.}\)

\[\therefore \text{ volume of wood } = (540 - 224) \text{ cu. in. } = 316 \text{ cu. in.}\]

If this box has no lid,

\[\text{internal height } = (6 - 1) \text{ in. } = 5 \text{ in.}\]

\[\therefore \text{ internal volume } = (8 \times 7 \times 5) \text{ cu. in. } = 280 \text{ cu. in.}\]

\[\therefore \text{ volume of wood } = (540 - 280) \text{ cu. in. } = 260 \text{ cu. in.}\]
Volume of Solid of uniform Cross-section

The diagram represents a solid 9 in. long so shaped that, if any cut is made through it at right angles to its length, the surface exposed to view is always the same size and shape, in this example an L shape; this surface is called the cross-section of the solid, and the solid is said to be of uniform cross-section. Solids of uniform cross-section are of common occurrence, e.g. a new pencil, a water-pipe, a bar of chocolate or a stick of Yarmouth rock, a ruler, a flight of steps, a lean-to shed, etc.

The L-shaped solid in the diagram can be built up by using cubic inch blocks.

By dividing the cross-section into two rectangles, we see that its area is

\[(8 \times 3) + (2 \times 4)\] sq. in., = 32 sq. in.

Therefore 32 cu. in. blocks are required for each inch in the length of the solid.

\[
\text{volume of solid} = (32 \times 9) \text{ cu. in.} = 288 \text{ cu. in.}
\]

Here, the number of units of volume has been found by multiplying the number of units of area of the cross-section by the number of units of length, and a similar argument may be used for finding the volume of any solid of uniform cross-section. The rule may be stated in the abbreviated form,

Volume of solid of uniform cross-section = (area of cross-section) \times length.

Note. Cross-area is a convenient abbreviation for area of cross-section.

EXERCISE 16

Find the internal dimensions of closed wooden boxes of the following external dimensions, and find the volume of space in each box:

1. 12 in. by 9 in. by 8 in.; wood 1 in. thick.
2. 10 in. by 8 in. by 6 in.; wood half an inch thick.

VOLUME OF SOLID OF UNIFORM CROSS-SECTION

Find the internal dimensions of open wooden boxes of the following external dimensions, and find the volume of water each box will hold:

3. 10 in. by 7 in.; wood 2 in. thick.
4. 9 in. by 6 in.; wood 1 inch thick.

Find the amount of wood used in making closed wooden boxes of the following external dimensions:

5. 16 in. by 12 in.; wood 1 in. thick.
6. 1 ft. by 9 in. by 5 in.; wood half an inch thick.

Find the amount of wood used in making open wooden boxes of the following external dimensions:

7. 20 in. by 15 in.; wood 1 in. thick.
8. 2 ft. by 18 in.; wood 2 in. thick.
9. Find the area of the inside surface of an open box made of wood 1 in. thick if the box is 10 in. long, 9 in. wide, 8 in. high, measured externally.

*10. A closed box, 8 in. by 7 in. by 6 in., has a coating of asbestos all round it, half an inch thick. What amount of asbestos is used? [The external surface is also a cuboid.]

The solids represented in the following diagrams are composed of rectangular blocks, dimensions in inches.

11. Find the area of the total surface of the given L-shaped solid.
12. Find the volume of the given L-shaped solid.

*13. What are the dimensions of the smallest cuboid from which the given L-shaped solid can be carved, and what is the volume of the portion which must be cut away?

*14. Find the area of the total surface of the given T-shaped solid.

*15. Find the volume of the given T-shaped solid.
EXERCISE 18

[Assume that the rates are uniform]

1. If 6 peaches cost 2s., find the cost of 4 peaches.
2. I can walk 3 mi. in 45 min.; how long do I take to walk 5 mi. at the same rate?
3. A car uses 6 gal. of petrol for 144 mi.; how far will it run with 5 gal.?
4. For 4d. I can buy 12 envelopes; how many can I buy for 1s. 3d.?
5. 2 dozen dried eggs cost 3s.; find the cost of 5 dozen.
6. 4 lb. of Gorgonzola cost 5s.; find the cost of 3 lb.
7. A hotel bill for 10 days is £7, 10s.; what is the charge for 1 week at the same rate?
8. The fare for 8 mi. is 1s.; what is the fare for 22 mi. at the same rate?
9. 21 lb. of freezing-salts cost 1s. 6d.; find the cost of 7 lb. of 56 lb.
10. In 20 min. a train travels 15 mi.; how far does it go in 4 min., in 12 min., at the same rate?
11. If 40 mi. is represented on a map by a line 16 in. long; what length on the map represents 5 mi., 25 mi.?
12. A man saves £12 in 40 weeks; how long does it take him to save (i) £3, (ii) £21 at the same rate?
13. A lorry uses 15 gal. of petrol for 250 mi.; how much will it use for (i) 50 mi., (ii) 350 mi.?
14. If 84 French francs are worth £1, what is the value of 700 French francs?
15. If 10 lb. of coffee costs 15 shillings, how much can be bought for a guinea?
16. A faulty cyclometer records 36 mi. for a journey of 42 mi. What does it record for 63 mi.?
17. The shadow of a man 6 ft. high is 10 ft. long at the same time that the shadow of a telegraph pole is 35 ft. long. Find the height of the pole.
18. A bankrupt who owes £550 only possesses £150. How much does a man to whom he owes £14 receive?
19. A carpet, 10 ft. by 7 ft., costs £14. Find the cost of a carpet of the same quality, 15 ft. by 8 ft.
20. A thin metal sheet, 2 ft. by 3 ft., weighs 4 oz. Find the weight of 1 sq. yd. of a sheet of the same kind.
21. If 100 cu. in. of copper weigh 30 lb., find the weight of a rectangular block of copper 1 ft. long, 6 in. wide, 5 in. high.
22. In a sale the price of a table is reduced from 28s. to 21s.; find the sale price of a chair, originally £1, if reduced at the same rate.
23. If 4 in. on a map represents 1 mi., what area on the map represents 9 sq. mi.?
24. If 16 in. on a map represents 40 mi., what area on the map represents 25 sq. mi.?

Inverse Variation

Example 3. 3 men can weed a field in 8 days. How long will 4 men take, if all work at the same rate?
3 men can weed the field in 8 days;
(but 1 man will take 3 times as long as 3 men take);
1 man can weed the field in 8 days = 3 = 24 days;
(but 4 men will take one-quarter of the time 1 man takes);
4 men can weed the field in 24 ÷ 4 = 6 days.

Note. The sentences in brackets show the reasoning which must be done mentally; they should not be written down.

Example 4. A certain sum of money is sufficient to pay the wages of 12 men for 15 days. For how many days is it sufficient for 10 men, if all receive the same daily wages?
12 men can be paid for 15 days;
1 man can be paid for (15 × 12) days = 180 days;
10 men can be paid for (180 ÷ 10) days = 18 days.

The unitary-method argument can only be used in cases of direct of inverse variation.
For example, if a football team scores 4 goals in 1 match, it is absurd to argue that it will score 40 goals in 10 matches.
In examples where the unitary-method argument can be used, it is always necessary to consider whether the quantity at the end of the sentence is increased or decreased by the change made in the quantity at the beginning of the sentence.
EASY UNITARY METHOD

In Exercises 19–21, if the unitary-method argument cannot be used, say so, and give the reason, but no other answer.

EXERCISE 19 (Oral)

[Assume that the rates are uniform unless this assumption is contrary to common sense]

1. 6 men can mow a field in 3 days; how long will it take 1 man? 2 men? 9 men?
2. 4 men can dig a trench in 5 days; how long will it take 1 man? 2 men? 10 men?
3. At 6 mi. an hour, a man takes 4 hr. for a journey; how long will he take at 1 mi. an hour? 3 mi. an hour? 12 mi. an hour?
4. A man runs 100 yd. in 10 sec.; how long will he take to run 1000 yd.?
5. Four taps running at the same rate fill a tank in 12 min.; how long will it take with 1 tap? 2 taps? 3 taps?
6. Two men can whitewash a ceiling in 120 min.; how long will it take 1 man? 3 men? 120 men?
7. The oats in a bin will feed 9 horses for 10 days; how long will they feed 3 horses? 15 horses?
8. The sun dries 6 towels on a clothes line in 30 min.; how long does it take to dry 1 towel? 8 towels?
9. A certain number of tennis balls have to be packed in boxes. If 6 balls are packed in a box, 10 boxes are required; how many boxes are used with 12 balls in each box? 4 balls in each box?
10. 2 in. of rain fall in 3 days; how much rain falls in 12 days?
11. A 10 horse-power car can do a certain journey in 3 hr.; how long will it take a 40 horse-power car?
12. I have enough money for my holiday to spend £8 a week for 6 weeks. How long will it last if I spend £4 a week? £12 a week?

EXERCISE 20

[Assume that the rates are uniform unless this assumption is contrary to common sense]

1. A man packs 20 boxes of peaches with 18 in each box; how many boxes are needed if only 12 are put in each box?
2. At 30 mi. an hour a train takes 6 hr. for a journey; how long will it take at 36 mi. an hour?

MISCELLANEOUS EXAMPLES

EXERCISE 21

[Assume that the rates are uniform unless this assumption is contrary to common sense]

1. If 12 balls cost 15 shillings, find the cost of 8 balls.
2. If 10 men can repair a road in 18 days, how long will 15 men take?
3. A man earns £25 in 10 weeks; how long will he take to earn £50 at the same rate of pay?
4. A watch gains 48 sec. in 10 days; what will it gain in 15 days?
EASY UNITARY METHOD

5. A steamer, doing 12 knots, takes 15 days for a journey. How long will it take at 10 knots? [1 knot = 1 nautical mile per hour.]

6. 20 tins of boot polish cost 4s. 2d.; find the cost of 2 dozen tins.

[7] A car, which runs 24 mi. to the gallon of petrol, uses 5 gal. for a journey; how much does a car, which runs 30 mi. to the gallon, use for the same journey?

**8.** A man weighs 10 stone when he is 20 years old; what will he weigh when he is 60?

**9.** On a certain map 2 in. represent 1 mi.; what area does 20 sq. in. on the map represent?

10. The railway fare for 20 mi. is half a crown; what is the fare for 36 mi.?

11. A tin sheet, 20 in. by 42 in., weighs 6 oz.; find the weight of a tin sheet, 35 in. by 60 in., of the same material and thickness.

12. If 4 cu. in. of copper weigh 21 oz., find the weight of a rectangular block of copper, 2 in. by 3 in. by 4 in.

13. The greatest safe load for a lift is 12 men each weighing 15 stone. How many people, each weighing 10 stone, can it take safely?

**14.** A batsman scores 10 runs in his first over; what does he score in his first 5 overs?

15. 3 turns of the winder of my watch keep it going for 12 hr. What is the effect of 5 turns?

**16.** When the temperature rises by 72° Fahrenheit, it rises 40° Centigrade. What is the full 90 degrees Centigrade corresponding to a fall of 45° Fahrenheit?

17. A cartwheel, perimeter 12 ft., makes 120 revolutions in going from A to B. How many revolutions does a carriage wheel, perimeter 9 ft., make in going from A to B and back again?

18. 3 similar lamps burn 4 gal. of oil in 100 hr.; how much oil will 5 such lamps burn in 60 hr.?

CHAPTER IV

FRACTIONS: ADDITION AND SUBTRACTION

Common Fractions. If 1d. is divided into 4 equal parts, each part is written as $\frac{1}{4}$ d., and 3 of these parts as $\frac{3}{4}$ d. is called a vulgar or common fraction or simply a fraction; 4 is called the denominator of the fraction, it is the number of parts into which the unit is divided; 3 is called the numerator of the fraction, it is the number of parts taken.

Fraction = \[\frac{\text{Numerator}}{\text{Denominator}}\]

EXERCISE 22 (Oral)

Write down the values of:

1. One-quarter of 1s.
2. One-third of 1s.
3. One-sixth of 1s.
4. Three-quarters of 1s.
5. Two-thirds of 1s.
6. Five-sixths of 1s.
7. One-half of 1 lb.
8. One-quarter of 1 lb.
9. One-eighth of 1 lb.
10. Two-quarters of 1 lb.
11. Three-quarters of 1 lb.
12. Six-eighths of 1 lb.
13. One-fifth of 1 hr.
14. One-tenth of 1 hr.
15. One-twentieth of 1 hr.
16. Two-fifths of 1 hr.
17. Four-tenths of 1 hr.
18. Eight-twentieths of 1 hr.

Write as fractions:

19. Four-fifths.
20. Seven-tenths.
22. One-ninth.
23. Four-sevenths.
25. Write in words $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{7}$.
26. Write in words $\frac{1}{6}$, $\frac{3}{5}$, $\frac{2}{3}$.

Write down the values of:

27. $\frac{1}{4}$ of £1.
28. $\frac{1}{5}$ of £1.
29. $\frac{1}{6}$ of £1.
30. $\frac{2}{5}$ of £1.
31. $\frac{3}{7}$ of £1.
32. $\frac{2}{3}$ of £1.
33. $\frac{1}{8}$ of £1.
34. $\frac{3}{4}$ of £1.
35. $\frac{3}{5}$ of 1 hr.
36. $\frac{1}{4}$ of 1 day.
37. $\frac{3}{7}$ of 1 day.
38. $\frac{2}{3}$ of 1 day.
39. $\frac{1}{6}$ of 1 wk.
40. $\frac{1}{7}$ of 1 wk.
41. $\frac{2}{3}$ of 1 wk.
42. $\frac{1}{4}$ of 1 day.
43. $\frac{3}{2}$ of 2 ft.
44. $\frac{2}{3}$ of 1 ft.
45. $\frac{3}{2}$ of 2 ft.
46. $\frac{1}{3}$ of 3 lb.
FRACTIONS: ADDITION AND SUBTRACTION

47. \(\frac{1}{2}\) of 3 lb.  48. \(\frac{1}{2}\) of 1 yd.  50. \(\frac{1}{2}\) of 1 hr.
51. \(\frac{1}{4}\) of 1 ch.  52. \(\frac{1}{2}\) of 1 gall.  55. \(\frac{1}{3}\) of 1 day
56. \(\frac{2}{3}\) of 1 lb.  58. \(\frac{1}{6}\) of 1 yd.  59. \(\frac{2}{3}\) of 2 tons.

Take a rectangular sheet of paper (long and narrow) and fold it in half with a firm crease. Shade one of the halves.

Fold the paper again so as to divide it into 4 equal parts; then the shaded area contains 2 of these parts and is therefore \(\frac{1}{2}\) of the sheet; thus \(\frac{1}{2}=\frac{1}{4}\). Repeat the process, dividing the sheet into 8 equal parts; this shows that \(\frac{1}{8}=\frac{1}{4}\); and so on.

Thus \(\frac{1}{2}=\frac{1}{4}=\frac{1}{8}\). Draw on a squared blackboard or on squared paper a rectangle, 3 units wide, 5 units high, and divide it into 3 equal columns, and shade 2 of them. The shaded area, see fig. (i), is \(\frac{2}{3}\) of the whole rectangle. Fig. (ii) represents the same rectangle divided into 15 equal parts, and the shaded area contains 10 of these parts; thus \(\frac{10}{15}=\frac{2}{3}\).

In the same way it can be shown that \(\frac{3}{6}=\frac{3}{4}=\frac{3}{8}\).

This may also be illustrated by finding in inches the lengths of these fractions of a line 1 yd. long.

Hence we have the following rule:
The value of a fraction is not altered by multiplying, or dividing, the numerator and denominator by the same number.

For example, \(\frac{1}{2}=\frac{2}{4}\); \(\frac{3}{4}=\frac{6}{8}\); \(\frac{1}{8}=\frac{3}{24}\); \(\frac{3}{8}=\frac{5}{12}\).

Example 1. What fraction is 9d. of 2s.?

2s. = 24d.  It is shorter to say,

\(\therefore\) 1d. = \(\frac{1}{2}\) of 2s.

\(9d. = \frac{9}{20}\) of 2s.

\(9d. = \frac{9}{8}\) of 2s.

\(9d. = \frac{9}{8}\) of 2s.

LOWEST TERMS

EXERCISE 23 (Oral)

What fraction is:

1. 3d. of 1s.
2. 4d. of 1s.
3. 9d. of 1s.
4. 10s. of £1.
5. 4s. of £1.
6. 6s. of £1.
7. 20s. of £1.
8. 12s. of £1.
9. 8 hr. of 1 day.
10. 20 min. of 1 hr.
11. 20 min. of 1 hr.
12. 18 min. of 1 hr.
13. 4 oz. of 1 lb.
14. 12 oz. of 1 lb.
15. 9 in. of 1 yd.
16. 27 in. of 1 yd.
17. 30 in. of 1 yd.
18. 8 cwt. of 1 ton.
19. 6 pt. of 1 gal.
20. 8 lb. of 1 st.
21. 6 sq. ft. of 1 sq. yd.

Complete the following:

22. \(\frac{9}{10}\) of \(\frac{1}{6}\) = \(\frac{1}{4}\) of \(\frac{1}{14}\) = \(\frac{1}{2}\) of \(\frac{9}{10}\)

23. \(\frac{3}{4}\) of \(\frac{1}{8}\) = \(\frac{1}{5}\) of \(\frac{9}{32}\) = \(\frac{1}{20}\) of \(\frac{21}{12}\)

24. Express each of the fractions \(\frac{1}{2}\), \(\frac{2}{3}\), \(\frac{3}{4}\), \(\frac{4}{5}\) in the form \(\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}\)

Express as simply as possible:

25. \(\frac{1}{4}\), \(\frac{5}{10}\), \(\frac{8}{16}\), \(\frac{6}{12}\), \(\frac{9}{18}\), \(\frac{10}{20}\), \(\frac{12}{24}\), \(\frac{15}{30}\), \(\frac{16}{32}\), \(\frac{20}{40}\), \(\frac{24}{48}\)

26. \(\frac{1}{9}\), \(\frac{2}{18}\), \(\frac{4}{36}\), \(\frac{3}{9}\), \(\frac{6}{18}\), \(\frac{12}{36}\), \(\frac{5}{15}\), \(\frac{10}{30}\), \(\frac{2}{8}\), \(\frac{4}{16}\), \(\frac{8}{32}\)

27. \(\frac{2}{3}\), \(\frac{3}{4}\), \(\frac{4}{5}\), \(\frac{3}{4}\), \(\frac{4}{5}\), \(\frac{5}{6}\), \(\frac{6}{7}\), \(\frac{7}{8}\), \(\frac{8}{9}\), \(\frac{9}{10}\)

Write down the value of \(x\) in the following equations:

28. \(\frac{2}{3} x = 4\)  46. \(\frac{7}{4} = \frac{34}{32}\)

29. \(\frac{1}{4} x = \frac{3}{10}\)  47. \(\frac{5}{7} = \frac{35}{35}\)  48. \(\frac{2}{10} = \frac{15}{x}\)  49. \(\frac{9}{21} = \frac{30}{x}\)

Lowest Terms. If the numerator and denominator of a fraction have a common factor (other than 1), we have seen that the fraction can be simplified by dividing the numerator and denominator by this factor. If we divide by the highest common factor, no further reduction is possible, and the resulting fraction is then said to be in its lowest terms.

Example 2. Reduce \(\frac{36}{48}\) to its lowest terms.

\(\therefore\) 84 = 7 x 12 = 2 x 3 x 7; \(\therefore\) 210 = 10 x 21 = 2 x 3 x 5 x 7

In practice, however, it is easier to make the reduction in steps.

\(\frac{36}{48} = \frac{36}{48}\) (dividing above and below by 7)

\(\frac{5}{5}\) (dividing above and below by 5)

Time is saved by crossing out or "cancelling"; but this must be done neatly.

20
5
EXERCISE 24

Reduce to their lowest terms:
1. \( \frac{8}{12} \)
2. \( \frac{9}{18} \)
3. \( \frac{5}{15} \)
4. \( \frac{4}{12} \)
5. \( \frac{5}{25} \)
6. \( \frac{4}{16} \)
7. \( \frac{7}{21} \)
8. \( \frac{3}{9} \)
9. \( \frac{2}{6} \)
10. \( \frac{6}{18} \)
11. \( \frac{11}{33} \)
12. \( \frac{10}{30} \)
13. \( \frac{8}{24} \)
14. \( \frac{14}{28} \)
15. \( \frac{5}{10} \)
16. \( \frac{16}{48} \)
17. \( \frac{5}{20} \)
18. \( \frac{3}{12} \)
19. \( \frac{12}{48} \)
20. \( \frac{2}{8} \)
21. \( \frac{8}{40} \)
22. \( \frac{4}{12} \)
23. \( \frac{5}{15} \)
24. \( \frac{2}{8} \)

Example 3. Arrange in ascending order of magnitude (that is to say, the smallest first, then the next smallest, and so on):

\( \frac{3}{7}, \frac{5}{7}, \frac{7}{10}, \frac{1}{2} \)

The least common multiple of the denominators 3, 4, 10 is 60.
Express each fraction so that its denominator is 60:

\( \frac{3}{7} = \frac{3 \times 8}{7 \times 8} = \frac{24}{56} \)
\( \frac{5}{7} = \frac{5 \times 8}{7 \times 8} = \frac{40}{56} \)
\( \frac{7}{10} = \frac{7 \times 6}{10 \times 6} = \frac{42}{60} \)
\( \frac{1}{2} = \frac{1 \times 30}{2 \times 30} = \frac{30}{60} \)

Arranging these in ascending order of magnitude, we have:

\( \frac{3}{7}, \frac{5}{7}, \frac{1}{2}, \frac{7}{10} \)

Thus to compare two or more fractions, re-write them so that all have the same denominator: this is called expressing them with a common denominator.

EXERCISE 25

Which fraction is the greater in the following pairs?
1. \( \frac{3}{4}, \frac{1}{2} \)
2. \( \frac{3}{4}, \frac{1}{2} \)
3. \( \frac{2}{3}, \frac{1}{3} \)
4. \( \frac{5}{7}, \frac{7}{9} \)
5. \( \frac{3}{4}, \frac{1}{2} \)
6. \( \frac{3}{4}, \frac{1}{2} \)
7. \( \frac{3}{4}, \frac{1}{2} \)
8. \( \frac{3}{4}, \frac{1}{2} \)
9. \( \frac{3}{4}, \frac{1}{2} \)
10. \( \frac{3}{4}, \frac{1}{2} \)
11. \( \frac{3}{4}, \frac{1}{2} \)
12. \( \frac{3}{4}, \frac{1}{2} \)
13. \( \frac{3}{4}, \frac{1}{2} \)
14. \( \frac{3}{4}, \frac{1}{2} \)
15. \( \frac{3}{4}, \frac{1}{2} \)
16. \( \frac{3}{4}, \frac{1}{2} \)

Arrange in ascending order of magnitude:
17. \( \frac{3}{4}, \frac{1}{2}, \frac{3}{4} \)
18. \( \frac{3}{4}, \frac{1}{2}, \frac{3}{4} \)
19. \( \frac{3}{4}, \frac{1}{2}, \frac{3}{4} \)
20. \( \frac{3}{4}, \frac{1}{2}, \frac{3}{4} \)
21. \( \frac{3}{4}, \frac{1}{2}, \frac{3}{4} \)
22. \( \frac{3}{4}, \frac{1}{2}, \frac{3}{4} \)
23. \( \frac{3}{4}, \frac{1}{2}, \frac{3}{4} \)
24. \( \frac{3}{4}, \frac{1}{2}, \frac{3}{4} \)

Arrange in descending order of magnitude:
25. \( \frac{3}{4}, \frac{1}{2}, \frac{3}{4} \)
26. \( \frac{3}{4}, \frac{1}{2}, \frac{3}{4} \)
27. \( \frac{3}{4}, \frac{1}{2}, \frac{3}{4} \)
28. \( \frac{3}{4}, \frac{1}{2}, \frac{3}{4} \)
29. \( \frac{3}{4}, \frac{1}{2}, \frac{3}{4} \)
30. \( \frac{3}{4}, \frac{1}{2}, \frac{3}{4} \)

EXERCISE 26 (Oral)

Express as mixed or whole numbers:
1. \( \frac{3}{4} \)
2. \( \frac{5}{6} \)
3. \( \frac{6}{3} \)
4. \( \frac{4}{3} \)
5. \( \frac{2}{3} \)
6. \( \frac{1}{2} \)
7. \( \frac{1}{4} \)
8. \( \frac{3}{4} \)
9. \( \frac{5}{10} \)
10. \( \frac{11}{12} \)
11. \( \frac{11}{12} \)
12. \( \frac{11}{12} \)

Express as improper fractions:
16. \( \frac{3}{4} \)
17. \( \frac{3}{4} \)
18. \( \frac{3}{4} \)
19. \( \frac{3}{4} \)
20. \( \frac{3}{4} \)
21. \( \frac{3}{4} \)
22. \( \frac{3}{4} \)
23. \( \frac{3}{4} \)
24. \( \frac{3}{4} \)
25. \( \frac{3}{4} \)

Write down the values of:
26. \( \frac{2}{6} \)
27. \( \frac{6}{12} \)
28. \( \frac{10}{8} \)
29. \( \frac{18}{12} \)
30. \( \frac{9}{6} \)
31. \( \frac{14}{7} \)
32. \( \frac{11}{5} \)
33. \( \frac{23}{5} \)

EXERCISE 27

Express as mixed or whole numbers:
1. \( \frac{9}{10} \)
2. \( \frac{9}{10} \)
3. \( \frac{9}{10} \)
4. \( \frac{9}{10} \)
5. \( \frac{9}{10} \)
6. \( \frac{9}{10} \)
7. \( \frac{9}{10} \)
8. \( \frac{9}{10} \)
9. \( \frac{9}{10} \)
10. \( \frac{9}{10} \)
11. \( \frac{9}{10} \)
12. \( \frac{9}{10} \)
13. \( \frac{9}{10} \)
14. \( \frac{9}{10} \)
15. \( \frac{9}{10} \)
16. \( \frac{9}{10} \)
17. \( \frac{9}{10} \)
18. \( \frac{9}{10} \)
19. \( \frac{9}{10} \)
20. \( \frac{9}{10} \)
21. \( \frac{9}{10} \)
22. \( \frac{9}{10} \)
23. \( \frac{9}{10} \)

Express as improper fractions:
19. \( \frac{9}{10} \)
20. \( \frac{9}{10} \)
21. \( \frac{9}{10} \)
22. \( \frac{9}{10} \)
23. \( \frac{9}{10} \)
24. \( \frac{9}{10} \)
25. \( \frac{9}{10} \)
26. \( \frac{9}{10} \)
27. \( \frac{9}{10} \)
28. \( \frac{9}{10} \)
FRACTIONS: ADDITION AND SUBTRACTION

Addition. Express the fractions so that they have a common denominator. Give the answer in its lowest terms.

Example 6. Simplify \( \frac{1}{6} + \frac{1}{6} \).
\[
\begin{align*}
\frac{1}{6} + \frac{1}{6} &= \frac{1+1}{6+6} = \frac{2}{12} = \frac{1}{6}.
\end{align*}
\]
The L.C.M. of the denominators 5, 6, 10 is 30.
Reduce \( \frac{1}{6} \) to its lowest terms.

EXERCISE 28

[Numbers 1–12 are suitable for oral work]

Simplify:

1. 2 sevenths + 3 sevenths.
2. 2 ninths + 5 ninths.
3. 2 ninths + 4 ninths.
4. 7 tenths + 7 tenths.
5. \( \frac{1}{3} + \frac{1}{3} \).
6. \( \frac{1}{8} + \frac{1}{8} \).
7. \( \frac{1}{10} + \frac{1}{10} \).
8. \( \frac{3}{8} + \frac{3}{8} \).
9. \( \frac{2}{5} + \frac{2}{5} \).
10. \( \frac{3}{10} + \frac{3}{10} \).
11. \( \frac{7}{15} + \frac{7}{15} \).
12. \( \frac{3}{7} + \frac{3}{7} \).
13. \( \frac{3}{8} + \frac{3}{8} \).
14. \( \frac{4}{5} + \frac{4}{5} \).
15. \( \frac{9}{10} + \frac{9}{10} \).
16. \( \frac{1}{4} + \frac{1}{4} \).
17. \( \frac{2}{3} + \frac{2}{3} \).
18. \( \frac{1}{2} + \frac{1}{2} \).
19. \( \frac{3}{6} + \frac{3}{6} \).
20. \( \frac{1}{2} + \frac{1}{2} \).
21. \( \frac{7}{10} + \frac{7}{10} \).
22. \( \frac{1}{6} + \frac{1}{6} \).
23. \( \frac{1}{4} + \frac{1}{4} \).
24. \( \frac{3}{8} + \frac{3}{8} + \frac{1}{8} \).
25. \( \frac{1}{2} + \frac{1}{2} + \frac{1}{2} \).
26. \( \frac{2}{6} + \frac{2}{6} + \frac{1}{6} \).
27. \( \frac{2}{3} + \frac{2}{3} + \frac{2}{3} \).
28. \( \frac{4}{6} + \frac{4}{6} + \frac{4}{6} \).
29. \( \frac{1}{2} + \frac{1}{2} + \frac{1}{2} \).
30. \( \frac{3}{5} + \frac{3}{5} + \frac{3}{5} \).
31. \( \frac{1}{3} + \frac{1}{3} + \frac{1}{3} \).
32. \( \frac{1}{6} + \frac{1}{6} + \frac{1}{6} \).

Example 7. Simplify \( \frac{2}{6} + \frac{1}{6} \).
\[
\begin{align*}
\frac{2}{6} + \frac{1}{6} &= \frac{2+1}{6+6} = \frac{3}{12} = \frac{1}{4}.
\end{align*}
\]
The L.C.M. of 5, 6, 10 is 30. Give the answer as a mixed number, do not leave it as an improper fraction. The first step can be omitted.

Example 8. Simplify \( \frac{1}{4} + \frac{1}{8} + \frac{2}{8} + \frac{3}{8} \).
\[
\begin{align*}
\text{Expression} &= \frac{1}{4} + \frac{1}{8} + \frac{2}{8} + \frac{3}{8} = \frac{1+2+2+3}{8} = \frac{8}{8} = 1.
\end{align*}
\]
In addition, first express any improper fraction as a mixed number, then add up all the whole numbers and write the fractions with a common denominator.

Example 9. Subtract \( \frac{1}{6} \) from \( \frac{3}{6} \).

First deal with the whole numbers, and write the fractions with a common denominator. Answer in lowest terms.

Example 10. Subtract \( \frac{2}{4} \) from \( \frac{8}{4} \).

We cannot subtract 9 from 4; take 1 of the 8 whole numbers and change it into \( \frac{4}{4} \); this gives whole numbers and \( \frac{1}{4} \) instead of \( \frac{3}{4} \).

Example 11. Simplify \( \frac{7}{14} - \frac{2}{4} \).

In subtraction, first express any improper fraction as a mixed number, then deal with the whole numbers.


FRACTIONS: ADDITION AND SUBTRACTION

EXERCISE 30 (Oral)

Simplify:
1. \(\frac{3}{4} - \frac{1}{4}\); 2. \(\frac{7}{8} - \frac{3}{8}\); 3. \(\frac{3}{5} - \frac{3}{5}\); 4. \(\frac{7}{8} - \frac{1}{8}\).
5. \(1 - \frac{1}{2}\); 6. \(1 - \frac{1}{3}\); 7. \(1 - \frac{1}{4}\); 8. \(1 - \frac{1}{5}\).
9. \(2 - \frac{1}{2}\); 10. \(2 - \frac{1}{3}\); 11. \(3 - \frac{1}{2}\); 12. \(4 - \frac{1}{2}\).
13. \(\frac{3}{4} - \frac{1}{4}\); 14. \(\frac{7}{8} - \frac{1}{8}\); 15. \(\frac{5}{6} - \frac{1}{6}\); 16. \(\frac{7}{8} - \frac{3}{8}\).
17. \(\frac{7}{10} - \frac{4}{10}\); 18. \(\frac{7}{10} - \frac{3}{10}\); 19. \(3 - \frac{1}{2}\); 20. \(6 - \frac{1}{2}\).
21. \(2\frac{1}{2} - \frac{1}{2}\); 22. \(4\frac{1}{2} - 1\frac{1}{2}\); 23. \(5\frac{1}{2} - 1\frac{1}{2}\); 24. \(4\frac{1}{2} - 4\frac{1}{2}\).
25. \(1\frac{1}{2} - \frac{1}{2}\); 26. \(1\frac{1}{2} - \frac{3}{4}\); 27. \(2\frac{1}{2} - \frac{1}{2}\); 28. \(3\frac{1}{2} - \frac{1}{2}\).
29. \(3\frac{1}{2} - 1\frac{1}{2}\); 30. \(5\frac{1}{2} - 2\frac{1}{2}\); 31. \(4\frac{1}{2} - 1\frac{1}{2}\); 32. \(5\frac{1}{2} - 3\frac{1}{2}\).

EXERCISE 31

Simplify:
1. \(\frac{1}{2} - \frac{1}{4}\); 2. \(\frac{1}{2} - \frac{1}{4}\); 3. \(\frac{3}{4} - \frac{1}{4}\); 4. \(\frac{3}{4} - \frac{1}{4}\).
5. \(\frac{1}{4} - \frac{1}{4}\); 6. \(\frac{1}{4} - \frac{1}{4}\); 7. \(\frac{1}{4} - \frac{1}{4}\); 8. \(\frac{1}{4} - \frac{1}{4}\).
9. \(\frac{1}{10} - \frac{1}{10}\); 10. \(\frac{1}{10} - \frac{1}{10}\); 11. \(\frac{1}{10} - \frac{1}{10}\); 12. \(\frac{1}{10} - \frac{1}{10}\).
13. \(\frac{1}{2} - \frac{1}{2}\); 14. \(\frac{1}{2} - \frac{1}{2}\); 15. \(\frac{1}{2} - \frac{1}{2}\); 16. \(\frac{1}{2} - \frac{1}{2}\).
17. \(\frac{1}{3} - \frac{1}{3}\); 18. \(\frac{1}{3} - \frac{1}{3}\); 19. \(\frac{1}{3} - \frac{1}{3}\); 20. \(\frac{1}{3} - \frac{1}{3}\).
21. \(\frac{1}{5} - \frac{1}{5}\); 22. \(\frac{1}{5} - \frac{1}{5}\); 23. \(\frac{1}{5} - \frac{1}{5}\); 24. \(\frac{1}{5} - \frac{1}{5}\).
25. \(\frac{1}{6} - \frac{1}{6}\); 26. \(\frac{1}{6} - \frac{1}{6}\); 27. \(\frac{1}{6} - \frac{1}{6}\); 28. \(\frac{1}{6} - \frac{1}{6}\).

Addition and Subtraction. In addition and subtraction sums, improper fractions should be expressed at once as mixed numbers.

Example 12. Simplify \(\frac{7}{8} - \frac{3}{8} - \frac{1}{8} + \frac{3}{8}\).

The expression
\[\frac{7}{8} - \frac{3}{8} - \frac{1}{8} + \frac{3}{8}\]
\[= \frac{7 - 3 - 1 + 3}{8}\]
\[= \frac{6}{8}\]
\[= \frac{3}{4}\].

Problems involving Fractions

Example 13. What fraction of a cake remains when \(\frac{1}{3}\) of the cake has been eaten?

Suppose the cake is divided into 9 equal slices, then 5 of the slices have been eaten and therefore (9 - 5) slices, = 4 slices, remain.

But each slice is \(\frac{1}{9}\) of the cake, : \(\frac{4}{9}\) of the cake remains.

Or we may say: when \(\frac{1}{3}\) of the cake has been eaten, (1 - \(\frac{1}{3}\)) of the cake remains, that is \(\frac{2}{3}\) of the cake remains.

Example 14. I read \(\frac{1}{3}\) of a book on Friday, \(\frac{1}{2}\) of it on Saturday, and the rest, 160 pages, on Sunday. How many pages are there in the book?

On Friday and Saturday I read \(\frac{1}{3}\) of the book, = \(\frac{1}{3}\) of the book;
: on Sunday I read \(\frac{1}{2}\) of the book, = \(\frac{1}{2}\) of the book.

4 ninths of the book is 160 pages,
: 1 ninth of the book is (160 ÷ 4) pages, = 40 pages,
: the whole book is (40 × 9) pages, = 360 pages.
EXERCISE 33

1. I buy a cake and eat $\frac{1}{8}$ of it; what fraction of the cake remains?

2. The weights of 3 parcels are $2\frac{3}{4}$ lb., $1\frac{5}{8}$ lb., $\frac{3}{2}$ lb. respectively; find their total weight.

3. A can contains 2 gall. of water. How much must I pour out to leave $\frac{1}{4}$ of a gallon?

4. From a stick 10 in. long, a piece 6 in. long is cut off; what length remains?

5. I buy 18 oranges, but $\frac{2}{3}$ of them are bad; how many are fit to eat?

6. A piece of flannel 8 ft. 6 in. long shrinks after washing to a length of 7 ft. 6 in.; how much shorter is it?

7. Add $\frac{1}{3}$ to the difference of $3\frac{1}{2}$ and 1$\frac{1}{3}$.

8. Subtract 1$\frac{1}{4}$ from the sum of $2\frac{3}{4}$ and 3$\frac{1}{6}$.

9. I use 20 tons of fuel (coal and coke) in a year; $\frac{5}{12}$ of this is coke; how many tons of coal do I use?

10. When I have travelled 6 miles, I have done $\frac{2}{3}$ of my journey. What is the length of the journey?

11. After using $\frac{2}{3}$ of the coal I bought, 15 tons remain; how many tons did I buy?

12. The perimeter of a rectangular tile is 15 in.; it is 4$\frac{3}{4}$ in. long; find its width.

13. $\frac{2}{3}$ of a school are boys, what fraction of the school consists of girls? If there are 165 girls, what is the number of boys?

14. After spending $\frac{1}{4}$ of his money, a boy has 15s. left; how much had he at first?

15. I increase my stride from $\frac{3}{4}$ of a yard to $\frac{5}{6}$ of a yard. What is the increase (i) as a fraction of a yard, (ii) in inches?

16. A, B and C buy a business; A pays $\frac{1}{2}$ of the cost price, B pays $\frac{2}{5}$ of it; what fraction of it does C pay? If C pays £700, what did the business cost?

17. A cargo of 600 tons of coal is shared between A, B, C, D. A receives $\frac{1}{3}$ of it; B and C each take $\frac{1}{6}$ of it. How many tons does D get?

18. A new road is paid for by 4 towns, A, B, C, D; A pays $\frac{4}{9}$ of the cost, B and C each pay $\frac{1}{3}$ of the cost. What fraction of the cost does D pay? What did the road cost if D paid £8000?

19. A tank is one-third full. After drawing off 7 gall. it is just one-quarter full. How much will the tank hold?

20. One tap can fill a bath in 6 min. What fraction of the bath can it fill in 1 min.? Another tap can fill the bath in 3 min. What fraction of the bath is filled in 1 min. when both taps are running? How long will it take to fill the bath if both taps are running?

21. One tap can fill a bath in 10 min., and another tap in 15 min. How long does it take to fill the bath if both taps are running?

22. Two men can mow a field in 8 days; one of them could do so by himself in 12 days; how long would the other man take by himself?
CHAPTER V
FRACTIONS: MULTIPLICATION AND DIVISION

MULTIPLICATION

Multiplication by an Integer and by a Unit Fraction.

Example 1
\( \frac{3}{5} \times 4 = \frac{4}{5} \times \frac{3}{1} = \frac{12}{5} \)

Thus \( \frac{3}{5} \times 4 = \frac{12}{5} \).

Example 2
\( \frac{2}{3} \times \frac{3}{3} = \frac{1}{3} \)

Thus \( \frac{2}{3} \times 1 = \frac{2}{3} \).

EXERCISE 34 (Oral)

Write down the values of:
1. \( \frac{3}{4} \) in. \( \times \) 2.
2. \( \frac{1}{2} \) in. \( \times \) 3.
3. \( \frac{2}{3} \) in. \( \times \) 4.
4. \( \frac{3}{4} \) in. \( \times \) 5.
5. \( \frac{2}{3} \) lb.
6. \( \frac{2}{3} \) lb.
7. \( \frac{3}{4} \) lb.
8. \( \frac{3}{5} \) lb.
9. \( \frac{3}{4} \) oz.
10. \( \frac{3}{4} \) oz.
11. \( \frac{3}{4} \) oz.
12. \( \frac{3}{4} \) oz.
13. \( \frac{3}{4} \) oz.
14. \( \frac{3}{4} \) oz.
15. \( \frac{3}{4} \) oz.
16. \( \frac{3}{4} \) oz.
17. \( \frac{3}{4} \) oz.
18. \( \frac{3}{4} \) oz.
19. \( \frac{3}{4} \) oz.
20. \( \frac{3}{4} \) oz.
21. \( \frac{3}{4} \) oz.
22. \( \frac{3}{4} \) oz.
23. \( \frac{3}{4} \) oz.
24. \( \frac{3}{4} \) oz.
25. \( \frac{3}{4} \) oz.
26. \( \frac{3}{4} \) oz.
27. \( \frac{3}{4} \) oz.
28. \( \frac{3}{4} \) oz.
29. \( \frac{3}{4} \) oz.
30. \( \frac{3}{4} \) oz.
31. \( \frac{3}{4} \) oz.
32. \( \frac{3}{4} \) oz.
33. \( \frac{3}{4} \) oz.
34. \( \frac{3}{4} \) oz.
35. \( \frac{3}{4} \) oz.
36. \( \frac{3}{4} \) oz.

Multiplication by any Fraction

The product of two (proper or improper) fractions is a fraction whose numerator is the product of the numerators and whose denominator is the product of the denominators.

If a mixed number occurs in a product, begin by expressing it as an improper fraction.

If "of" occurs between two fractions, replace it by \( \times \).

Simplify products of fractions by dividing above and below by any common factors before multiplying up.

Example 3
\( 1\frac{1}{2} \times 1\frac{1}{3} = \frac{3}{2} \times \frac{4}{3} = \frac{4}{3} \)

Example 4
\( 1\frac{1}{4} \times 4 = \frac{5}{4} \times \frac{4}{1} = \frac{20}{4} = 5 \)

Example 5. Simplify \( \frac{1}{4} \) of \( 1\frac{1}{2} \).
First divide above and below by 11, this leaves \( \frac{1}{2} \). Next divide above and below by 3, the quotient of the numerator is 1, and this must be written down.

Example 6. Simplify \( 1\frac{1}{3} \times 2\frac{1}{2} \times \frac{1}{3} \).
Cancelling leads to the quotients 1, 1 of the numerator and denominator, and these must be written down.

EXERCISE 35

Simplify:
1. \( \frac{1}{4} \times \frac{1}{2} \)
2. \( \frac{2}{3} \times \frac{2}{3} \)
3. \( \frac{3}{5} \times \frac{3}{5} \)
4. \( \frac{4}{6} \times \frac{4}{6} \)
5. \( \frac{5}{8} \times \frac{5}{8} \)
6. \( \frac{6}{9} \times \frac{6}{9} \)
7. \( \frac{7}{9} \times \frac{7}{9} \)
8. \( \frac{8}{12} \times \frac{8}{12} \)
9. \( \frac{9}{12} \times \frac{9}{12} \)
10. \( \frac{10}{12} \times \frac{10}{12} \)
11. \( \frac{11}{12} \times \frac{11}{12} \)
12. \( \frac{12}{12} \times \frac{12}{12} \)
13. \( \frac{13}{12} \times \frac{13}{12} \)
14. \( \frac{14}{12} \times \frac{14}{12} \)
15. \( \frac{15}{12} \times \frac{15}{12} \)
16. \( \frac{16}{12} \times \frac{16}{12} \)
17. \( \frac{17}{12} \times \frac{17}{12} \)
18. \( \frac{18}{12} \times \frac{18}{12} \)
19. \( \frac{19}{12} \times \frac{19}{12} \)
20. \( \frac{20}{12} \times \frac{20}{12} \)
21. \( \frac{21}{12} \times \frac{21}{12} \)
22. \( \frac{22}{12} \times \frac{22}{12} \)
23. \( \frac{23}{12} \times \frac{23}{12} \)
24. \( \frac{24}{12} \times \frac{24}{12} \)
25. \( \frac{25}{12} \times \frac{25}{12} \)
26. \( \frac{26}{12} \times \frac{26}{12} \)
27. \( \frac{27}{12} \times \frac{27}{12} \)
28. \( \frac{28}{12} \times \frac{28}{12} \)

Division by an Integer. \( \frac{3}{4} \div 3 \) is the same as \( \frac{3}{4} \times \frac{1}{3} \), this equals \( \frac{3}{12} \). The number \( \frac{1}{3} \) is called the reciprocal of 3; hence the rule:
To divide by an integer, multiply by its reciprocal.
Division by a Fraction

\[
\frac{\frac{2}{3}}{\frac{4}{5}} = \left(\frac{2}{3} \times 5\right) \div \left(\frac{4}{5} \times 3\right) \\
= \left(\frac{2 	imes 5}{3 	imes 4}\right) \\
= \frac{5}{6} \\
= \frac{\frac{5}{6}}{\frac{4}{5}}
\]

is called the reciprocal of \(\frac{4}{5}\); hence the rule: To divide by a fraction, multiply by its reciprocal; in other words,

To divide one fraction by another (proper or improper) fraction, turn the second fraction upside down and multiply.

In division, as in multiplication, mixed numbers should be expressed as improper fractions.

Example 7

\[
3\frac{1}{2} \div 5 = \frac{7}{2} \div \frac{1}{5} \\
= \frac{7}{2} \times \frac{5}{1} \\
= \frac{35}{2} \\
= \frac{17}{2}
\]

Example 8

\[
1\frac{1}{3} \div \frac{2}{3} = \frac{4}{3} \div \frac{2}{3} \\
= \frac{4}{3} \times \frac{3}{2} \\
= \frac{2}{1} \\
= 2
\]

Example 9

\[
\frac{3}{4} \div \frac{4}{5} = \frac{3}{4} \times \frac{5}{4} \\
= \frac{15}{16}
\]

**Exercise 36 (Oral)**

1. Divide \(\frac{4}{5}\) by 3, 4, 5.
2. Divide \(\frac{3}{5}\) by 3, 4, 5.
3. Divide \(\frac{7}{8}\) by 4, 5, 8.
4. Divide \(\frac{3}{4}\) by 2, 3, 9.
5. Divide \(\frac{1}{2}\) by 3, 4, 6.
6. Divide \(\frac{2}{3}\) by 3, 6, 9.

Write down the values of:

9. \(3 \div \frac{1}{2}\)
10. \(2 \div \frac{1}{3}\)
11. \(5 \div \frac{1}{4}\)
12. \(7 \div \frac{1}{5}\)
13. \(1 \div \frac{1}{4}\)
14. \(1 \div \frac{1}{5}\)
15. \(1 \div \frac{1}{6}\)
16. \(1 \div \frac{1}{7}\)
17. \(\frac{1}{2} \div \frac{1}{3}\)
18. \(\frac{1}{3} \div \frac{1}{4}\)
19. \(\frac{1}{5} \div \frac{1}{6}\)
20. \(\frac{1}{6} \div \frac{1}{7}\)
21. \(\frac{1}{7} \div \frac{1}{8}\)
22. \(\frac{1}{8} \div \frac{1}{9}\)
23. \(\frac{1}{9} \div \frac{1}{10}\)
24. \(\frac{1}{10} \div \frac{1}{11}\)
25. Write down the reciprocal of (i) \(\frac{1}{7}\); (ii) \(\frac{2}{3}\); (iii) \(\frac{3}{4}\); (iv) \(\frac{4}{5}\); (v) \(\frac{2}{3}\); (vi) \(\frac{3}{4}\).
FRACIONS: MULTIPLICATION AND DIVISION

[23] $\frac{5}{2} \times 4\frac{1}{2} - 3\frac{1}{3} \times 3\frac{1}{4}$.

[24] $(3\frac{1}{2} - 1\frac{1}{4}) \div (2\frac{1}{2} - \frac{1}{2})$.

[25] $1\frac{1}{4} \times 7 + (12\frac{7}{9} - 11\frac{1}{3})$.

*26. $(\frac{8}{4} + \frac{1}{2}) \times (\frac{5}{4} - \frac{1}{2})$.

[27] $(1\frac{1}{8} - 3\frac{1}{3}) + (3\frac{3}{4} - 2\frac{3}{8})$.

[28] $8\frac{1}{4} - 4\frac{1}{4} - 1\frac{1}{4} \times 8 \div 3$.

[29] $(\frac{4}{3} - \frac{1}{3}) \div (\frac{4}{3} + \frac{1}{3})$.

[30] $2\frac{1}{4} - 3\frac{1}{3} + 1\frac{1}{2} + 1\frac{1}{2} - 3(\frac{3}{4} \times 3\frac{1}{2})$.

Expression of one Quantity as a Fraction of another

Example 12.
Express 8s. 9d. as a fraction of 1½ guineas.

8s. 9d. = $\frac{89}{100}$s.; 1½ guineas = $\frac{3}{2}$ 21s.

\[\therefore \text{ required fraction} = \frac{89}{100} \times \frac{3}{2} = \frac{267}{200}\text{ of 1½ guineas.}\]

Alternatively, express each sum in threepences:

8s. 9d. = $(8 \times 4 + 3)$ threepences = 35 threepences,

1½ guineas = 31s. 6d. = 126 threepences,

\[\therefore \text{ required fraction} = \frac{35}{126} = \frac{5}{18}\text{ of 1½ guineas.}\]

Example 13.
Express 2 cwt. 40lb. as a fraction of 2 tons 13 cwt. 4 lb.

Reduce each quantity to lb.

<table>
<thead>
<tr>
<th>cwt.</th>
<th>lb.</th>
<th>tons</th>
<th>cwt.</th>
<th>lb.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>40</td>
<td>2</td>
<td>13</td>
<td>4</td>
</tr>
<tr>
<td>112</td>
<td>224</td>
<td>20</td>
<td>40</td>
<td>5936</td>
</tr>
<tr>
<td>224</td>
<td>254</td>
<td>53</td>
<td>5940</td>
<td></td>
</tr>
<tr>
<td>112</td>
<td>53</td>
<td>106</td>
<td>5936</td>
<td></td>
</tr>
</tbody>
</table>

\[\therefore \text{ required fraction} = \frac{224}{5936} = \frac{1}{26} \text{ of 2 tons 13 cwt. 4 lb.}\]

EXERCISE 39

Express the first number or quantity as a fraction of the second:

1. $\frac{3}{4} : 12$;
2. $7\frac{1}{2} : 81$;
3. $\frac{3}{2} : 4\frac{1}{2}$;
4. $3\frac{1}{2} : 12$;

5. $8\frac{1}{3} : 19$;
6. $10\frac{1}{2} : 19$;
7. $6\frac{1}{2}$.
8. $3\frac{1}{2}$;
9. $1\frac{1}{2}$.

[10] 16s. 8d. = £5.
[11] 8s. 9d.; 6s. 3d.
[12] £1 7s.; £2 5s.
[13] 8s. 2d.; £1 11s. 6d.
[14] £1 2s. 6d.; £5 5s.
[15] £1 8s. 4d.; 11s. 8d.
[16] 12s. 3d.; £1 2s. 9d.
[17] 2 ft. 8 in.; 3 yd. 1 ft.
[18] 3 yd. 2 in.; 14 yd. 2 ft.
[20] 3 qr. 15 lb.; 2 cwt. 7 lb.

PROBLEMS INVOLVING FRACTIONS

21. 4 fur. 20 yd.; 1 mi. 240 yd.
22. 1 ch. 1 yd. 1 ft.; 14 ch. 9 yd. 1 ft.
23. £1 6s. 3d.; £5 16s. 8d.
24. £2 1s. 3d.; £3 13s. 4d.

Example 14.
(i) If 5 yd. of silk cost 1·5s., find the cost of 4 yd.
(ii) If $\frac{3}{4}$ yd. of silk costs 3s. 9d., find the cost of $\frac{2}{3}$ yd.

(i) 5 yd. cost 1·5s.
2 ·5 yd. costs $\frac{3}{4}$s.
2 ·5 yd. costs $(\frac{3}{4} \times \frac{3}{4})s.$

(ii) 4 ·5 yd. cost $(3\frac{3}{4} + \frac{3}{4})s.$
2 ·5 yd. costs $(\frac{3}{4} \times \frac{3}{4})s.$

In the working of (ii), 1 yd. costs more than $\frac{3}{4}$ yd.; division by the proper fraction $\frac{3}{4}$ causes an increase. Also $\frac{3}{4}$ yd. costs less than 1 yd.; multiplication by the proper fraction $\frac{3}{4}$ causes a decrease.

Example 15.
A boy has 4s. 8d.; what fraction of this is left after he has spent 3s. 4d.?

He starts with 56d. and then spends 40d.; this leaves 16d.
\[\therefore \frac{4}{9} \text{ of his money is left;} \therefore \frac{5}{9} \text{ of his money is left.}\]

Example 16.
After a boy has spent $\frac{2}{9}$ of his money, he has 6s.
left; how much had he at first?

After spending $\frac{2}{9}$ of his money, $\frac{7}{9}$ of his money is left.
\[\therefore \frac{7}{9} \text{ of his money is 6s.} \therefore \frac{7}{9} \text{ of his money is 9s.}\]
\[\therefore \frac{7}{9} \text{ of his money is } \frac{7}{9} \times 9s. = \frac{7}{9} \times 9s.\]
\[\therefore \text{ he had 13s. 6d. at first.}\]

Example 17.
How many $\frac{3}{4}$-lb. packets can be made up from 40 lb. of tea, and how much remains?

The number of packets is the number of times $\frac{3}{4}$ lb. is contained in 40 lb.
\[\text{number of packets} = 40 \div \frac{3}{4} = 40 \times \frac{4}{3} = \frac{160}{3} = 53\frac{1}{3};\]
53 complete packets can be made up, and this leaves enough tea for $\frac{1}{3}$ of a packet, that is $\frac{1}{3} \times \frac{3}{4} = \frac{1}{4}$ lb.
\[53 \text{ packets can be made up, and } \frac{1}{4} \text{ lb. of tea remains.}\]
EXERCISE 40

1. A boy's stride is 30 in.; what fraction is this of 1 yd?

2. What fraction is 2 l. d. of a shilling?

3. Express 12 s. 6 d. as a fraction of £1 10 s.

4. 1 pt. of water weighs 1 lb.; find the weight of (i) 1 gall, (ii) 3 gal. of water.

5. What must be added to 3 to make 7?

6. Find the area of a mat 4 1/2 ft. long, 2 3/4 ft. wide.

7. There are 480 pupils in a school. If there are 270 boys, find what fraction of the school consists of girls.

8. If 3 lb. of tea costs 10 d., find the cost of 2 lb. of tea.

9. Find the cost of 3 lb. of tobacco at 15 s. a lb.

10. The product of two numbers is 10; one of them is 3 1/2, what is the other?

11. A man sleeps 7 1/2 hr. every day; for what fraction of each day is he awake?

12. How many 3/4 lb. packets can be made up from 15 lb. of tea?

13. By what must 2 1/2 be multiplied to make 3 1/2?

14. In a black and white tile, the area of the white portion is 5 1/4 sq. in., and of the black portion is 2 3/4 sq. in.; what fraction of the surface of the tile is white?

15. Find the volume of a rectangular block 3 1/2 in. long, 3 1/2 in. wide, 1 1/2 in. high.

16. How many jars, each of which holds 7 1/4 lb. of jam, can be filled from a vessel containing 210 lb. of jam?

17. The area of the floor of a room, 11 1/2 ft. wide, is 180 sq. ft.; find the length of the room.

18. Which of the expressions, 3/4, 2/3, 5/6, is the greatest, and by how much?

19. A man can weed 3 1/4 of a field in 12 days; how long will he take for the whole of it?

20. 3/8 of a tank can be filled in 11 1/2 min.; how long will it take to fill the whole tank?

21. A man buys a table for £9 and sells it for 10 guineas. What fraction of the cost price is his profit?
CHAPTER VI

DECIMALS: NUMERATION, ADDITION, AND SUBTRACTION

Decimal and Common Fractions. A decimal is merely a fraction whose denominator is a power of 10.

EXERCISE 41 (Oral)

Read off and write down as decimals:

<table>
<thead>
<tr>
<th>Thousands</th>
<th>Hundreds</th>
<th>Tens</th>
<th>Units</th>
<th>Tenths</th>
<th>Hundredths</th>
<th>Thousandths</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>2</td>
<td>8</td>
<td></td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>2</td>
<td>7</td>
<td>8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>3</td>
<td>0</td>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>9</td>
<td>2</td>
<td>6</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>9</td>
<td>1</td>
<td>0</td>
<td>3</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>0</td>
<td>3</td>
<td>6</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>2</td>
<td>1</td>
<td>7</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>3</td>
<td>2</td>
<td>6</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>5</td>
<td>4</td>
<td>6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Make a table as above and show in it the following:

16. 7.36. 17. 4.08. 18. 0.5. 19. 0.03. 20. 0.009. 21. 18.4. 22. 20.3. 23. 30.04. 24. 5.017. 25. 6.008. 26. 204.03. 27. 300.72. 28. 19.083. 29. 1.0008. 30. 1.040.05. 31. 7 tenths. 32. 3 hundredths. 33. 8 thousandths. 34. \(7 + \frac{3}{10} + \frac{2}{1000}\). 35. \(204 + \frac{1}{100}\). 36. \(80 + \frac{9}{10} + \frac{3}{1000}\).

Metric System. The unit of length in the metric system is 1 metre (1 m.), which is a little more than a yard (about 39.4 inches). Fractions of the unit are denoted by the following prefixes:

- \(\frac{1}{10}\) metre = 1 decimetre (dm.).
- \(\frac{1}{100}\) metre = 1 centimetre (cm.).
- \(\frac{1}{1000}\) metre = 1 millimetre (mm.).

EXERCISE 42 (Oral)

Write down the following lengths in metres:

<table>
<thead>
<tr>
<th>Metres, m.</th>
<th>Decimal metres, dm.</th>
<th>Centimetres, cm.</th>
<th>Millimetres, mm.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
<td>7</td>
<td>5</td>
</tr>
<tr>
<td>8</td>
<td>5</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>9</td>
<td>8</td>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>10</td>
<td>7</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>11</td>
<td>4</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>12</td>
<td>1</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

13. Express in mm. the lengths in Nos. 9–12.
14. Express in cm. the lengths in Nos. 5–8.
15. Express in dm. the lengths in Nos. 3–7.
54 DECIMALS: NUMERATION, ADDITION, SUBTRACTION

Make a table as on p. 53, and show in it the following lengths, and then express each length in metres:

16. 2 m. 8 dm. 17. 4 dm. 5 cm. 18. 3 m. 5 dm. 4 cm. 19. 6 dm. 7 cm. 5 mm. 20. 4 m. 8 cm. 6 mm. 21. 5 m. 2 dm. 9 mm.

Express in metres:

22. 5 dm.; 2 m. 8 dm. 23. 4 cm.; 3 m. 7 cm. 24. 6 mm.; 1 m. 2 mm. 25. 5 m. 4 dm. 6 cm. 26. 3 dm. 7 cm. 5 mm. 27. 8 m. 6 dm. 3 mm. 28. 1 m. 4 cm. 6 mm. 29. 10 m. 2 cm. 5 mm. 30. Express in mm.: 7 cm. 2 mm.; 8 dm. 3 mm.; 2 m. 4 cm.

Express in cm.: 4 cm. 2 mm.; 6 dm. 8 cm.; 3 dm. 4 mm.

Express in dm.: 3 dm. 4 cm.; 4 m. 6 cm.; 5 dm. 8 mm.

Express as compound quantities (m., dm., cm., mm.):

33. 2:85 m.; 3:04 m. 34. 1:605 m.; 4:082 m. 35. 0.72 m.; 0:063 m. 36. 1:004 m.; 0:205 m. 37. 735 mm.; 420 mm. 38. 608 mm.; 1240 mm. 39. 6.3 cm.; 20.5 cm. 40. 106.4 cm.; 300.5 cm. 41. 7.8 dm.; 6.04 dm. 42. 25.3 dm.; 40.05 dm.

Just as small lengths are measured in the metric system in tenths of a metre, hundredths of a metre, thousandths of a metre, so large lengths are measured in tens of metres, hundreds of metres and thousands of metres.
The following prefixes are used:

10 deka D.
100 hektö H.
1000 kilo K or k.

10 metres = 1 dekametre (Dm.). 10 m. = 1 Dm.
100 metres = 1 hectometre (Hm.). 10 Dm. = 1 Hm.
1000 metres = 1 kilometre (Km. or km.). 10 Hm. = 1 Km.

Notice the use of the capital letter in 1 Dm. (10 metres) to distinguish it from 1 dm. (one metre).
The units most commonly used are kilometres, metres, centimetres, millimetres. Lengths are usually expressed in terms of one unit only, very rarely in terms of more than two units.

METRIC SYSTEM

Since a metre is about 39 inches, it is easy to show that a kilometre is about 3:207 km. or as 3207 m. or as 3000 m.

Example 1. Express 3 km. 2 Hm. 7 m. (i) in km., (ii) in cm., (iii) as a compound quantity in km. and m.
The given length can be written in the form:

<table>
<thead>
<tr>
<th>km.</th>
<th>Hm.</th>
<th>Dm.</th>
<th>m.</th>
<th>cm.</th>
<th>mm.</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2</td>
<td>7</td>
<td>0</td>
<td>7</td>
<td>0</td>
</tr>
</tbody>
</table>

This shows that it can be expressed either as 3:207 km. or as 3207 m. or as 3 km. 207 m.

EXERCISE 43 (Oral)
Write down the following lengths (i) in km., (ii) in m.:

<table>
<thead>
<tr>
<th>km.</th>
<th>Hm.</th>
<th>Dm.</th>
<th>m.</th>
<th>cm.</th>
<th>mm.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>4</td>
<td>7</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>1</td>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>5</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td></td>
<td>8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>7</td>
<td>3</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>8</td>
<td>2</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>4</td>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>6</td>
<td>2</td>
<td>7</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>5</td>
<td>3</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>2</td>
<td></td>
<td>3</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>5</td>
<td></td>
<td>6</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td></td>
<td>2</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>1</td>
<td></td>
<td>1</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>4</td>
<td></td>
<td></td>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

18. Express as compound quantities in km. and m. the lengths in Nos. 1-4.
17. Express as compound quantities in m. and cm. the lengths in Nos. 9-11.

Make a table as above, and show in it the following lengths, and then express each length in km.:

18. 2 km. 4 Hm. 3 Dm. 19. 3 km. 5 Dm. 7 m.
20. 1 km. 7 m. 2 dm. 21. 4 km. 5 m. 4 cm.
22. 6 Hm. 2 m. 5 dm. 22. 4 Dm. 3 m. 7 cm.
23. 8 m. 2 dm. 3 cm. 25. 2 m. 4 cm. 5 mm.
56 DECIMALS: NUMERATION, ADDITION, SUBTRACTION

Express in metres:
26. 2 Dm. 5 m.; 6 Hm. 3 Dm. 27. 1 km. 5 Hm.; 2 km. 4 Dm.
28. 2 km. 4 Hm.; 7 Dm. 5 m. 29. 3 km. 7 m. 5 dm.

Express in kilometres:
30. 4 km. 3 Hm.; 2 km. 5 Dm. 31. 1 km. 8 m.; 7 Hm. 4 Dm.
32. 5 Hm. 2 m.; 6 Dm. 4 m. 33. 5 m. 7 dm.; 2 m. 5 cm.
34. 324 m.; 510 m. 35. 600 m.; 708 m. 36. 1428 m.; 3800 m.
37. 83 m.; 40 m. 38. 6 m.; 9-3 m. 39. 64-5 m.; 0-75 m.

Express as compound quantities in km. and m.:
40. 3-84 km.; 6-2 km. 41. 8-07 km.; 1-003 km.

---

Multiplication and Division by Powers of 10

Multiplication. 8 tenths × 10 = 8 units, that is 0.8 × 10 = 8;
7 hundredths × 10 = 7 tenths, that is 0.07 × 10 = 0.7; and so on.

Thus to multiply a decimal by 10, we move each figure 1 place to the
left, treating the decimal point as fixed.

Suppose the numbers are set out in columns:

<table>
<thead>
<tr>
<th>Hundreds</th>
<th>Tens</th>
<th>Units</th>
<th>Tenths</th>
<th>Hundredths</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.87</td>
<td>0.8</td>
<td>0.7</td>
<td>0.07</td>
<td></td>
</tr>
<tr>
<td>8.7</td>
<td>8</td>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.5-46</td>
<td>3</td>
<td>5</td>
<td>4-6</td>
<td></td>
</tr>
<tr>
<td>354-6</td>
<td>35</td>
<td>4</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>16-04</td>
<td>1</td>
<td>6</td>
<td>0-4</td>
<td></td>
</tr>
<tr>
<td>160-4</td>
<td>16</td>
<td>0</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>20-05</td>
<td>2</td>
<td>0</td>
<td>0-5</td>
<td></td>
</tr>
<tr>
<td>20.05</td>
<td>2</td>
<td>0</td>
<td>0-5</td>
<td></td>
</tr>
<tr>
<td>304-8</td>
<td>3</td>
<td>0</td>
<td>4-8</td>
<td></td>
</tr>
<tr>
<td>3048</td>
<td>3</td>
<td>0</td>
<td>4-8</td>
<td></td>
</tr>
</tbody>
</table>

Thus to divide a decimal by 1000, we move each figure 2 places to the
left, treating the decimal point as fixed.

Similarly for multiplication by 1000:
7-13 × 1000 = 7130; 713 × 10 = 7130.

Thus to multiply a decimal by 1000, we move each figure 3 places to the
left, treating the decimal point as fixed.

Hence we have the following rule:

To MULTIPLY a decimal by 10, 100, 1000, . . . , move each figure 1, 2, 3 . . . places to the LEFT, treating the decimal point as fixed.

Any empty place between the figures and the decimal point must be marked by a zero.

Division. 7 units ÷ 10 = 7 tenths, that is 7 ÷ 10 = 0.7;
8 tenths ÷ 10 = 8 hundredths, that is 0.8 ÷ 10 = 0.08; and so on.

Suppose the numbers are set out in columns:

<table>
<thead>
<tr>
<th>Hundreds</th>
<th>Tens</th>
<th>Units</th>
<th>Tenths</th>
<th>Hundredths</th>
<th>Thousandths</th>
</tr>
</thead>
<tbody>
<tr>
<td>7-84</td>
<td>7</td>
<td>8</td>
<td>4</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>7-84 ÷ 10=</td>
<td>7</td>
<td>8</td>
<td>4</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>0-49</td>
<td>4</td>
<td>0</td>
<td>4</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>0-49 ÷ 10</td>
<td>= 4</td>
<td>0</td>
<td>4</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>20-05</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>20.05 ÷ 10</td>
<td>= 2</td>
<td>0</td>
<td>0</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>304-8</td>
<td>3</td>
<td>0</td>
<td>4</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>3048 ÷ 10=</td>
<td>3</td>
<td>0</td>
<td>4</td>
<td>8</td>
<td></td>
</tr>
</tbody>
</table>

Thus to divide a decimal by 10, we move each figure 1 place to the
right, treating the decimal point as fixed.

Dividing by 10 is the same as dividing by 10 × 10:
354-7 ÷ 100 = 3547 ÷ 10 = 35-47.

Thus to divide a decimal by 100, we move each figure 2 places to the
right, treating the decimal point as fixed.

Similarly for division by 1000,
8-639 ÷ 1000 = 0-8639 ÷ 1000 = 0-008639 ÷ 10 = 0-0008639.
DECIMALS: NUMERATION, ADDITION, SUBTRACTION

Hence we have the following rule:

To DIVIDE a decimal by 10, 100, 1000, ... move each figure 1, 2, 3, ... places to the RIGHT, treating the decimal point as fixed.

Any empty place between the figures and the decimal point must be marked by a zero.

Example 2. Express as decimals: (i) \( \frac{62}{100} \); (ii) \( \frac{9}{100} \); (iii) \( \frac{5}{10} \).

(i) \( 621 \div 100 = 0.621 \)  
(ii) \( 5 \div 100 = 0.05 \)  
(iii) \( 840 \div 1000 = 0.84 \).

Example 3. Express as fractions having powers of 10 as denominators: (i) 0.7; (ii) 0.03; (iii) 0.042.

(i) \( 0.7 = \frac{7}{10} \);  
(ii) \( 0.03 = \frac{3}{100} \);  
(iii) \( 0.042 = \frac{42}{1000} \).

Example 3 shows how any decimal can be written down at sight as a common fraction: the numerator is the number formed by the digits of the decimal disregarding initial zeros, the denominator is 1000, ... where the number of zeros is the same as the number of figures after the decimal point. Fractions thus obtained will often not be in their lowest terms.

Thus \( 0.0705 = \frac{705}{10000} \); this can be reduced to \( \frac{43}{700} \).

And \( 10.206 = \frac{10206}{1000} \) or \( \frac{10206}{10000} \), which reduces to \( \frac{5103}{5000} \).

Example 4. Express 0.0625 as a common fraction in its lowest terms.

\( 0.0625 = \frac{1}{16} = \frac{125}{2000} = \frac{25}{400} = \frac{1}{16} \).

Example 5. Express 4.82 kilometres in metres.

4.82 km. = (4.82 \times 1000) m. = 4820 m.


230 m. = (230 \div 1000) km. = 0.23 km.

Note: Examples 5, 6 may be worked by using columns:

<table>
<thead>
<tr>
<th>Example 5</th>
<th>km</th>
<th>Hm</th>
<th>Dm</th>
<th>m</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>4</td>
<td>2</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>Example 6</td>
<td>2</td>
<td>3</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Note: = 4820 m.  
=0.23 km.

MULTIPLICATION AND DIVISION BY POWERS OF 10

EXERCISE 44 (mainly Oral)

Multiply each of the following numbers by 10, by 100, by 1000:

1. 41; 416.  
2. 0.3; 0.387.  
3. 50.2; 0.0502.

Divide each of the following numbers by 10, by 100, by 1000:

4. 60; 641.  
5. 7.2; 7.03.  
6. 0.91; 0.0805.

Write down the values of the following:

7. 6.075 \times 10.  
8. 8.45 \div 10.  
9. 76 \times 100.

10. 0.97 \times 10.  
11. 5.21 \times 1000.  
12. 520 \div 100.

13. 0.03 \times 10.  
14. 70 \div 1000.  
15. 60.01 \times 10.

16. 0.603 \div 1000.  
17. 30.07 \div 100.  
18. 320 \div 10000.

Write down as decimals:

19. \( \frac{2}{5} \); 20. \( \frac{3}{8} \).  
21. \( \frac{1}{10} \); 22. \( \frac{3}{8} \).  
23. \( \frac{1}{50} \).

24. \( \frac{2}{5} \); 25. \( \frac{1}{10} \); 26. \( \frac{3}{8} \).  
27. \( \frac{1}{50} \); 28. \( \frac{3}{8} \).

29. \( \frac{1}{100} \); 30. \( \frac{1}{1000} \).

Express as common fractions in their lowest terms:

31. 0.5.  
32. 0.6.  
33. 0.25.  
34. 0.75.  
35. 0.075.

36. 0.35.  
37. 0.016.  
38. 0.004.  
39. 0.09.  
40. 0.13.

41. 0.007.  
42. 0.042.

Express in metres:

43. 6.4 km.  
44. 0.08 km.  
45. 485 cm.  
46. 36 cm.

47. 9 cm.  
48. 8 mm.  
49. 180 dm.  
50. 0.307 km.

Express in kilometres:

51. 365 m.  
52. 72 m.  
53. 9 m.  
54. 80 m.

55. 27 cm.  
56. 25 dm.  
57. 470 m.  
58. 750 mm.

Complete the following:

60. 7.3 \( \div \ldots = 7.30 \).  
61. 0.085 \( \times \ldots = 8.5 \).  
62. 2.3 \( \div \ldots = 0.48 \).

63. 0.390 \( \div \ldots = 0.59 \).  
64. 2.3 \( \div \ldots = 0.023 \).
Express as common fractions or mixed numbers, in their lowest terms:

65. 0.375. 66. 0.125. 67. 0.32. 68. 0.065.
69. 1.0045. 70. 4.875. 71. 0.0625. 72. 1.096.
73. 2.075. 74. 5.0085. 75. 3.0064. 76. 1.0125.
77. 0.425. 78. 1.0256. 79. 3.0275. 80. 2.008.

Addition and Subtraction

Example 7. Add:

<table>
<thead>
<tr>
<th>m.</th>
<th>dm.</th>
<th>cm.</th>
<th>mm.</th>
<th>metres</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>7</td>
<td>3</td>
<td>5</td>
<td>2.735</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>1</td>
<td>8</td>
<td>4.018</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
<td>2</td>
<td>2</td>
<td>4.072</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>5</td>
<td>0</td>
<td>2.065</td>
</tr>
</tbody>
</table>

**Sum**

7 2 9 0 = 7.290

.: the sum is 7 m. 2 dm. 9 cm. or 7.29 m.

In all addition sums, start by adding upwards; then check by adding downwards.

Example 8. Subtract the second quantity from the first:

<table>
<thead>
<tr>
<th>m.</th>
<th>dm.</th>
<th>cm.</th>
<th>mm.</th>
<th>metres</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>5</td>
<td>7</td>
<td>2</td>
<td>9.572</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>3</td>
<td>...</td>
<td>2.830</td>
</tr>
</tbody>
</table>

**Diff.**

6 7 4 2 = 6.742

.: the difference is 6 m. 7 dm. 4 cm. 2 mm. or 6.742 m.

Example 9. Subtract the second quantity from the first:

<table>
<thead>
<tr>
<th>m.</th>
<th>dm.</th>
<th>cm.</th>
<th>mm.</th>
<th>metres</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>3</td>
<td>...</td>
<td>...</td>
<td>6.300</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>3</td>
<td>7</td>
<td>1.537</td>
</tr>
</tbody>
</table>

**Diff.**

4 7 6 3 = 4.763

.: the difference is 4 m. 7 dm. 6 cm. 3 mm. or 4.763 m.

After a little practice it will be found unnecessary to insert the zeros printed in italics. The answer may be checked by adding together the two lower lines.

---

Exercise 45

Add together, and express each answer in terms of the highest unit named:

1. m. dm. cm. [2] km. Hm. Dm. [3] dm. cm. mm.
   1 7 4 3 2 9 5 0 6
   2 5 3 1 0 7 7 8
   1 8 6 2 9 8 1 4 5

[4] 2 4 5 5. 6 ... 8 6. 2 3 4
   1 2 3 7 5 ... 7 7 4
   3 1 2 8 1 2 6 9 2

Add together:

7. 4.62 8. 2.83 [9] 5.72 10. 6.75
   2.34 5.47 1.38 4.3

   7.13 4.909 0.958 0.099

[15] 37.18 16. 4.087 *17. 0.045 *18. 53.07
   0.77 3.95 1.709 17.8
   21.09 0.07 0.086 3.56
   6.94 5.893 9.14 20.07

19. 0.872 24. 2.45 3.008 4.16 1.75
   30. 16.32 20. 0.4 5.207 0.918 1.01

*31. 0.0736 6.094 1.0809 0.0074 0.9107
   *32. 0.0804 0.00924 0.1072 0.02065 0.3018

Subtract the second quantity from the first, and express each answer in terms of the highest unit named:

23. m. dm. cm. [24] dm. cm. mm. [25] km. Hm. Dm.
   5 7 6 6 1 5 4 0 5
   1 8 4 1 7 8 1 2 8

[26] 3 ... 4 27. 8 ... 5 28. 4 ... ...
   1 6 8 ...

Find the value of:

28. 5.8 - 1.3. 30. 3.4 - 1.8. 31. 4 - 1.2.
   29. 7.4 - 0.7. [31] 2 - 0.6.
   35. 8 - 0.09. [36] 1 - 0.1.
62  DECIMALS: NUMERATION, ADDITION, SUBTRACTION

Subtract the second number from the first:

38. 8.76  39. 0.837  40. 6.84  [41] 0.675
   5.31          0.237          1.59           0.087
---   ---      ---      ---         ---
42. 7.6       [43] 7.63      44. 8.3      45. 1
   2.38      0.683      1.074           0.586
---   ---      ---      ---         ---
[46] 4.08      [47] 1.23      48. 0.729  [49] 0.915
   1.3          0.7          0.68           0.08
---   ---      ---      ---         ---

Find the value of:

50. 7.89 - 1.425.  [51] 12.78 - 9.19.  52. 3.07 - 0.895.
53. 5.01 - 0.724.  *54. 0.1 - 0.0807.  *55. 0.01 - 0.0002.
56. 7.2 - 1.4 + 2.8 - 0.9.  [57] 5 - 0.3 - 0.76 - 0.04.
*58. 10 - 0.05 - 0.06 + 0.01.  *59. 0.1 - 0.05 - 0.007 + 1.
*60. 100 - 0.7 - 0.05 + 0.1.  *61. 1.01 - 0.1 - 0.001 + 10.

---

MISCELLANEOUS EXAMPLES

EXERCISE 46

[Give answers as decimals, not as common fractions]

1. In the diagram (not drawn to scale), ABC is a straight line. If the distance of A from B is 3.7 cm., and of B from C is 2.3 cm., find the distance of A from C.

2. In the diagram (not drawn to scale), ABC is a straight line. If the distance of A from C is 5 cm., and of A from B is 2.75 cm., find the distance of B from C.

3. A fence is made of 10 hurdles each 1.65 m. long; find its length.

4. A pile of 10 equal note-books is 12.5 cm. high; find the thickness of each notebook in mm.

5. A nail 2.3 cm. long is driven into a board 1.7 cm. thick; if the head of the nail is level with the top of the board, find what length of the nail projects outside the board in mm.

6. 10 swings of a pendulum are timed to take 8.5 sec.; find the time of one swing.

7. A boy's temperature is 98.6 degrees at 4 p.m. If it rises 2.5 degrees in the next 5 hr., what is it at 9 p.m.?

8. The lengths of the sides of a triangle are 5.75 cm., 6.8 cm., 7.15 cm.; find the perimeter (i) in cm., (ii) in m.

9. A tank 4.5 dm. deep contains water to a depth of 18.5 cm.; how far in cm. is the water-level below the top of the tank?

10. A pile of 100 sheets of paper is 2.4 cm. high; find the thickness of each sheet in millimetres.

11. Express (i) 1 cm. in kilometres; (ii) 25 mm. in metres.

12. If 1 yd. = 0.915 m., express 100 yd. in kilometres.

13. A book is 5 cm. thick and each cover is 1.5 mm. thick; how thick is the book without its covers?

14. The rainfall in the 4 weeks of February was 0.45 in., 1.07 in., 0.6 in., 0.88 in.; find the total rainfall for the month.

15. A man walks 2.25 metres a second; find in kilometres the distance he walks in 100 seconds.

16. A rectangle is 10.4 cm. long, 7.6 cm. wide; find its perimeter in decimetres.

17. From a piece of tape 20 m. long, 100 pieces each 7.5 cm. long are cut off; how much remains?

18. A frontage of 108 m. is divided into 10 equal sections; what is the width of 1 section, of 2 sections?

19. If 1 metre is taken as 39.4 in. approximately, express 1 mm. in inches.

20. The external diameter of a hollow metal pipe is 8.16 cm., and the metal is 1.7 cm. thick; find the internal diameter.

21. The external dimensions of an open wooden box are 10.7 cm. by 8.2 cm. by 6 cm. high, and the wood is 8 mm. thick. Find in centimetres the internal dimensions of the box.

22. How many hundredths are there in 0.025, and how many thousandths in 0.0105?

23. If 1 cm. = 0.394 in., find in inches the perimeter of a rectangle 1 m. long, 1 dm. wide.

24. An unstretched spring is 48 cm. long, and the spring stretches 15 cm. for each ounce weight supported by it. Find in decimetres the length of the spring when it is supporting a body of weight 10 oz.
MULTIPLICATION BY INTEGERS UP TO 12

Example 3. Multiply 63.57 by 12.

Put the 2 in 12 under the right-hand digit 7 of the multiplicand.

In multiplying, use the "12 times" table.

\[ \therefore 63.57 \times 12 = 762.84. \]

Rough Check: \(60 \times 12 = 720; \therefore 762.84 \) is a reasonable answer.

Example 4. Multiply 0.843 by 1200.

\[ \begin{align*} 0.843 \times 12 &= 10.116, \\ \therefore 0.843 \times 1200 &= 1011.6. \end{align*} \]

Rough Check: \(9.6 \times 1200 = 9600 (\text{about 1000}) ; \therefore 1011.6 \) is a reasonable answer.

EXERCISE 48

Multiply:

1. 3.74 by 2, 3, 7.  2. 0.4 by 2, 3, 6.  3. 0.5 by 3, 4, 8.
4. 0.7 by 5, 9, 12.  5. 0.8 by 4, 5, 11.  6. 0.9 by 6, 11, 12.
7. 0.02 by 3, 4, 5.  8. 0.06 by 2, 8, 12.  9. 0.07 by 5, 11, 12.
10. 0.005 by 6, 7, 12.  11. 0.009 by 4, 10, 11.  12. 0.001 by 8, 11, 100.
13. 0.2 by 30, 400.  14. 0.6 by 70, 9000.  15. 0.8 by 50, 6000.
16. 0.03 by 70, 800.  17. 0.05 by 40, 300.  18. 0.007 by 60, 700.

Example 1. Multiply 6.28 metres by 7.

The process may be explained by comparing it with the multiplication of a compound quantity.

\[ \text{Dm. m. dm. cm. } \times \text{m. metres.} \]

\[ 6 \quad 2 \quad 8 \quad \text{m.} \]

\[ 3 \quad 9 \quad 6 \quad \text{m.} \]

\[ 43.96 \text{ m.} \]

The working should always be arranged so that the units digit of the multiplier is under the right-hand digit of the multiplicand.

Example 2. Multiply 0.00845 by 6.

Put the 6 under the right-hand digit 5 of the multiplicand.

Proceed as before: the decimal point comes under the decimal point in the multiplicand.

\[ 0.00845 \times 6 = 0.0507. \]

Rough Check: \(0.008 \times 6 = 0.048; \therefore 0.0507 \) is a reasonable answer.

Notice that the answer is given as 0.0507, not 0.05070.

MULTIPLICATION BY ANY NUMBER

Method I. Counting the Decimal Places.

Multiply as in ordinary multiplication, taking no notice of the decimal points. Then mark off in the product as many decimal places as there are in the multiplier and multiplicand together.

Example 5. Multiply 52.836 by 74.9.

Rough Estimate: \(50 \times 70 = 3500.\)

This method is based on the following principle:

\[ 52.836 \times 74.9 = \frac{52836}{749} = 72.634 \]

\[ \therefore 52.836 \times 74.9 = \frac{395741.64}{395741.64} \]

\[ = 395741.64 \]

\[ = 395741.64 \]

\[ e \]
Example 6
2410 \times 0.0001832 = 438.1

Example 7
187.5 \times 0.0368

2410
1832
241
368
368

7328
11250
0.0001832
690000

(0 + 2) decimal places
(1 + 4) decimal places
product
6.90000

Method II. Automatic placing of decimal point in each partial product.

Example 8
52.836 \times 74.9

Example 9
0.728 \times 0.0325

decimal points
0.728
0.0325

\underline{\downarrow}

52.836
-0.2184
1456
3698.52
211.344
47.5524

3957.4164

Product

Product

Check the position of the decimal point by counting the number of decimal places.

Example 10
Multiply 52.836 by 74.9.

Rough Estimate: 50 \times 70 = 3500.

52.836 \times 74.9 = 3957.4164.

Example 11

Example 12
2410 \times 0.0001832

Rough Estimate: 2000 \times 0.0002 = 0.4.

Product = 241 \times 0.01832

187.5 \times 0.0368

Rough Estimate: 2000 \times 0.004 = 0.8.

Product = 1.875 \times 3.68

0.01832
2.41
0.03664
7328
1832

Product

1875
368
5625
1250
15000
690000

6.90000

Product = 6.9.
EXERCISE 50 [Nos. 1-26 are intended for Oral work]

Write down the values of:

1. 0.2 × 0.3  2. 0.7 ÷ 0.4  3. 8 ÷ 0.6  4. 6 ÷ 0.5
5. 0.9 ÷ 0.1  6. 0.03 ÷ 0.6  7. 0.02 ÷ 0.1  8. 0.15 ÷ 0.4
9. (0.3)^2  10. (0.1)^2  11. (0.05)^2  12. (0.008)^2
13. (0.01)^2  14. (0.2)^2  15. 12 ÷ 0.7  16. 400 ÷ 0.5
17. 1.1 × 0.11  18. 210 ÷ 0.01  19. 15 ÷ 0.04  20. 1.2 ÷ 0.02
21. 0.1 ÷ 0.2 ÷ 0.3  22. 0.7 ÷ 20 ÷ 0.03  23. 0.01 ÷ 0.1 ÷ 0.1
24. (0.2)^2 ÷ 0.4  25. (1.1)^2 ÷ 0.05  26. (0.003)^2 ÷ 100

Find the values of:

27. 2.14 × 3.2  28. 10.3 × 4.1  29. 243 × 1.5
30. 3120 × 7.9  31. 4.35 × 1.6  32. 62.5 ÷ 2.4
33. 3.62 ÷ 0.13  34. 0.218 ÷ 0.27  35. 0.0617 × 0.56
36. 4.73 × 140  37. 0.0804 × 1900  38. 230 ÷ 0.028
39. 3625 × 0.056  40. 101 ÷ 0.101  41. 0.102 ÷ 0.031
42. 980 ÷ 0.79  43. 1900 ÷ 0.909  44. 2.034 × 14.6
45. 37.06 ÷ 0.384  46. 4.375 ÷ 0.052  47. 98.27 ÷ 36.8
48. 0.00709 ÷ 209.8  49. 5.19 ÷ 3.714  50. 0.0809 ÷ 0.908
51. 1.23 ÷ 0.14 ÷ 150  52. 0.018 ÷ 5.4 ÷ 1300
53. 130 ÷ 22.5 ÷ 0.0464  54. 101 ÷ 0.11 ÷ 0.1001
55. 207.049 ÷ 38.012  56. 0.078245 ÷ 400.36

Short Division


\[
\frac{94.72}{23.68}
\]

Example 14. Divide 0.375 by 4.

\[
\frac{0.375}{0.09375}
\]

EXERCISE 51 (Oral)

Express as decimals:

1. 4.6 ÷ 2  2. 5.8 ÷ 2
3. 7.8 ÷ 3  4. 9.1 ÷ 7
5. 0.84 ÷ 4  6. 0.72 ÷ 6
7. 0.63 ÷ 9  8. 0.32 ÷ 8
9. 1.38 ÷ 3  10. 2.4 ÷ 8
11. 8.4 ÷ 12  12. 7.7 ÷ 11
13. 0.045 ÷ 5  14. 0.096 ÷ 7
15. 0.3 ÷ 2  16. 0.3 ÷ 5
17. 0.2 ÷ 2  18. 0.6 ÷ 4
19. 0.3 ÷ 5  20. 0.4 ÷ 8
21. 1 ÷ 2  22. 2 ÷ 4
23. 6 ÷ 8  24. 3 ÷ 5
25. 14 ÷ 4  26. 21 ÷ 4
27. 21 ÷ 5  28. 15 ÷ 4
29. 20 ÷ 6  30. 10 ÷ 8
31. 1 ÷ 8  32. 3 ÷ 8
33. 0.01 ÷ 4  34. 0.015 ÷ 6
35. 0.03 ÷ 4  36. 0.5 ÷ 8

Common Fractions and Decimals. A fraction represents the result of dividing the number in the numerator by that in the denominator. For example, \( \frac{3}{4} \) is written as 0.75. But if we divide 3 by 4, using decimals, we obtain 0.75; \( \therefore \frac{3}{4} = 0.75 \).

Example 15. Express \( \frac{3}{4} \) as a decimal.

Divide 5 by 8; \( \therefore \frac{5}{8} = 0.625 \).

Short division should be used to find the quotient of one number divided by another whenever the divisor can be converted into a whole number, not greater than 12, by multiplying or dividing by any power of 10. Write the division sum as a fraction with one decimal point under the other and draw a line down the page to mark the new positions of the decimal points. Fill up any blank spaces in the numerator between the figures and the decimal points with zeros.

Example 16. Find the value of \( \frac{9}{100} \).

\[
\frac{9}{1000} = 0.009 \]

\( \therefore \frac{9}{1000} = 0.009 \)

\( \therefore \frac{9}{0.006} = 1500 \).

Example 17. Find the value of \( \frac{0.000805}{0.007} \).

\[
\frac{0.000805}{0.007} = \frac{0.007}{0.007} = 0.115
\]

Example 18. Find the value of \( \frac{56.4}{12} \).

\[
\frac{56.4}{12} = 4.7
\]

\( \therefore \frac{56.4}{12} = 0.047 \).
Express as decimals:

1. $\frac{1}{2} \div \frac{1}{4}$
2. $\frac{1}{3} \div \frac{1}{9}$
3. $\frac{1}{4} \div \frac{1}{5}$
4. $\frac{2}{3} \div \frac{1}{6}$
5. $\frac{3}{4} \div \frac{1}{8}$
6. $\frac{5}{6} \div \frac{1}{3}$

Express as common fractions in their lowest terms:

1. 0.375
2. 0.048
3. 0.925
4. 0.018
5. 0.025
6. 0.125

Complete the following:

10. $\frac{1}{4} \div \frac{1}{2}$
11. $\frac{1}{9} \div \frac{1}{3}$
12. $\frac{2}{3} \div \frac{1}{6}$
13. $\frac{3}{4} \div \frac{1}{8}$
14. $\frac{4}{5} \div \frac{1}{2}$
15. $\frac{5}{6} \div \frac{1}{3}$

Transform the following fractions so that the denominator becomes a whole number not greater than 12:

16. $\frac{17}{30}$
17. $\frac{3}{40}$
18. $\frac{12}{72}$
19. $\frac{9}{8}$
20. $\frac{1}{0.09}$

Express as decimals (or whole numbers):

21. 0.48
22. 0.06
23. 0.003
24. 0.74
25. 0.29
26. 0.31
27. 0.204
28. 0.21
29. 0.9
30. 0.05
31. 0.006
32. 0.08
33. 0.7
34. 0.134
35. 0.101
36. 0.121
37. 0.011

Write down as decimals, **without any working on paper**

38. $\frac{1}{4} + \frac{1}{3}$
39. $\frac{1}{2} + \frac{1}{8}$
40. $\frac{1}{12} + \frac{1}{2}$
41. $\frac{1}{30}$

Write down as common fractions, **without any working on paper**

42. 0.4
43. 0.8
44. 0.08
45. 0.125
46. 0.25
47. 0.625
48. 0.875

**Approximations**

**Example 19.** Express $\frac{1}{3}$ as a decimal.

However many zeros are added to the dividend, the process of division never ends, and we obtain a succession of 3's in the quotient.

The number 0.8333... is called a **recurring decimal** and is written for short 0.$\overline{83}$, which is read "eight point eighty three, recurring," a dot being placed over the figure which recurs.

**Example 20.** Divide 0.253 by 7.

0.036, 0.428, 0.571, 0.714, 0.857, 1...

The process of division never ends, and the group of figures, 142857, continually recurs in the quotient, which is written 0.036142857...

In practical work, recurring decimals are never used. It is impossible to measure the length of a line exactly or to find the exact weight of an object. If the length of a line is stated to be 2.37 inches, it is meant only that its length is nearer to 2.37 than to 2.36 in. or 2.38 in., and we speak of the length as given in inches correct to 2 places of decimals. A more precise measurement may show that the length is 2.368 inches correct to 3 places of decimals. This agrees with the first statement because 2.368 is closer to 2.37 than to 2.36. In practical measurements, all answers are approximate, the important thing is that they should be reliable as far as they go.

The symbol $\approx$ is used to mean "is approximately equal to."

Thus from Example 19, $\frac{1}{3} \approx 0.8; \frac{2}{3} \approx 0.67; \frac{3}{3} \approx 1$; these statements are correct as far as they go, namely to 1 place, 2 places, 3 places of decimals.

**Example 21.** Express 13.72504 correct to (i) the nearest whole number, (ii) 1 place of decimals, (iii) 2 places, (iv) 3 places, (v) 4 places.

1. 137 is nearer to 14 than to 13
2. 13.7 is nearer to 14 than to 13
3. 13.72 is nearer to 14 than 13
4. 13.725 is exactly half-way between them
5. 13.725 is nearer to 14 than 13
6. 13.7250 is nearer to 14 than 13
7. 13.72504 is nearer to 14 than 13
8. 13.72504 is nearer to 14 than 13
9. 13.72504 is nearer to 14 than 13
10. 13.72504 is nearer to 14 than 13
EXERCISE 54

Express the following numbers correct to (i) 1 place of decimals, (ii) 2 places of decimals:

1. 3.548; 6.274; 1.906.  
2. 8.379; 4.847; 0.666.
3. 2.963; 7.498; 9.092.
4. 6.798; 3.982; 7.096.

Express the following numbers correct to (i) the nearest whole number, (ii) 3 places of decimals:

5. 6.80749; 5.48685.  
6. 18.62971; 24.70963.
7. 9.71255; 37.30968.
8. 45.51547; 99.69971.

Express the following fractions (i) as recurring decimals, (ii) as decimals correct to 2 places of decimals:

9. \( \frac{1}{3} \); \( \frac{3}{5} \); \( \frac{6}{7} \).  
10. \( \frac{1}{10} \); \( \frac{1}{20} \); \( \frac{1}{30} \).
11. \( \frac{1}{11} \); \( \frac{1}{17} \); \( \frac{1}{31} \).
12. \( \frac{1}{13} \); \( \frac{1}{16} \); \( \frac{1}{41} \).

Express as decimals correct to 2 places of decimals:

15. \( \frac{3}{4} \); \( \frac{2}{3} \); \( \frac{5}{7} \).  
16. \( \frac{1}{5} \); \( \frac{1}{10} \); \( \frac{1}{3} \).
17. \( \frac{1}{11} \); \( \frac{1}{17} \); \( \frac{1}{31} \).
18. \( \frac{1}{13} \); \( \frac{1}{16} \); \( \frac{1}{41} \).

Express as decimals correct to 3 places of decimals:

20. 50 ÷ 9; \( \frac{24}{12} \); \( \frac{25}{11} \); \( \frac{26}{10} \); \( \frac{27}{7} \).

Significant figures may be defined as those which must be retained for any position of the decimal point.

For example, 4.03 m. = 403 cm. = 4030 mm. = 0.00403 km.

In each of these forms, the zero between the 4 and 3 remains and is "significant," but the zero at the end in 4030 mm. and the zeros at the beginning in 0.00403 km. are merely due to the change in position of the decimal point caused by the change in the unit of length and are therefore not "significant."

Example 22. Express, correct to 3 significant figures,

(i) 2.0762; (ii) 0.0020762.

(i) Each figure in 2.0762 is significant. Taking the first 3 figures, we obtain 2.07, but 2.076 is nearer 2.08 than 2.07; 
the value is 2.08, correct to 3 significant figures.

(ii) The zeros at the beginning in 0.0020762 are not significant. Taking the first 3 significant figures, we obtain 0.002, but 0.0020762 is nearer 0.00208 than 0.00207; 
the value is 0.00208, correct to 3 significant figures.

Example 23. The length of a bench is measured as 3.6 m., correct to the nearest cm. To how many significant figures is the length known?

It is given that the length differs from 3.6 m., or 360 cm., by less than 0.5 cm., and therefore lies between 360.5 cm. and 359.5 cm. Hence the length can be expressed as 360 cm., or 3.60 m., correct to three significant figures, and may be written \(3.60 \pm 0.005\) m.

Note: The phrase "correct to 3 significant figures" is often abbreviated to "correct to 3 figures."

EXERCISE 55 (Omicron)

Write down the values of the following, correct to the number of significant figures given in brackets:

1. 1.8634 (3).  
2. 27.485 (3).  
3. 0.7476 (2).
4. 4.732 (3).  
5. 6.448 (3).  
6. 5.447 (2).
7. 938 (1).  
8. 2714 (1).  
9. 40.636 (3).
10. 0.70562 (3).  
11. 0.0804 (2).  
12. 0.1032 (2).
13. 0.6972 (2).  
14. 60.045 (3).  
15. 100.472 (3).

Write down the following quantities to as many significant figures as the data justify:

16. A length 2.4 dm., correct to the nearest mm.
17. A length 0.37 km., correct to the nearest m.
18. (i) \(6.27 \pm 0.01\) m.; (ii) \(6.27 \pm 0.05\) m.

State how many of the zeros you think are significant in Nos. 19, 20:
19. \(1.82 \pm 0.8\); 1 hr. = 3600 sec.
20. A salesman's salary is £100 a year and in addition he earns from commissions about £350 a year.
21. The population of a town is stated to be 207009; express it correct to (i) 5, (ii) 4, (iii) 2 significant figures, also to 1 significant figure.

Express as decimals correct to 2 significant figures:

22. \( \frac{1}{3} \); \( \frac{1}{5} \); \( \frac{1}{2} \); \( \frac{1}{7} \); \( \frac{1}{9} \); \( \frac{1}{9} \); \( \frac{1}{9} \); \( \frac{1}{9} \);
23. \( \frac{5}{9} \); \( \frac{5}{9} \); \( \frac{5}{9} \); \( \frac{5}{9} \);
24. \( \frac{2}{7} \); \( \frac{2}{7} \); \( \frac{2}{7} \); \( \frac{2}{7} \);
25. \( \frac{1}{2} \); \( \frac{1}{2} \); \( \frac{1}{2} \); \( \frac{1}{2} \).

A method of showing that a number such as 4300 is correct to 3 significant figures is to write it in the form \(4300 \pm 10^4\). Use this method to represent the following:

26. 600, if correct to 2 significant figures.
27. 93,000,000, if correct to 4 significant figures.
28. 5,600,000, if correct to 5 significant figures.
Division by Factors. If only an approximate value of the quotient is required, the difficulty of calculating the remainder does not occur, and therefore the process of repeated short division may be used with advantage if the divisor can be expressed in simple factors.

Example 24. Evaluate \(\frac{0.23789}{0.063}\), correct to 2 places of decimals.

Since 2 places are required in the answer, keep 3 places in the working.

\[
\begin{array}{c|c}
0.237 & 9 \\
0.063 & 1 \\
\hline
7 & 237 \\
6 & 3 \\
\hline
9 & 33 \text{984...} \\
\hline
3 & 7 \text{76...}
\end{array}
\]

Quotient = 3.78, correct to 2 places.

Example 25. Express \(\frac{40}{8}\) as a decimal correct to 3 places.

Since 3 places are required in the answer, keep 4 places in the working.

\[
\begin{array}{c|c|c}
10 & 9 & 10 \\
9 & 9 & 10 \\
\hline
6 & 10 & 9 \\
7 & 1 & 8 \text{166...} \\
\hline
0 & 2 \text{595...}
\end{array}
\]

Quotient = 0.260, correct to 3 places.

Exercise 56

Find the values of:

1. \(29.4 \div 14\).
2. \(4.32 \div 160\).
3. \(0.108 \div 15\).
4. \(0.0315 \div 1.8\).
5. \(0.008 \div 2.5\).
6. \(0.252 \div 210\).
7. \(0.012 \div 0.32\).
8. \(1.35 \div 0.056\).
9. \(3.15 \div 4200\).
10. \(2.31 \div 0.066\).
11. \(2.31 \div 12600\).
12. \(11 \div 97 \div 0.84\).

Evaluate correct to 2 places of decimals:

13. \(0.275 \div 0.42\).
14. \(12.35 \div 7.7\).
15. \(191 \div 360\).
16. \(0.1 \div 0.063\).
17. \(0.064 \div 0.35\).
18. \(9.5072 \div 1.32\)

Express as decimals correct to 3 places of decimals:

19. \(\frac{4.44}{0.001}\).
20. \(\frac{0.8}{12.5}\).
21. \(\frac{0.15}{0.005}\).
22. \(\frac{0.035}{0.002}\).

Division by any Number

Method I. Standard Form

The division sum is written as a fraction which is then transformed so that the denominator contains exactly one digit (not zero) to the left of the decimal point. To do this, write the fraction so that one decimal point is under the other, and draw a vertical line behind the first digit (not zero) of the denominator. The position of the decimal point in the quotient should always be found by obtaining an approximate answer by short division; a final rough check by multiplication is also desirable.

Example 26. Divide \(0.8463 \div 0.031\).

\[
\begin{array}{l}
0.8463 \\
\hline
0.031 \\
\hline
27.3 \\
\hline
3.184 \text{63} \\
\hline
3.184 \text{63} \\
\hline
226 \\
\hline
217 \\
\hline
93 \\
\hline
93
\end{array}
\]

Rough Check: \(0.031 \times 27.3 = 0.8463 \approx 0.9; \ 0.8463 \approx 0.8\).

Example 27. Divide \(0.13962 \div 35.8\).

\[
\begin{array}{l}
0.13962 \\
\hline
35.8 \\
\hline
0.0039 \\
\hline
0.003 \text{9} \\
\hline
0.0039 \\
\hline
1074 \\
\hline
3222 \\
\hline
3222
\end{array}
\]

Rough Check: \(35.8 \times 0.0039 = 0.13962 \approx 0.14\).

Example 28. Evaluate \(0.00724 \div 0.0892\), correct to 3 places of decimals.

\[
\begin{array}{l}
0.00724 \\
\hline
0.0892 \\
\hline
0.0811 \\
\hline
8.92 \text{0} \\
\hline
7136 \\
\hline
892
\end{array}
\]

Rough Check: \(0.0892 \times 0.081 = 0.07072; \ 0.00724 \approx 0.07072\).

Note on Remainders. The 1 in 1480 is under the 4 in 0.7240, and this was in the fifth place of decimals in the original dividend 0.00724. Therefore in the actual remainder the 1 in 1480 must also be in the fifth place of decimals.

\[
\text{\text{.. remainder} = 0.000148.}
\]

Method II. Divisor a whole number

Write the division sum as a fraction so that one decimal point is under the other, and draw a vertical line behind the last digit of the denominator.
Example 29. Divide $0.8463$ by $0.031$.

\[
\begin{array}{c|c|c|c}
\hline
0.8463 & 31 & 0.031 & 9 \hline
0.031 & 27 & . & . \hline
0.8463 & 273 & \hline
\end{array}
\]

Put the $2$ in the quotient above the $2$ in the first product $62$.

\[273 \div 0.031 = 27.3\]

\[\text{Rough Check: } 0.031 \times 27.3 = 0.03 \times 27 = 0.8463\]

\[93\]

Example 30. Divide $0.13962$ by $35.8$.

\[
\begin{array}{c|c|c|c|c|c|c|c|c}
\hline
0.13962 & 35.8 & 13962 & 558 \hline
0.0039 & 358 & 1.3962 & 1074 \hline
0.00039 & 558 & 0.0039 & 1074 \hline
\end{array}
\]

Put the $3$ in the quotient above the $4$ in the first product $1074$. The decimal point in the quotient comes in the same column as the decimal point in the dividend; zeros must be inserted in the blank spaces between the first figure of the quotient and the decimal point.

\[0.13962 \div 35.8 = 0.0039\]

\[\text{Rough Check: } 35.8 \times 0.0039 = 40 \times 0.004 = 0.16; \quad 0.13962 = 0.14\]

Example 31. Evaluate $0.00724 \div 0.0892$ correct to $3$ places of decimals.

\[
\begin{array}{c|c|c|c|c|c|c|c|c}
\hline
0.00724 & 892 & 0.0892 & 72.4 \hline
0.00724 & 892 & 0.0892 & 72.4 \hline
0.0892 & 0.0892 & 0.0892 & 0.0892 \hline
0.0892 & 0.0892 & 1.040 & 1.040 \hline
\end{array}
\]

Work the quotient to four decimal places.

\[0.00724 \div 0.0892 = 0.081\]

\[\text{Rough Check: } 0.0892 \times 0.081 = 0.00724 ; \quad 0.00724 \approx 0.007\]

\[1480\]

For a note on the calculation of the actual remainder, if this is required, see p. 75.


EXERCISE 57

[Check the answers by approximate multiplication]

Find the values of:

1. $22.1 \div 1.3$. 2. $0.377 \div 2.9$. 3. $0.0368 \div 2.3$. 4. $803.6 \div 5.6$. 5. $2.958 \div 8.7$. 6. $0.1269 \div 9.4$. 7. $0.0783 \div 58$. 8. $0.0783 \div 58$. 9. $5.75 \div 92$. 10. $5.75 \div 92$. 11. $0.736 \div 1.15$. 12. $0.036 \div 1.15$.

Decimisation of Money. Note the following facts:

1 florin = $2s. = £(0.01); \quad 1s. = £(0.05)$ and $6d. = £(0.025)$. Thus $9s. = £(0.09 \times 9) = £(0.45)$; and $9s. 6d. = £(0.45 + 0.025) = £(0.475)$.

Example 32. Express $4\frac{3}{4}$d. as a decimal of 1s.

\[4\frac{3}{4} = 4.75 \times 12 = 4.5 + 12s. = 124.5 - 0.375s.

Example 33. Express £0.725 in s. d.

\[£0.725 = (0.725 \times 20) = 14.5s. = 14s. 6d.

Gr as follows:

\[£0.7 = 14s., \quad £0.025 = 6d.; \quad £0.725 = 14s. 6d.

\[\frac{.}{\text{3}}\]
EXERCISE 58 (Oral)

Express as decimals in pence:
1. ½d.; ¼d.; ⅛d.
2. 3½d.; 10½d.; 6⅝d.

Express as decimals in shillings:
3. 6d.; 3d.; 9d.
4. 1½d.; 7½d.; 2⅝d.

Express as decimals in £:
5. 5s.; 10s.; 5s.
6. 14s.; 18s.; 24s.
7. 3s.; 7s.; 15s.
8. 13s.; 17s.; 21s.
9. £3 10s.; £7 15s.
10. £2 6s.; £5 11s.
11. £1 3s.; £2 9s.
12. 6s. 6d.; 12s. 6d.
13. 7s. 6d.; 11s. 6d.
14. 8s. 6d.; 14s. 6d.

Express in shillings, or shillings and pence:
15. £0.1; £0.3; £0.7.
16. £0.05; £0.25; £0.45.
17. £0.025; £0.075.

Example 34. Express £3 13s. 9d. as a decimal of £.
First reduce 9d. to shillings by dividing by 12; then bring down 13s.
9d. = 0.75s.; 13s. 9d. = 13.75s.

Next reduce 13.75s. to £ by dividing by 20;
13. 9d. = £0.6875.

The working should be arranged in the abbreviated form shown; the figures in bold type are not part of the original quotations, but are brought down from the data.

Example 35. Express £3 13s. 9d. as a decimal of £5.
From Example 34, £3 13s. 9d. = £3.6875:

\[
\frac{3.6875}{5} = 0.7375; \]

\[
\therefore \, £3.13s.9d. = 0.7375 of £5.
\]

Example 36. Express £2 7s. 5⅝d. as a decimal of £1, correct to 3 places.
5⅝d. = 5.75d. Proceed as in Example 34.

\[
\frac{125.75}{100} = 0.7575; \]

\[
\therefore \, £2.7s.5⅝d. = £2.7575, correct to 3 places.
\]

Note. Unless the pence and farthings are equivalent to a number of farthings which is divisible by 3, the sum of money cannot be expressed as a terminating decimal of £.

EXERCISE 59

Example 37. Reduce £4-729 to £ s. d., to the nearest penny.

Leaving the £ as it is, and reduce £0-729 to shillings by multiplying by 20, giving 14-550s. Leave the 14s. as it is, and reduce 0-550s. to pence.

\[
\therefore \, £4.729 = £4 14s. 7d. to nearest penny.
\]

Example 38. Find the value of 0.816 of £4, to the nearest penny.

0.816 of £4 = £0.816 \times 4 = £3.264.

Proceed as in Example 37.

\[
\therefore \, 0.816 of £4 = £3 5s. 3d., to nearest penny.
\]

EXERCISE 59

Example 39. Express as a decimal of £:
1. 3s. 9d.
2. 6s. 3d.
3. 18s. 6d.
4. 4s. 5d.
5. 5s. 4⅝d.
6. 15s. 1½d.
7. 3s. 5⅝d.
8. 10d.
9. 2½d.
10. 12½d.
11. £2 7s. 3⅝d.
12. £8 17s. 8⅛d.

Express as a decimal of £1, correct to 3 places:
13. 4s. 5d. [14] 16s. 7d.
15. 3s. 6⅝d.
16. 11s. 5⅛d.
17. £1 4s. 11d. [18] £3 16s. 4d.
19. £5 11s. 8¾d. [20] £7 4s. 7¾d.
20. £3 10s. 2⅝d.

Find the value, correct to the nearest penny, of:
21. £0.63. [22] £0.85.
22. £0.738. [23] £0.914.
23. £0.042. [24] £0.807.
25. £0.0095. [26] £2.7036.
26. £3.4917.
27. £2.7094. [28] £0.3028.

Express the first sum as a decimal of the second, correct to 3 places:
29. 18s. 7d.; £2.
30. 9s. 5d.; £5.
31. 13s. 8¾d.; £4.
32. £1 6s. 10d.; £4.
33. £3 16s. 8¾d.; £10.
34. £1 17s. 6⅛d.; £5.

Find the value, correct to the nearest penny, of:
35. 0.263 of £2.
36. 0.817 of £5.
37. 0.6038 of £4.
38. 0.7148 of £2 10s.
39. 0.307 of £1 10s.
40. 0.916 of £7 10s.
DEIMALISATION OF MONEY BY THE THREE-PLACE METHOD. The following method for decimalising money at sight, correct to 3 places of decimals of £1, should be omitted unless it is learnt easily:—

\[
\begin{align*}
£ 0.05 & = 1s.; \quad £ 0.025 = 6d. = 24 \text{ farthings;} \\
\therefore \text{ 1 farthing} & = £ 0.001 + \frac{1}{8} \text{ of } £ 0.001.
\end{align*}
\]

Hence \(2\frac{3}{4}\text{d.} = 11 \text{ farthings} = £ 0.011 + \frac{1}{4} \text{ of } £ 0.001\)

\[= £ 0.011, \text{ to 3 places, because } \frac{1}{4} \text{ is less than } \frac{1}{8}.\]

But \(3\frac{1}{4}\text{d.} = 13 \text{ farthings} = £ 0.013 + \frac{1}{8} \text{ of } £ 0.001\)

\[= £ 0.014, \text{ to 3 places, because } \frac{1}{4} \text{ is greater than } \frac{1}{8}.\]

And \(9\frac{3}{4}\text{d.} = 37 \text{ farthings} = £ 0.037 + \frac{3}{8} \text{ of } £ 0.001\)

\[= £ 0.039, \text{ to 3 places, because } \frac{3}{8} \text{ is greater than } \frac{1}{8}.\]

We therefore have the following rule:—

To express pence as a decimal of £1, correct to 3 places, use the fact that 1 farthing = £0.001, and for amounts between 3d. and 9d. add £0.001, and for amounts between 9d. and 1s. add £0.002.

Example 39. Express 7s. 5\(\frac{1}{4}\)d. as a decimal of £1, correct to 3 places.

\[7s. = £0.75; \quad 5\frac{1}{4}\text{d.} = 22 \text{ farthings} = £0.022 + £0.001; \quad \therefore 7s. 5\frac{1}{4}\text{d.} = £0.773, \text{ to 3 places}.\]

Example 40. Express £4.898 in £ s. d., to the nearest farthing.

\[£0.85 = 17s.; \quad £0.048 = 46 \text{ farthings} = 11\frac{1}{4}\text{d.}, \text{ because this sum lies between 9d. and 1s.}\]

\[\therefore £4.898 = £4.17s. 11\frac{1}{4}\text{d.}, \text{ to nearest farthing}.\]

EXERCISE 60

[If additional practice is required, Exercise 59 should be used]

Write down as decimals of £1, to 3 places:

1. 5s. 2d. 2. 16s. 5d. 3. 8s. 10d. 4. 9s. 10d.
5. 1s. 1\(\frac{1}{4}\)d. 6. 7s. 7\(\frac{1}{4}\)d. 7. 15s. 11\(\frac{1}{4}\)d.
8. 6s. 6\(\frac{3}{4}\)d. 9. 12s. 1\(\frac{3}{4}\)d. 10. 11s. 10\(\frac{1}{4}\)d.
11. 3s. 9\(\frac{3}{4}\)d. 12. 9s. 5\(\frac{3}{4}\)d. 13. 8s. 9\(\frac{1}{4}\)d.
14. 14s. 8\(\frac{3}{4}\)d. 15. 7s. 11\(\frac{1}{4}\)d. 16. 16s. 3\(\frac{3}{4}\)d.

DEIMALISATION OF COMPOUND QUANTITIES

Write down in £ s. d., to the nearest farthing:

17. £0.257. 18. £0.411. 19. £2.659. 20. £1.729.
21. £3.884. 22. £5.362. 23. £4.585. 24. £6.843.
25. £1.96. 26. £2.491. 27. £3.63. 28. £5.374.
29. £2.687. 30. £3.265. 31. £1.481. 32. £2.569.

DEIMALISATION OF OTHER COMPOUND QUANTITIES

Example 41. Express 8 cwt. 3 qr. 18 lb. as a decimal of 1 ton, correct to 4 places of decimals.

First reduce 18 lb. to qr. by dividing by 28; work this quotient to 4 places of decimals; then bring down 3 qr.

Reduce 3-6428 qr. to cwt. by dividing by 4; then bring down 8 cwt. Reduce cwt. to tons by dividing by 20 and work this quotient to 5 places.

\[\therefore 8\text{ cwt. 3 qr. 18 lb. }= 0.44553 \text{ ton, correct to 4 places.}\]

Note. The figures in bold type are not part of the original quotations, but are brought down from the data.

Example 42. Reduce 2-7328 mi. to mi., ch., yd., to the nearest yard.

Leave the 2 mi. as it is and reduce 0-7328 mi. to ch. by multiplying by 80, giving 58-624 ch.

Leave the 58 ch. as it is and reduce 0-624 ch. to yd. by multiplying by 22 = 2 x 11.

\[\therefore 2-7328 \text{ mi. } = 2 \text{ mi. 58 ch. 14 yd., to the nearest yard}. \]

[Give the answers to Nos. 1-15 correct to 3 places of decimals]

EXERCISE 61

Express as a decimal of 1 ton:

1. 9 cwt. 2. 17 cwt. 2 qr. 3. 14 cwt. 3 qr. 21 lb.
4. 16 cwt. 2 qr. 10 lb. 5. 2 qr. 15 lb. 12 oz. 6. 8 cwt. 3 st. 7 lb.

Express as a decimal of 1 mile:

7. 5 fur. 6 ch. 8. 6 fur. 8 ch. 11 yd. 9. 1240 yd.
DECIMALS: MULTIPLICATION AND DIVISION

Express the first quantity as a decimal of the second:

10. 2 ft. 5 in.; 1 yd. .......................... [11] 37 min. 20 sec.; 1 hr.
12. 3 qt. 1 pt.; 1 gall. ...................... *13. 7 lb. 10 oz.; 1 st.
14. 3000 oz.; 1 cwt. ......................... 15. 4 ch. 18 yd. 2 ft. 3 in.; 1 ch.

Express in tons, cwt., qr., lb., to the nearest lb.:


Express as compound quantities, correct to the nearest unit of the lowest given denomination:

21. 7 8/26 hr. (hr., min., sec.). .......................... [22] 4.316 gall. (gall., qt., pt.),

Express the first quantity as a decimal of the second, correct to 3 places:

25. 13 cwt. 3 qr.; 2 tons. .......................... [26] 2 ft. 10 in.; 5 yd.
27. 6 fur. 3 ch.; 4 mi. .......................... [28] 3000 yd.; 10 mi.
29. 2000 lb.; 5 tons. .......................... 30. 41 3/5 hr.; 5 days.

Express as compound quantities, correct to the nearest unit of the lowest given denomination, the values of:

31. 0 2/3 of 2 mi. (mi., fur., yd.), *32. 0 8/14 of 2 3/4 hr. (hr., min., sec.),
33. 0 9/17 of 5 gall. (gall., qt., pt.),
*34. 0 6/18 of 7 3/4 tons (tons, cwt., qr.).

Averages. If a man spends £600 a year, we may say that his average monthly expenditure is £(600 ÷ 12), that is £50. This does not mean that actually he spends £50 each month, but that his total expenditure for the year is the same as it would have been if he had spent £50 a month.

Similarly, if a motorist travels 100 miles in 2 3/4 hours, we may say that his average speed for the journey is (100 ÷ 2 3/4) miles per hour, that is 40 miles per hour, or is 3 3/8 mile per minute. This does not mean that he travels 3 3/8 mile each minute, or even that he travels 40 miles in each hour. It merely means that the journey takes him the same time as if he had travelled 40 miles each hour or 3 3/8 mile each minute.
DECIMALS: MULTIPLICATION AND DIVISION

14. A batsman scored 1237 runs in 43 completed innings; find his average correct to 1 place of decimals.

[15] A bowler took 136 wickets for 2854 runs; find the average number of runs per wicket correct to 1 place of decimals.

16. A dealer buys 1 ton of tea for £170; find the average price per lb. correct to 3rd.

17. A man earns £400 a year; find his average daily earnings to the nearest penny, taking 1 year = 365 days.

*18. A man buys 10 tons of coal at 48s. per ton and 14 tons at 45s. per ton; find the average price per ton for the whole amount.

19. A man drives at 30 m.p.h. for 2 hr. and then at 40 m.p.h. for half an hour; find his average speed for the whole journey.

*20. A man buys 1000 articles at 3d. each; he sells 700 of them at 4½d. each, and the rest at 2½d. each. Find his average profit per article sold.

MISCELLANEOUS EXAMPLES

EXERCISE 63

1. Write down correct to 2 places of decimals:
   (i) 2.719; (ii) 5.403; (iii) 8.597; (iv) 4.096.

2. Add: 0.3 x 0.2 to 4 x 0.06.

3. Express as common fractions, in their lowest terms:
   (i) 0.65; (ii) 0.375; (iii) 0.208; (iv) 0.012.

4. Express £2.675 in £ s. d.

5. Express as decimals: (i) 7/8; (ii) 481/23 x 38.

6. Evaluate: (i) 0.605 x 0.24; (ii) 8.89 + 0.14.

7. Reduce £3.7s. 9d. to the decimal of £.

8. Express 4½ in. as the decimal of 1 yd.

9. The perimeter of a triangle is 18 cm., and the lengths of two of its sides are 5.47 cm., 6.68 cm.; find the third side.

[10] The rainfall in 4 successive weeks was 1.24 in., 0.83 in., 0.55 in., 2.18 in.; find the average rainfall per week.

*11. Find to the nearest penny the value of 0.1083 of £7 10s.

12. Reduce 7 cwt. 1 qr. 14 lb. to the decimal of 1 ton.

MISCELLANEOUS EXAMPLES

13. Express 2.625 tons in tons, cwt., lb.

14. Find the cost in francs of 5 m. 40 cm. of ribbon at 1.75 francs per metre.

15. Express £2.19s. 4½d. as a decimal of £.

*16. Express 7128 in. as a decimal of 1 mi.

17. Express 5/3 of 15s. 4d. as a decimal of £.


[Decimalise each fraction separately.]

19. Given that 1 yd. = 0.9144 m., express in metres, to the nearest decimetre, the height of the Chrysler Building in New York, 1032 ft.

*20. Simplify 2.54 x 2.7 - 0.45.

21. A boy bicycles 11.5 km. in 40 min.; find his average speed in kilometres per hour.

*22. Simplify 3.559248 ÷ 0.504.

*23. The diameter of the revolving plate of a gramophone is 9 in. By how much, to the nearest 1/64 in., does a French record 0.23 m. in diameter overlap the plate? [1 metre = 39.37 inches.] 


Simplify the following expressions by multiplying numerator and denominator by a convenient power of 10, and reducing the fractions to their lowest terms:

25. 0.75

26. 0.05 x 0.15

27. 0.5 x 0.75

28. 0.25 x 3.5

29. 0.05 x 2.75

30. 3.15 x 2.8

31. 0.36 x 4.9

*31. 15 million farthings weigh 42 tons; find the weight of one farthing in ounces correct to 2 places of decimals.

32. Use the identity, a² - b² = (a + b)(a - b), to evaluate

33. Express 2 m. 35 cm. in yards, feet, inches, correct to the nearest inch. [1 metre = 39.37 inches.]

*34. How many strips of tape each 3/2 in. long can be cut from a length of 9 yd., and how much remains?

35. 1 cu. ft. of water weighs 62.3 lb.; find in ounces the weight of 1 cu. in. of water, correct to 1/10 oz.
36. Nice is 922 km. from Paris. Express the distance in miles to the nearest mile. [1 mile = 1.6093 kilometres.]

*37. The charge for borrowing £85 for a year is £4-45. What is the charge, to the nearest penny, for borrowing £100 for a year at the same rate?

*38. A man leaves his house at 9.15 a.m. and reaches a town, 46 mi. away, at 10.50 a.m. Find his average speed in miles per hour, correct to 1 place of decimals.

CHAPTER VIII

METRIC SYSTEM

Weight. The unit of weight in the metric system is 1 gram (gm.).

1 gram is the weight of 1 cubic centimetre of water,
under certain conditions of temperature and pressure.

All metric tables are constructed on the same principle (p. 53).

10 grams = 1 dekagram (Dg.).
100 grams = 1 hectogram (Hg.).
1000 grams = 1 kilogram (Kg.).

1000 kilograms = 1 metric ton or tonne.

The units most commonly used are kilograms, grams, and milligrams. Weights are usually expressed in terms of one unit only, very rarely in terms of more than 2 units. A kilogram, often called for short a kilo, is approximately 2½ lb.

Areas and Volumes. The tables are derived from the table for lengths:

100 (=10^2) sq. mm. = 1 sq. cm.
100 sq. cm. = 1 sq. dm.
100 sq. dm. = 1 sq. m.

and so on.

1000 (=10^3) cu. mm. = 1 cu. cm.
1000 cu. cm. = 1 cu. dm.
1000 cu. dm. = 1 cu. m.

and so on.

1 cubic centimetre is usually written 1 c.c.
7. Draw on a piece of squared paper ruled in tenths of an inch a square of side 5 cm. How many small squares does it contain? Hence express approximately 1 sq. cm. in sq. in.

8. Point out some surface in the room about 1 sq. m. in area. Estimate the area of the door in sq. m.

9. Express 35 cm. (i) in dm., (ii) in m.; and express 24 dm. in m.

10. Express 150 sq. cm. (i) in sq. dm., (ii) in sq. m.; and express 480 sq. dm. in sq. m.

11. Point out objects in the room whose sizes are about (i) 1 c.c.; (ii) 1 cu. dm.; (iii) 1 cu. m.

12. Express in cu. dm., 750 c.c., 45 c.c., 8 c.c., 1250 c.c.

13. Express in c.c., 1.5 cu. dm., 0.75 cu. dm., 0.08 cu. dm.

14. How many c.c. are there in 1 litre, 12 litres, 0.4 litre, 0.06 litre?

15. How many litres are there in 2.5 cu. dm., 280 c.c., 35 c.c.?

16. What is the weight of (i) 10 c.c. of water; (ii) 2 cu. dm. of water; (iii) 3 litres of water?

17. How many c.c. of water weigh (i) 1 kg.; (ii) 50 gm.?

18. Point out an object that weighs about 1 kg.

19. 5 kg. = 11 lb.; a boy weighs 110 lb., what is this in kg? A man weighs 70 kg., what is this in lb.? in stones?

20. 1 litre = 1.75 pt.; express approximately in pints (i) 4 litres, (ii) 2 cu. dm., (iii) 500 c.c.

21. Express (i) 10 fr. 50 c. in francs; (ii) 12 dollars 5 cents in dollars.

22. Any volume of copper is about 9 times as heavy as the same volume of water, or for short copper is about 9 times as heavy as water. Find approximately the weight of 10 cu. dm. of copper (i) in kg.; (ii) in lb. [See No. 19.]

23. Mercury is about 13.6 times as heavy as water. Find approximately the weight of 1 litre of mercury in kg.

24. Brass is about 8 times as heavy as water. Find approximately the volume in c.c. of a lump of brass which weighs 1 kg.

METRIC SYSTEM: ADDITION AND SUBTRACTION

EXERCISE 65

1. Express in grams:
   (i) 12 gm. 8 c.g.; (ii) 15 mg.; * (iii) 4 Dg. 5 c.g.

2. Express in kilograms:
   (i) 3 Hg. 5 Dg.; (ii) 48 gm.; (iii) 6 Dg. 5 d.

Add and give the answers in grams:
3. 5 gm. 7 d.; 8 gm. 9 c.g.; 5 d. 4 c.g.

4. 3 d. 8 mg.; 7 c.g. 5 mg.; 2 g.m. 7 mg.

5. 1 Kg. 148 gm.; 2 Kg. 62 gm.; 2 Hg. 4 Dg.

Subtract the second quantity from the first and give the answers in Kg.:
6. 7 Kg. 375 gm.; 2 Kg. 180 gm. [7] 2 Kg. 3 Dg.; 850 gm.

7. 5 Hg.; 75 gm. [8] 8 Dg.; 24 gm. 6 d.

10. Add 35 Dg., 37 Hg., 105 gm.; answer in Kg.

11. Subtract 15 mm. from 1 dm.; answer in metres.

12. Subtract 2570 gm. from 3 Kg.; answer in Kg.

13. Express in francs the sum of 10 fr. 25 c., 7 fr. 65 c., 12 fr. 5 c., 9 fr. 75 c.; and subtract the sum from 50 fr.

Find the values of:
14. 6 gm. 7 d. 3 c.g. + 5 gm. 8 c.g. 4 mg. + 2 gm. 14 mg. (in gm.).

15. 4 ft. 7 in. = 6 ft. 15 c.g. + 2 Dl. 3 l. 2 dl. 8 cl. (in l.).

16. 6 Kg. 4 Dg. 5 gm. 7 d. - 3 Hg. 8 gm. 9 d. 5 c.g. (in Kg.).

17. Subtract 275 sq. cm. from 1 sq. m. (answer in sq. m.).

18. Subtract 50 c.c. from 1 cu. dm. (answer in cu. dm.).

19. Three parcels weigh 2 Kg. 150 gm., 1 Kg. 75 gm., 3 Kg. 875 gm., respectively. Find their total weight in Kg.

20. A bottle contains 1 litre of milk; how many c.c. remain when 350 c.c. have been poured out?

21. A flask weighs 37-53 gm. when empty and 118-28 gm. when full of water. Find (i) the weight of the water; (ii) the capacity of the flask.

22. From a sheet of cardboard 1 dm. square, a rectangular portion 8 cm. by 5 cm. is removed. Find in sq. dm. the area of what remains.

23. The capacities of three bottles are 250 c.c., 375 c.c., 525 c.c., respectively. Find their total capacity in litres.
24. The external diameter of a hollow metal pipe is 8·1 cm., and the internal diameter is 6·9 cm. Find the thickness of the metal.
25. The internal measurements of a rectangular tin are 8 cm. by 6 cm. by 5 cm.; express its capacity in litres, and find the weight in Kg. of the water it will hold.
26. An empty flask weighs 32·5 gm.; find the total weight if 0·18 litre of water is poured into it.
27. A litre of petrol weighs 0·7 Kg. Find in Kg. the total weight if 2 litres of petrol are put into a tin which weighs 270 gm. when empty.

**Multiplication and Division**

**Example 1.** Find in sq. m. the area of a rectangle of length 4 m. 75 cm. and breadth 3 m. 40 cm.

- The length is 4·75 m.,
- The breadth is 3·4 m.,
- \[ \text{Area} = 4\text{.75} \times 3\text{.4} \text{ sq. m.} = 16\text{.15 sq. m.} \]

**Note.** 16·15 sq. m. = 16 sq. m. 1500 sq. cm.

Formal "practice" is reserved for Chapter X, but simple examples may with advantage be worked by practice methods at this stage; see Example 2.

**Example 2.** Given that 1 yd. = 0·9144 m., express 3 yd. 2 ft. 5 in. in m., cm., to the nearest cm.

\[
\begin{align*}
\text{m.} & \quad 0\cdot9144 & \quad 1 \text{ yd.} \\
\text{3} & \quad 2\cdot7432 & \quad 3 \text{ yd.} \\
\text{1 ft.} = \frac{1}{3} \text{ of 1 yd.} & \quad 0\cdot3048 & \quad 1 \text{ ft.} \\
\text{4 in.} = \frac{1}{4} \text{ of 1 ft.} & \quad 0\cdot1016 & \quad 4 \text{ in.} \\
\text{1 in.} = \frac{1}{4} \text{ of 4 in.} & \quad 0\cdot0254 & \quad 1 \text{ in.} \\
\text{Add} & \quad 3\cdot4798 & \quad 3 \text{ yd. 2 ft. 5 in.} \\
\text{Total} & \quad 3\cdot4798 & \quad 3 \text{ yd. 2 ft. 5 in.} \\
\end{align*}
\]

\[ \therefore \text{3 yd. 2 ft. 5 in. = 3·4798 m. = 3 m. 48 cm. to nearest cm.} \]

**EXERCISE 66**

1. Find in cm. the height of a pile of 9 notebooks if each is 1 cm. 7 mm. thick.
2. 76 fr. 50 c. is shared equally between 9 boys; how much does each receive?
MISCELLANEOUS EXAMPLES

EXERCISE 67

1. A rectangular field is 386 m. long, 274 m. wide; express its perimeter in km.

2. Express in mg. the difference between 0.86 dg. and 0.24 gm.

3. Find in gm. the weight of 35 cl. of water.

4. A racing car travels 350 m. in 10 sec.; express this speed in km. per hour.

5. The weights of three articles are 27 gm. 45 cg., 108 gm. 65 cg., 8 Dg., respectively. Find their total weight in Kg.

6. Water is coming out of a tap at the rate of 75 c.c. per sec.; how many litres come out in 2[1/2] min.? How many pints per minute? [1 litre = 1.76 pt.]

7. A wooden box without a lid is 2.5 dm. long, 1.5 dm. wide, 6.5 cm. high, external measurements. The wood is 7.5 mm. thick; find the internal measurements.

8. How many cu. cm. blocks can be cut from a cuboid, 0.6 m. long, 0.25 m. wide, 1.4 dm. high?

9. Aluminium is 2[1/2] times as heavy as water; find in kg. the weight of 0.6 cu. dm. of aluminium.

10. Taking 1 ton = 1016 Kg., express 6 cwt. 3 qr. in Kg. to the nearest Kg.

11. Find in grams the weight of half a litre of oil which is 0.9 times as heavy as water. How many oz. to nearest oz.? (See No. 16.)

12. How many kilolitres of water can a tank 15 m. long, 8 m. wide, 4.5 m. deep, hold?

13. A photograph 14 cm. high, 8.5 cm. wide is mounted on a card so that there is a margin 25 mm. wide all round. Find the perimeter of the card in dm.

14. Express 11 oz. in gm. to the nearest gm. [1 lb. = 453.6 Kg.]

15. A pan is 4 dm. long, 35 cm. wide, 7.5 mm. deep, internal measurements. Find its capacity in litres.

16. Express 7 Hg. 8 Dg. in lbs., oz., to the nearest oz.

17. The diameter of a cigarette, 7 cm. long, is 7.5 mm. How many cigarettes can be packed in a box which measures internally 7 cm. by 15 cm. by 3 cm.?

18. Given 1 mi. = 1.609 km., express 1240 yd. in km., to the nearest Dm.

19. A flask weighs 64-27 gm. when empty and 150-35 gm. when full of water. Find its weight when it is half full of water.

20. The water in a bottle 3/4 full weighs 52.8 gm.; what would be the weight of the water if the bottle was 1/2 full?

CHAPTER IX

GRAPHS

The object in representing facts graphically is to convey information rapidly. Thus the table in Example 2, p. 94, gives the number of deaths per week from road accidents in Great Britain for a certain period. It is quicker to see how this number went up and down by glancing at the graph than by reading through the table.

Example 1 (for Oral work). A motor-car is fitted with a gauge which shows the number of gallons of petrol in the tank. When full, the tank holds 5 gal. A motorist starts out at 10 a.m. and notes the readings on the gauge at hourly intervals. The result is shown in the diagram given above:

Use a ruler to interpret the facts shown in the diagram, and answer the following:—

(i) What is the length in inches of the line which represents (a) 2 gall., (b) 3 gall.?

(ii) How much petrol is represented by an upright line of length (a) 1 in., (b) 1-5 in.?

(iii) How much petrol did he start with? How much had he at 1 p.m. and at 4 p.m.?

(iv) During what time is it probable that the car was not running?

(v) About what time did he fill up the tank?

(vi) The car averages 24 mi. to the gallon; estimate the distances travelled in successive hours.

(vii) What meaning, if any, can you give to an upright inserted midway between the last two points?

(viii) Can you insert with fair accuracy an upright for 10.30 a.m., for 1.30 p.m., for 2.30 p.m.?
**Use of Squared Paper.** It was necessary to use a ruler to interpret the facts given by the petrol-gauge graph because it was drawn on plain paper; it saves time and trouble to use squared paper.

**Example 2.** The number of deaths per week from road accidents in Great Britain for the midsummer of 1934 was as follows:

<table>
<thead>
<tr>
<th>Week ending</th>
<th>June 30</th>
<th>July 7</th>
<th>July 14</th>
<th>July 21</th>
<th>July 28</th>
<th>Aug. 4</th>
<th>Aug. 11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deaths</td>
<td>142</td>
<td>180</td>
<td>143</td>
<td>143</td>
<td>157</td>
<td>148</td>
<td>160</td>
</tr>
</tbody>
</table>

Represent these facts by a graph.

First draw a line across the squared paper and mark on it points to represent the given weekly periods.

Next draw a line up the paper and graduate it to show the number of deaths. Since in no week the number is less than 140 or greater than 180, it is unnecessary to show any graduation outside these limits.

These two lines are called the **axes of reference**, and the graduations show the chosen scales.

![Road Accidents graph]

**EXERCISE 68 (Oral)**

State which quantity should be measured along the axis drawn across the page for the following graphs:

1. Postage on parcels of various weights.
2. Number of passengers on the Underground at different times of day.
3. A boy's age and height.
4. The H.P. of a motor-car and the tax on it.
5. A travel graph: distance from home and time of day.
6. Record times for races of various lengths.
7. A man's age and his weight.
8. Stretch of a spiral spring produced by various loads.
9. A mark reducer: original marks and scaled marks.
10. A graph to convert miles to kilometres.

What scales would you choose and what would be the smallest and largest graduations to represent the following ranges of values, for the given lengths of axes?

11. Length 10 in.: from 7 to 53.
12. Length 10 in.: from 135 to 1050.
13. Length 10 in.: from 56 to 23.8.
14. Length 6 in.: from 45 to 100.
15. Length 8 in.: from 100 to 250.
16. Length 5 in.: from 0 to 1.
17. Length 5 in.: from 65 yd. to 295 yd.
18. Length 6 in.: from 2.75 lb. to 3.25 lb.
19. Length 7 in.: from 0.28 m. to 0.54 m.
20. Length 6 in.: from £500,000 to £800,000.

EXERCISE 69

Represent on squared paper the following statistics. State in each case whether any meaning can be attached to intermediate uprights, and if so, whether intermediate uprights can be inserted with fair accuracy without further data.

Give each diagram a title and write along each axis what that axis represents, and show how it is graduated.

1. The average diameter of oak trees of different ages:

<table>
<thead>
<tr>
<th>Age in years</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>50</th>
<th>70</th>
<th>100</th>
<th>150</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter in inches</td>
<td>5</td>
<td>10</td>
<td>14</td>
<td>23</td>
<td>32</td>
<td>41</td>
<td>54</td>
</tr>
</tbody>
</table>

Estimate (i) the diameter for an age of 40 years;
(ii) the age at which the diameter is 38 in.

[2] The average daily receipts of a certain grocer:

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Receipts in £</td>
<td>13</td>
<td>11</td>
<td>14</td>
<td>16</td>
<td>12</td>
<td>18</td>
</tr>
</tbody>
</table>

Which day was probably "early closing"?

3. The average weight of boys of different ages:

<table>
<thead>
<tr>
<th>Age in years</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight in lb.</td>
<td>79</td>
<td>85</td>
<td>92</td>
<td>...</td>
<td>114</td>
<td>129</td>
<td>142</td>
<td>146</td>
</tr>
</tbody>
</table>

What is the average weight at age 14?

4. The marks obtained out of 200 by a boy in successive fortnights of a term:

<table>
<thead>
<tr>
<th>Number of fortnight</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marks obtained</td>
<td>125</td>
<td>110</td>
<td>104</td>
<td>162</td>
<td>140</td>
</tr>
</tbody>
</table>

LOCUS GRAPHS

•5. Cars sold by a company in successive quarters of 1933, 1934:

<table>
<thead>
<tr>
<th>Quarter</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number sold</td>
<td>420</td>
<td>560</td>
<td>620</td>
<td>510</td>
<td>440</td>
<td>630</td>
<td>740</td>
<td>580</td>
</tr>
</tbody>
</table>

What seems to be (i) the best month, (ii) the worst month for selling cars?

Locus Graphs. In representing a set of statistics by a graph it is not necessary to draw the whole of the uprights; the usual custom is to mark only the top point of each upright, and to leave the rest to the imagination.

Example 8 (for Oral work). The given diagram shows the temperature at stated times of a boy with a feverish cold. If his temperature had been taken more frequently, there would be more points marked on the graph; but there are sufficient to give a good idea of how his temperature changes.

Make a copy of this diagram and draw a smooth curve through the marked points. This curve probably represents with fair accuracy the locus of the top points of the uprights which correspond to the boy's temperature at intermediate times; we therefore call it a locus-graph. It shows at a glance the boy's temperature (approximately) at any time between 8 a.m. and 8 p.m.

D 7
Use your figure to answer the following questions:

(i) What is approximately his temperature at 9 a.m., 5 p.m.?

(ii) At what time is his temperature 101°, 101.7°?

(iii) At what time is his temperature 100°?

The top points of successive uprights are often joined by straight lines in order to guide the eye rapidly from one point to the next; in such cases the intermediate points on the lines do not usually represent intermediate values. The road-accidents graph in Example 2, p. 94, appeared in a newspaper where it was shown with the top points joined by straight lines in order to make the rise and fall in the number of deaths more vivid, but in this case intermediate points on the lines have no meaning.

Example 4. The height of the barometer in inches is recorded at hourly intervals on a certain day as follows:

<table>
<thead>
<tr>
<th>Time</th>
<th>9 a.m.</th>
<th>10 a.m.</th>
<th>11 a.m.</th>
<th>12</th>
<th>1 p.m.</th>
<th>2 p.m.</th>
<th>3 p.m.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inches</td>
<td>29-55</td>
<td>29-70</td>
<td>29-77</td>
<td>29-70</td>
<td>29-90</td>
<td>29-72</td>
<td>29-15</td>
</tr>
</tbody>
</table>

We select the times at which the height is observed, the time-axis is therefore taken across the page, unit, 1 in. represents 2 hr., the first graduation is 9 a.m.

All readings lie between 29 in. and 30 in.; they are marked by crosses in the graph. If an automatic recording machine had been employed, the pointer of the machine would have marked not only these isolated points but also a continuous curve passing through them, thus forming the locus graph shown in the next diagram.

![Barograph Graph](image)

Use the printed diagram to answer the following:

(i) What is the height of the barometer at 9.36 a.m., 12.24 p.m., 1.24 p.m., 2.48 p.m.?

(ii) At what times is the height of the barometer 29-65 in., 29-80 in., 29-45 in.?

(iii) Between what times was the barometer rising?

(iv) Between what times was the barometer above 29-65 in.?

(v) How much did the barometer fall between 1.30 p.m. and 2.30 p.m.?

(vi) How much did the barometer rise between 9.30 a.m. and 10.30 a.m.?

(vii) What inferences can you draw from noticing that a special part of the graph slopes downwards and that one part slopes downwards more steeply than another part?

EXERCISE 70

1. The following table gives the distances in which a train can be stopped for various velocities:

<table>
<thead>
<tr>
<th>Velocity in m.p.h.</th>
<th>30</th>
<th>40</th>
<th>45</th>
<th>50</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance in yards</td>
<td>100</td>
<td>176</td>
<td>223</td>
<td>276</td>
<td>400</td>
</tr>
</tbody>
</table>

Find from a graph (i) how much farther a train runs after the brakes are put on hard when the speed is 35 m.p.h., 55 m.p.h.; (ii) how fast a train is travelling if it can be stopped in 200 yd.

*2. If £100 is allowed to accumulate at 4% per annum compound interest, the amount is as follows:

<table>
<thead>
<tr>
<th>Number of years</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>20</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amount in £</td>
<td>100</td>
<td>122</td>
<td>148</td>
<td>219</td>
<td>324</td>
</tr>
</tbody>
</table>

Find from a graph the amount after (i) 15 years, (ii) 25 years, (iii) 33 years.

After what time will £100 amount to £350?

At 4% simple interest, the amounts of £100 after 10, 20, 30 years are £140, £180, £220, respectively; draw the corresponding simple interest graph on the same figure. After what time does the amount of £100 at compound interest exceed that at simple interest, at 4% per annum, by £30?

3. Expectation of life of an Englishman at different ages:

<table>
<thead>
<tr>
<th>Age in years</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expectation in years</td>
<td>33.2</td>
<td>26.5</td>
<td>19.9</td>
<td>13.6</td>
<td>8.6</td>
<td>8.2</td>
</tr>
</tbody>
</table>

Find from a graph (i) how much longer an Englishman may expect to live at the age of 34, 53, 66; (ii) at what age the expectation of life is 22, 16, 11 years.
[4] The annual premium for a Life Assurance of £1000 varies with the age of the insurer when he makes the first payment:

<table>
<thead>
<tr>
<th>Age</th>
<th>25</th>
<th>30</th>
<th>35</th>
<th>40</th>
<th>45</th>
</tr>
</thead>
<tbody>
<tr>
<td>Premium in £</td>
<td>141</td>
<td>164</td>
<td>192</td>
<td>23</td>
<td>272</td>
</tr>
</tbody>
</table>

Find from a graph (i) the premiums for starting at the age of 32, 38; (ii) the age at which a man started who had to pay a premium of £25 15s.

*5. The number of hours in the longest day in a year depends on the latitude:

<table>
<thead>
<tr>
<th>Lat. in degrees</th>
<th>25</th>
<th>35</th>
<th>45</th>
<th>50</th>
<th>55</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of hours</td>
<td>13:6</td>
<td>14:4</td>
<td>15:4</td>
<td>16:1</td>
<td>17:1</td>
<td>18:5</td>
</tr>
</tbody>
</table>

Find from a graph (i) the number of hours in the longest day in latitudes 30°, 52°; (ii) the latitude in which the longest day is 15 hr., 18 hr.

6. The time of a complete oscillation of a pendulum depends on its length; the following results hold in London:

<table>
<thead>
<tr>
<th>Length in feet</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time in seconds</td>
<td>1:11</td>
<td>1:57</td>
<td>2:02</td>
<td>2:21</td>
<td>3:18</td>
<td>3:11</td>
</tr>
</tbody>
</table>

Find from a graph (i) the time for lengths 2 ft. 6 in., 4 ft. 9 in.; (ii) the length to give a time, 2 sec.

The pendulum of a clock should make complete oscillations every 2 sec. if the clock is keeping time. How should the length be corrected if a complete oscillation takes 2:1 sec.? What alteration in length is required to reduce the time of a complete oscillation from 1:3 sec. to 1:2 sec.?

[7] The heights of a shell fired from a howitzer at various times after projection are as follows:

<table>
<thead>
<tr>
<th>Time in seconds</th>
<th>5</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>35</th>
<th>40</th>
<th>50</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height in hundreds of feet</td>
<td>15</td>
<td>28</td>
<td>45</td>
<td>52</td>
<td>49</td>
<td>35</td>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

Find from a graph (i) the height after 15 sec., (ii) the times when the height is 4300 ft., (iii) the length of the time for which the height is more than 5000 ft.
**13.** The curves A, B, C show the number of people killed of ages from 0 to 70 in 1932 in road accidents. The three curves refer to motorists, pedestrians, and pedal cyclists. Which is curve A? Which is curve B? Give short reasons for your answer, and state in general terms the notable facts established by these graphs.

**STRAIGHT LINE GRAPHS**

**Example 5 (for Oral work).** The diagram shows some points on the travel graph of a steamer; it must _not_ be called the path of the steamer.

(i) How far did the steamer travel in 2 hr., 4 hr., 6 hr.?
(ii) Use a ruler to see whether the marked points lie on a straight line. What does this mean? What is the speed of the steamer?
(iii) _Make a copy of the diagram and draw on it_ travel graphs for speeds of 20 m.p.h., 25 m.p.h., 5 m.p.h.
(iv) Draw on your copy the travel graph of a man who goes at 10 m.p.h. for 1 hr., then at 20 m.p.h. for 3 hr., then at 15 m.p.h. for 2 hr.
(v) What is the meaning of the statement that one part of a travel graph is steeper than another part?

Since 5 kg. = 11 lb., we can obtain a point on the graph by marking off the distance ON along the kg. axis which represents 5 kg., and then moving upwards the distance NA which represents 11 lb.; then A is a point on the graph, and the required graph is the straight line which joins A to the origin O.

To express 2.6 kg. in lb., mark off the distance OP along the kg. axis which represents 2.6 kg.; by measurement we see that the distance PB from P up to the graph represents 5.7 lb.; ∴ 2.6 kg. = 5.7 lb.

To express 8 lb. in kg., mark off the distance QQ along the lb. axis which represents 8 lb.; by measurement we see that the distance QC from Q across to the graph represents 3.6 kg.; ∴ 8 lb. = 3.6 kg.
EXERCISE 71

[All answers should be based on readings from the graphs]

1. Interpret the given travel graph, stating the different speeds in miles per hour. What is the average speed for the whole journey? How can you tell without any calculation which part of the graph corresponds to the greatest speed?

2. A spiral spring is suspended from one end, and its length is measured when different weights are attached to the other end:

<table>
<thead>
<tr>
<th>Weight in gm.</th>
<th>10</th>
<th>15</th>
<th>30</th>
<th>50</th>
<th>75</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length in cm.</td>
<td>22</td>
<td>24</td>
<td>30</td>
<td>38</td>
<td>48</td>
</tr>
</tbody>
</table>

Draw a graph to show the relation between the length and the load, and use it to find:

(i) the length if the load is 20 gm., 40 gm., 65 gm.;
(ii) the load if the length is 23 cm., 28 cm., 42 cm.;
(iii) the natural length, i.e., the length when there is no load.

Is the graph a straight line? If so, what does this mean? The corresponding graph for another spiral spring is a steeper straight line; what does this mean?

3. Draw the travel graph of a man, travelling due north, who starts at 11 a.m. and walks at 4 m.p.h. for half an hour, then bicycles at 12 m.p.h. for 1½ hr., then stops for ½ hr., then motors at 32 m.p.h. for 1½ hr. How far has he gone at 2.30 p.m.? What is the time when he has travelled 10 mi., 28 mi.? Find his total average speed.

4. Draw a graph for converting miles to kilometres, for distances up to 5 mi., given that 1 mi. = 1.6 km. Use the graph to express 2.5 mi. in km., and 6.2 km. in miles.

5. The diagram shows the travel graphs OCG, OQS of two men, X and Y respectively, who start from the same house at 10 a.m. and proceed on the same road.

(i) Describe the journey represented by OCG in detail, giving the various speeds. Can you tell at a glance when the man is moving fastest, when he is resting? How far does he go altogether? When does he start to come home?

(ii) Repeat (i) for the graph OQS.

(iii) When does X overtake Y?

(iv) When and where do X and Y pass one another?

(v) How far is X ahead of Y when he turns back?

(vi) How far does X go while Y is resting?

6. Draw a graph for converting degrees Centigrade to degrees Fahrenheit, given that 0° Centigrade, 100° Centigrade are equivalent to 32° Fahrenheit, 212° Fahrenheit respectively. Use the graph to express in degrees Fahrenheit, 35° Centigrade, 70° Centigrade, and to express in degrees Centigrade, 140° Fahrenheit, 185° Fahrenheit.
[7] Draw a graph for converting speeds in miles per hour to speeds in feet per second, up to 60 m.p.h., given that 30 m.p.h. is the same speed as 44 ft. per second. Use the graph to express in feet per second, 22 m.p.h., 46 m.p.h., and to express in miles per hour, 20 ft. per second, 75 ft. per second.

*8. Marks in an examination run from 32 to 74. Draw a graph for scaling them to run from 0 to 100. What is the scaled mark if the original mark is 43? What is the original mark if the scaled mark is 83? If A obtains 20 more marks than B in the examination, how many more marks does A get than B after the scaling?

9. A motorist leaves home at 9 a.m.; the mileage recorded by his cyclometer, originally set at zero, is as follows:

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mileage</td>
<td>2</td>
<td>5</td>
<td>9</td>
<td>12</td>
<td>16</td>
<td>19</td>
<td>24</td>
<td>31</td>
<td>37</td>
</tr>
</tbody>
</table>

Find from a graph (i) the readings at 9.36, 9.54, 10.15 a.m.; (ii) the distance travelled between 9.35 a.m. and 10.5 a.m.; (iii) the time when the distance travelled is 14 mi., 21 mi., 33 mi.

10. A cyclist starts from A at noon and rides steadily at 12 m.p.h. towards B, 60 mi. away. At 2 p.m. a motorist leaves B for A and travels at 36 m.p.h. Find from a graph when the two meet and their distance from A at this time.

11. A pedestrian sets off to walk along a road at 4 m.p.h.; a cyclist starts from the same place an hour later and rides along the same road to a town 18 mi. away at 10 m.p.h., waits there for half an hour and then returns at the same speed. When and where does he meet the pedestrian on his return journey?

12. A few years ago income tax was levied as follows: no tax on the first £150 of a man’s income, on the next £250 the tax was 2s. 6d. in the £, and on income beyond this was 5s. in the £. Draw a graph to show the tax payable on any income up to £600. What was the income of a man whose tax was (i) £15, (ii) £50?

Draw rough graphs to illustrate Nos. 13-16.

13. A travel graph: a boy walks for 5 min., runs for 2 min., stands still for 3 min., and then returns to his starting-point in a car.

14. The inland postage for letters of various weights: charge up to 2 oz., 1½d.; for each additional 2 oz., or part of it, 1d. more.

16. The bank-balance of a man whose salary is paid only three times a year, namely at the end of March, of July, of December. Explain any special features the graph possesses.

17. Three men A, B, C travel from X to Y by the same road. A walks the distance in 5 hr. B cycles, starting 1½ hr. after A and arriving 1 hr. before him. C motors, starting 3 hr. after B and arriving 2 hr. before him. Show by a graph that B and C pass A at the same place.

18. X, Y are two places 32 mi. apart. A leaves Y at 1.10 p.m. and drives at a rate which would bring him to X at 2.30 p.m. for a meeting. On the way he meets B, who left X at 1 p.m. and has driven at 30 m.p.h. They talk for 10 min. and A hears that the meeting is at 2.15 p.m. At what rate must A drive for the rest of the journey to be in time. Solve graphically.

19. To travel from one house to another I can either motor from door to door at an average speed of 20 m.p.h., or I can go by train at an average speed of 45 m.p.h.; but, if I travel by train, it is necessary to allow 20 min. at each end for the journeys between the houses and stations. Find graphically the shortest distance for which it is quicker to go by train, assuming that the distance between the stations is the same as that between the houses.

20. A cyclist starts at 10 a.m. to ride to a place 8 mi. away, riding at 12 m.p.h. till he has a puncture. He waits 10 minutes trying to repair it and then walks on at 4 m.p.h. He arrives at his destination at 11.15 a.m. How far had he ridden when the puncture occurred?
THREE-MINUTE AND FOUR-MINUTE ORAL PRACTICE

ORAL PRACTICE 1–8 (Ch. I–III)

[Nothing must be written down except the answer]

**Oral Practice 1 (3 min.)**

   
   108
   
   84
   
   19
   
   5. Express 2 lb. 8 oz. in oz.
   
   6. 1 ft. 8 in. + 9 in.
   
   7. 9 cwt. × 5, in tons, cwt.
   
   8. Express $2^4 \times 2^4 \times 2^4$ as a power of 2.
   
   9. Cost of 1 sq. yd. at 4d. per sq. ft.
   
   10. 3 marbles weigh 2 oz., what do 1 dozen weigh?

**Oral Practice 2 (3 min.)**

   
   3. 7536 ÷ 8.
   
   4. Express 90 in. in ft., in.
   
   5. Express 1 mi. 540 yd. in yd.
   
   6. 8 yd. × 3, in ch., yd.
   
   7. 2 tons 16 cwt. × 8.
   
   8. How many cu. inches in a 5-inch cube?
   
   9. 3 men can mow a field in 12 days, how long will 9 men take?
   
   10. Write in index form the cube of 3.

**Oral Practice 3 (3 min.)**

1. Add: 518  2. 809 × 12.
   
   3. 100001 ÷ 11.
   
   4. Express 3 gall. in pints.
   
   5. $(30 \times 40 \times 50) \div 10$.
   
   6. 2 cwt. = 50 lb., in cwt., lb.
   
   7. 4 lb. 4 oz. × 8, in qr., lb.
   
   8. Find in ft. the perimeter of a rectangle, 10 in. by 8 in.
   
   9. Cost of half a lb. of tobacco at 9d. an oz.
   
   10. Express in powers of prime factors, $6^2 \times 10^2$.

**Oral Practice 4 (3 min.)**

   
   73
   
   18368
   
   8.
   
   5. 5 ft. – 1 ft. 8 in.
   
   6. 5 fur. × 7, in mi., fur.
   
   47
   
   82
   
   7. 2 hr., 30 min. × 10.
   
   8. Cost of 15 envelopes, if 5 envelopes cost 4d.?
   
   9. Write in index form the square root of $2^n \times 3^4$.
   
   10. The area of a field 40 yd. wide is 4840 sq. yd.; what is its length?

**Oral Practice 5 (4 min.)**

   
   3. 100908 ÷ 12.
   
   703
   
   4. Express 1000 yd. in mi., yd.
   
   64
   
   5. Express half an acre in sq. yd.
   
   8
   
   6. 3 qt. 1 pt. × 6, in gall., qt.
   
   215
   
   7. 7 yd. 2 ft. 10 in. ÷ 11.
   
   96
   
   8. Cost of a carpet 3 yd. long, 8 ft. wide at 2s. per sq. ft.?
   
   9. Write in index form the L.C.M. of $2 \times 3^4$, $2^4 \times 5^2$, $3^2 \times 5$.
   
   10. A journey takes 10 hours at 24 miles per hour; how long does it take at 40 miles an hour?

**Oral Practice 6 (4 min.)**

   
   3098
   
   382574
   
   4. Express 10 ac. in sq. yd.
   
   72815
   
   5. 1 yd. = 2 ft. 5 in.
   
   6. 7 cwt. 3 qr. × 5, in tons, cwt., qr.
   
   7. 2 mi. 3 fur. 6 ch. × 7.
   
   8. Write in index form the square of 5.
   
   9. 2 qt. of water weigh 5 lb.; what is the weight of 2 gall. of water?
   
   10. A tank 6 ft. long, 2 ft. wide, holds 48 cu. ft.; what is its depth?
THREE- AND FOUR-MINUTE ORAL PRACTICE

Oral Practice 7 (4 min.)

1. Add: 804
2. Subtract: £ s. d.
3. 3. 20086 ÷ 11.
4. Express 279 lb. in cwts., lb.
5. Express 4 ch. 15 yd. in yd.
6. 3 lb. 6 oz. × 7, in lb., oz.
7. \((3^3 \times 3^8) ÷ (3^5 \times 3^3)\).

8. Cost of 12 articles at \(\frac{7}{8}\) d. each.
9. How many times is 5 in. contained in 1 yd., and how much is over?
10. A car travels 8 miles in 12 minutes; what is its speed in miles per hour?

Oral Practice 8 (4 min.)

1. Add: 28
2. Subtract: 613072
3. Subtract: £ s. d.
4. Express 1000 yd. in fur., yd.
5. Express 3 gall. 2 qt. in pints.
6. £3 11s. 8d. × 9.
7. 7 qr. 9 lb. 5 oz. ÷ 3.
8. Find the missing digit if \(\overline{783}\) is divisible by 11.
9. Find the total area of the four walls of a room, 12 ft. long, 8 ft. wide, 7 ft. high.
10. 8 men can mow a field in 9 days, how long will 6 men take? / 2

Oral Practice 9 (3 min.)

1. \(\frac{7}{8}\) of £1, in £ s. d.
2. \(\frac{6}{8} + \frac{1}{6}\).
3. Add: 1:03

4. 0·05 × 70.
5. 0·258 ÷ 3.
6. Express \(\frac{7}{8}\) as a decimal.
7. £0·45, in s.
8. Share 59 fr. 50 c. equally among 7 boys.
9. What must be added to \(\frac{2}{3}\) to make \(\frac{1}{2}\)?
10. After travelling 8 mi., I have done \(\frac{3}{4}\) of my journey. How long is my journey?

Oral Practice 10 (3 min.)

1. \(\frac{2}{5}\) of 3 lb., in lb. oz.
2. \(\frac{3}{10} - \frac{1}{6}\).
3. Subtract: 10

4. 0·2 ÷ 0·3.
5. 0·378 ÷ 70.
6. Express 2 m. 5 cm. in m.
7. \(\frac{7}{8}\) as a decimal to 2 places.
8. What fraction is \(\frac{4}{5}\) of \(\frac{7}{3}\)?
9. Cost of 12 m. at 8 fr. 50 c. per metre?
10. \(\frac{1}{2}\) acre is worth £40; what is \(\frac{2}{3}\) acre worth?

Oral Practice 11 (3 min.)

1. \(\frac{3}{4}\) of 2 yd., in yd. ft.
2. 7 ÷ \(\frac{3}{4}\).
3. Add: 0·82

4. 0·009 × 200.
5. 0·2 ÷ 25.
6. Express 0·05 as a fraction.
7. 0·625s., in pence.
8. Complete \(\frac{2}{7} = \frac{6}{\underline{...}} = \frac{8}{35} = \frac{2}{...}\).
9. Area of mat, 1·2 m. long, 0·9 m. wide?
10. Taking 1 yd. = 0·915 m., express 100 yd. in km.

Oral Practice 12 (3 min.)

1. \(\frac{3}{4}\) of £2, in £ s. d.
2. \(\frac{3}{8} + \frac{1}{2}\).
3. Add: 0·37

4. 0·45 × 0·4.
5. 0·108 ÷ 12.
6. \(\frac{3}{8}\) as a decimal to 2 places.
7. 0·85 ton, in cwt.
8. What must be added to \(\frac{3}{8}\) to make \(\frac{5}{8}\)?
9. Taking 1 litre = 1\(\frac{3}{4}\) pints, express 4 l. in qt., pt.
10. After spending \(\frac{1}{3}\) of my money, I have 2s. left; what have I spent?

Oral Practice 13 (4 min.)

1. \(\frac{3}{4}\) of 2 yd., in yd., ft., in.
2. \(\frac{3}{10} - \frac{3}{16}\).
3. 10 - 0·08 - 0·02.
4. 0·625 × 0·8.
5. 0·09 ÷ 0·4.
6. £0·375, in s. d.
7. Express 0·08 as a fraction.
8. The perimeter of a rectangle is 1 ft. 9 in.; its length is 7\(\frac{1}{2}\) in. What is its breadth?
9. Taking 1 m. = 1·0936 yd., express \(\frac{1}{2}\) km. in yards.
10. 1 c.c. of petrol weighs 0·85 gm.; what is the weight of 2 litres in kg.
1. 9 in. \times 1\frac{1}{2}, \text{ in ft., in.} \quad 2. \quad \frac{1}{4} + \frac{1}{6} + \frac{1}{3}.
3. Add: 0.35 and then subtract 0.8 from the sum. 1.65
4. 0.2 \times 0.4 \times 0.6. \quad 5. \quad 0.297 \div 1.1. \quad 6. \quad \text{Express 13s. as a decimal of £1.}
7. Subtract 5 sq. dm. from 1 sq. m.; answer in sq. dm.
8. By what must \frac{1}{3} be multiplied to make 1\frac{1}{3}?
9. A pile of 200 sheets is 3.5 cm. high; what is the thickness of
   1 sheet? Answer in mm.
10. Taking 1 pt. = 0.568 litre, express 1 qt. in c.c.

Oral Practice 16 (4 min.)
1. \frac{3}{16} of £2, in s. d. \quad 2. \quad 1\frac{1}{2} \times 1\frac{1}{3}. \quad 3. \quad \text{Add: 0.109}
   \quad 0.083 \quad 0.776
4. 0.1 \times 0.02 \div 0.5. \quad 5. \quad \frac{3}{4} as a decimal to 2 places.
6. Express 3 kg. 45 gm. in kg.
7. Volume of a rectangular block, 4.5 cm. by 2 cm. by 1.5 cm.?
8. Zinc is 7 times as heavy as water; what is the weight of
   1.5 cu. dm. of zinc in kg.
9. 12 articles are sold for 5s. 9d.; how much is this per article?
10. \frac{2}{9} of a barrel contains 10 gall.; how much will the barrel hold?

Oral Practice 16 (4 min.)
1. \frac{1}{3} of 1 guinea in s. d. \quad 2. \quad 1\frac{1}{2} - \frac{1}{3} - \frac{1}{4}. \quad 3. \quad \text{Subtract: 0.104}
   \quad 0.095
4. (0.03)\(^2\). \quad 5. \frac{15}{13} dm. + 52 cm., in m.
6. 1\frac{1}{2} as a decimal to 2 places.
7. By what must 0.004 be multiplied to make 2?
8. How many articles at 2d. each can be bought for 1s., and how
   much money is over?
9. If copper is 9 times as heavy as water, how much c.c. of copper
   weigh 450 gm.
10. A photograph is mounted on a card 10 in. high, 8 in. wide,
    so that there is a margin \frac{1}{2} in. wide all the way round. What
    is the area of the photograph?
TESTS IN COMPUTATION

4. Express 3-2794 cwt. in cwt., lb., oz., to the nearest oz.
5. What must be added to the sum of 5 Hg, 7 Dg., 6 Dg., 8 gm.; 487 dg. to make 1 Kg. Answer in Kg.
6. How many pieces of tape each 8-5 cm. long can be cut from a length of 4 m., and how much remains?

Test 5

1. (i) $4\frac{1}{2} - 1\frac{3}{8} - 1\frac{1}{8}$; (ii) $1 - \frac{3}{4} + \frac{2}{3}$.
2. (i) $0.475 \times 0.0604$; (ii) $6.426 \div 0.315$.
3. Express 3 fur. 7 ch. 11 yd. as a fraction of 21 mi.
4. Find correct to 3 places of decimals the value of $\frac{1}{2} \times 3 + \frac{4}{5}$.
5. If 1 gallon = 4-546 litres, find the capacity in c.c. of 3 pt., to the nearest c.c.
6. From a sheet of tin 1 m. square, a rectangular portion 35 cm. by 8 cm. is removed. Find in sq. m. the area of what remains.

Test 6

1. (i) $6\frac{3}{4} - 1\frac{2}{3} - 2\frac{1}{9}$; (ii) $(3\frac{3}{4} - 1\frac{1}{6}) \div (2\frac{2}{3} - 1)$.
2. (i) $0.6067 \times 90.8$; (ii) $0.00972 \div 13.6$.
3. Express 1 qr. 17 lb. 8 oz. as a fraction of 1 cwt.
4. Evaluate $0.375$ of £3 + $\frac{3}{4}$ of £2 10s. + £5 67s.
5. Given 1 mi. = 1609 km., express 1 km. in miles, to 3 places of decimals.
6. A wooden cuboid 7 cm. by 6 cm. by 5 cm. weighs 0-1281 kg.; find in gm. the weight of 1 c.c. of the wood.

Test 7

1. (i) $2\frac{3}{10} - 1\frac{2}{8} + 2\frac{3}{8}$; (ii) $4\times \frac{1}{3} - 6\frac{1}{3} \div 3$.
2. (i) $0.5075 \times 0.00008$; (ii) $2.01 \div 0.00375$.
3. Express 17 yd. 2 ft. 5 in. as a decimal of 133 ch., to 4 places.
4. Find to the nearest penny the value of £4 17s. 5d. $\times 2.37$.
5. Simplify $(1\frac{3}{4} \times 10\frac{2}{3} - 10\frac{2}{3}) \div (1\frac{3}{4} - 3\frac{1}{2} \div 2\frac{1}{3})$.
6. Given 1 kg. = 2-205 lb., express 67-9 gm. in oz., to 2 places of decimals.

REVISION PAPERS

PAPERS 1–8 (Elementary work)

Paper 1

1. What number multiplied by 17 gives 5253?
2. How many years are there from the beginning of A.D. 849 to the end of A.D. 1934?
3. I buy 3 articles for 24d., 3½d., 4½d. respectively; what change do I receive from one shilling?
4. (i) Divide £78 12s. by 16. (ii) Multiply £108 9s. 5d. by 120.
5. A charabanc takes 45 passengers; how many passengers can be carried by 18 charabans?
6. If £1 is divided equally between some boys, each gets 1s. 4d.; how many boys are there?

Paper 2

1. (i) Add: 1278; 809; 5914; 7048; 35; 6143. (ii) Subtract 8946 from one hundred thousand.
2. The product of two numbers is 4301; two of them are 11 and 17, find the third number.
3. In 1933 there were 139 wet days in Winchester; how many more days were dry than wet?
4. A rectangular field is 436 yards long, 378 yards wide. Find the length in chains of the fence which encloses it.
5. Find the cost of 19 chairs at 16s. 4d. each.
6. A roller makes 118 revolutions in travelling 1298 feet. Find its circumference.

Paper 3

1. What is the excess of the sum of 785 and 439 over the difference between 901 and 208?
2. How many 3d. articles can be bought for £2 11s. 9d.?
3. Oranges are sold at 8 for sixpence. What is the price of 4 dozen, of 1 gross (12 dozen)?
4. To-day is Friday. What day of the week will it be in 900 days time? What day of the week was it 900 days ago?
5. A wheel is 16 ft. in circumference. How many revolutions does it make in travelling 4 furlongs?
6. An advertisement claims that a car runs 35 miles on 1 gallon of petrol, but actually it only does 28 miles to the gallon. How much more petrol is required for 1960 miles than would be needed if the claim were correct?

Paper 4

1. What must be taken from the sum of 9073 and 6284 to leave 10008?
2. Multiply 7084 by 3102. Divide the product by 121.
3. A man buys 150 shirts for £96 and sells them at 18s. 9d. each. Find the profit.
4. How many 1½d. stamps can be bought for £4 17s. 3d.?
5. How many pieces of string, each 20 in. long, can be cut from a length of 75 yd.?
6. A school contains 585 pupils. If there are 29 more girls than boys, find the number of boys.

Paper 5

1. (i) Write down the odd multiples of 19 which are less than 120.
   (ii) Find the quotient and remainder when 84184 is divided by 231.
2. Write down the cost of 1 dozen articles (i) at 5½d. each, (ii) at 7¾d. each, and the cost of 240 articles at 11½d. each.
3. A grocer buys 45 cases each containing 56 lb. of tea and makes the tea up into 7-lb. packets. How many packets will there be, if there is no wastage?
4. How many tickets at 1s. 8d. each can be bought for £3 15s.?
5. A man walks 175 yd., taking strides of 30 in. each. How many fewer steps would he take if he increased his stride by 5 in.?
6. In a trial lasting one minute, a car travels at the rate of 9 yd. a second for 18 seconds, then 17 yd. a second for 26 seconds, and 23 yd. a second for the rest of the time. How far did it go in the minute?

Paper 6

1. The populations of the seven parishes into which a town is divided are 7219, 5863, 6015, 3186, 9074, 8117, 4928 respectively. How much is the total population short of 45,000?
2. Multiply £2 14s. 10d. by 43.
3. I buy 18 tons of coal and burn 12 cwt. of coal a week. How long will the coal last?
4. A hurdle is 8 ft. long. How many are required to enclose a rectangular piece of ground 136 yd. long, 112 yd. wide?
5. How many cigars at 1s. 9d. each can be bought for £2, and how much money remains?
6. 38 cars are parked side by side with 2 ft. between each and the next. If the breadth of each car is 3 ft., find in yards the total breadth of the row of cars, from one end of the row to the other end.
Revised Papers

Paper 7

1. Add: 507163, 294308, 707195, 8259, 30574.

2. (i) Subtract £8 13s. 6d. from £14 8s. 5½d.
   (ii) Divide £37 3s. 9d. by 21.

3. A man is paid 2s. 7½d. an hour ordinary time and 3s. 10d. an hour overtime. What does he get altogether for 10 hours work of which 2 hours is reckoned overtime?

4. The rungs of a 21-rung ladder are 15 in. apart, and the top and bottom rungs are 1 ft. from the ends. Find the length of the ladder in feet.

5. A man buys 6000 articles at 2½d. each and sells them at 3¾d. each. What profit does he make?

6. A 12-acre field is cut up into allotments each of area 950 sq. yd. How many allotments are there and what area is left over? (1 acre = 4840 sq. yards)

Paper 8

1. What must be added to the difference between 15004 and 9007 to make ten thousand?

2. A man's pace is 30 in. How many paces does he take in walking 1 mile?

3. The total amount required to pay 4 bills is £241 3s. 1d.; three of the bills are £18 12s. 9d., £74 17s. 8d., £94 9s. 10d.; how much is the fourth bill?

4. A man starts with a salary of £250 a year and receives an increase of £18 a year after each year. What is his salary for (i) his 5th year, (ii) his 20th year?

5. If a lorry runs 15 mi. on a gallon of petrol, and if the petrol costs 1s. 2d. a gall., find the cost of the petrol for a journey of 330 mi.

6. A man buys a consignment of brushes for £7 17s. 6d. and makes a profit of £5 12s. 6d. by selling them at 4½d. each. How many brushes did he buy?

Paper 9

1. Find in prime factors the L.C.M. of 270 and 504.

2. (i) By how many yards is 4 mi. short of 8000 yd.?
   (ii) How many chairs 21 in. wide can be put in a row 14 ft. long?

3. If 5 lb. of tea cost 8s. 4d., find the cost of 3 lb. Find also the cost of 1 ton.

4. The area of a rectangular field, 385 yd. long, is 14 ac. Find its breadth.

5. A cheap return ticket is issued for the price of a single ticket and a quarter. What is the cost of a cheap return ticket for a journey for which the single ticket costs £1 7s. 8d.?

6. The diagram represents the cross-section of a girder 5 ft. long; the corners are right-angled and the dimensions are shown in inches. Find (i) the area of the cross-section in sq. in., (ii) the volume of the girder in cu. ft., cu. in.

Paper 10

1. Find by prime factors the square root of 2304.

2. (i) Express 12475 oz. in cwt., lb., oz.
   (ii) How many times is 6 cwt. 3 qr. contained in 14 tons 17 cwt.?

3. Lace costs 10d. per yard, find the cost of a piece 11 ft. 6 in. long.

4. A car uses 15 gal. of petrol for a journey of 360 mi. How much does it use for 312 mi. at the same rate and how much does this amount of petrol cost at 1s. 5d. per gallon?

5. Find the area of a gravel path 8 ft. wide which runs all round a rectangular lawn 54 ft. long, 40 ft. wide.

6. How many rectangular blocks measuring 3 in. by 4 in. by 5 in. can be packed in a box measuring 4 ft. by 5 ft. by 6 ft. internally? Find also their weight in cwt. if the blocks weigh 28 lb. per cu. ft.
Paper 11

1. (i) Add: 6 fur. 7 ch. 18 yd.; 5 fur. 6 ch. 14 yd.; 7 fur. 3 ch. 8 yd.; 4 fur. 4 ch. 16 yd.
   (ii) Divide 77 tons 12 cwt. 2 qr. by 45.

2. A man’s income is £985 a year. How much does he save in a year in which he spends £591 more than he saves?

3. Find the cost of a carpet 5 yd. long, 4 yd. wide, at 2s. 6d. per sq. ft.

4. (i) 8 peaches cost 3s.; find the cost of 10 peaches.
   (ii) A man walks 110 yd. a minute; how long will he take to walk 15 mi.?

5. A truck is loaded with 624 boxes each weighing 35 lb.; find the weight of the load in tons, cwt.

6. The diagram represents the cross-section of a girder; the corners are right-angled and the dimensions are shown in inches. Find (i) the area of the cross-section in sq. in., (ii) the volume of a 4-ft. length of the girder in cu. ft., cu. in.

Paper 12

1. Find the cube root of 46656.

2. (i) Reduce 5 tons 7 cwt. 36 lb. to lb.
   (ii) Reduce 9280 yd. to mi., ch., yd.

3. Three whole tickets for a certain journey cost £1 14s. 6d.; find the cost of 6 tickets, one of which is half-price, for the same journey.

4. If the price of ribbon is lowered by 1/3d. a yard, find the decrease in price per mile.

5. A room 20 ft. long, 16 ft. wide, is carpeted so as to leave a border 2 ft. wide all round the edge. Find the cost of the carpet at 7s. 6d. per sq. yd. Find also the cost of covering the margin at 10½d. per sq. ft.

6. A block of wood measuring 1 ft. 4 in. by 1 ft. 3 in. by 1 ft. 6 in. is cut down into the form of a cube of edge 1 ft. 2 in.; how many cu. in. of wood are cut away?

Paper 13

1. Express 10584 in prime factors. What is the least number by which it must be multiplied to give (i) a perfect square, (ii) a perfect cube?

2. (i) Subtract 6 ch. 17 yd. 2 ft. from 1 fur. 15 yd.
   (ii) Divide 1028 tons 14 cwt. 1 qr. by 47.

3. A train travels 36 mi. in 45 min.; how far will it go in an hour and a quarter at the same rate?

4. Find the difference between the weight of 215 boxes each 1 lb. 14 oz. and the weight of 197 boxes each 2 lb. 3 oz.

5. A rectangular tin box with a lid is 7 in. long, 3 in. wide, 2 in. high. Find the area of tin sheeting used to make the box.

6. A metal block is 4 ft. long, 3 ft. wide, 16 in. high. If the metal weighs 490 lb. per cu. ft., find the weight of the block in tons, cwt.

Paper 14

1. Express $12 \times 16 \times 21 \times 28$ as the product of powers of prime numbers. Find its square root.

2. (i) How many times is £1 5s. 8d. contained in £29 10s. 4d.
   (ii) Multiply 13 yd. 2 ft. 10 in. by 47. (Answer in ch., yd., etc.)

3. A machine, kept working continuously, crushes 45 tons of ore a day; how many lb. of ore per minute is this?

4. I have enough money to take a three weeks’ holiday if I spend 25 shillings a day. How long a holiday can I take if I spend 35 shillings a day?

5. The cover of a safety matchbox is 7 cm. long, 4 cm. wide, 2 cm. high, and its ends are open. Find the area of the exterior surface of the cover.

6. The volume of a rectangular block is 1680 cu. in. (i) Find its height if it is 15 in. long, 7 in. wide. (ii) Find the area of the base if it is 1 ft. high.

Paper 15

1. Find the H.C.F. of 1512 and 6468.

2. (i) Express 70,000 sq. in. in sq. yd., sq. ft., sq. in.
   (ii) What is the smallest amount that must be added to £100 to obtain an exact multiple of £1 17s. 3d.?
3. 24 men can repair a road in 15 days. How long will it take 20 men to do so if all work at the same rate?

4. A man buys articles at 3s. 8d. per dozen and sells them at 5½d. each. Find the profit on the sale of 100 dozen. Find also the profit if he buys 50 dozen but is only able to sell 45 dozen of them.

5. Find the area of the ground plan of the house represented by the diagram; the corners are right-angled and the dimensions are shown in yards.

6. A rectangular tank, 12 ft. by 10 ft., and 7 ft. deep, contains water to a depth of 5 ft. How many blocks of cement, 4 in. by 4 in. by 3 in., can be put into the tank before the water overflows?

Paper 18

1. Lines are drawn so as to divide a rectangle 31 ft. 6 in. long, 25 ft. 8 in. wide into equal squares. Find the least possible number of squares.

2. (i) How many days are there in the years 1931 to 1940, inclusive?
   (ii) Find the cost of 17 tons of coal at £2 8s. 4d. per ton.

3. How many rails 32 ft. long are required for a double-track railway (i.e. with up-lines and down-lines) between two places 12 mi. apart?

4. On a map, 2 in. represents 1 mile. What area on the map represents 20 sq. mi. How many acres are represented by an area of half a square inch on the map?

5. A rectangular lawn 45 yd. long, 22 yd. wide, is surrounded by a path 4 ft. 6 in. wide. Find the area of the path in sq. yards. If a wall 5 ft. high runs along the outer edge of the path, find the area of the interior surface of the wall.

6. A closed box is 2 ft. long, 1 ft. 3 in. wide, 11 in. high, and is made of wood half an inch thick. Find the volume of the wood in cu. inches.

Paper 17

1. In 1932, the B.B.C. received £1,179,031 from licences and £237,834 from publications. The expenditure was £657,935, and in addition £50,000 was contributed to the Treasury. What balance remained?

2. (i) Simplify 3½ - 1¾ of 1½ + 2½.
   (ii) Express 8s. 9d. as a fraction of £1.

3. (i) Express 0·0325 as a vulgar fraction.
   (ii) Evaluate 0·0755 × 1·84.

4. A man motors 80 mi. in 2 hr. 40 min.; find his average speed in miles per hour.

5. Taking 1 yd. = 0·9144 m., express 1 ft. 5 in. in cm., correct to one place of decimals.

6. A beam of timber is 20 ft. long, 8 in. wide, 9 in. deep; the timber weighs 48 lb. per cu. ft. Find the weight of the beam in lb., and express it as a decimal of a ton, to 3 places.

Paper 18

1. (i) Simplify 2½ + 3½ - 1¼.
   (ii) After a boy has spent ¾ of his money, he has 7s. left; how much had he at first?

2. (i) Express 1000 as a decimal. (ii) Evaluate 0·18513 ÷ 30·6.

3. (i) Express £4·8875 in s. d.
   (ii) A field is 66 yd. long, 52 yd. wide; express its area as a decimal of 1 ac., to 3 places of decimals.

4. Find the cost of 5 yd. 40 cm. of ribbon at 1 fr. 75 c. per metre.

5. If 1 rupee = 1s. 4¼d., find the number of rupees that can be bought for £13 10s. 10d.

6. The diagram represents the cross-section of an iron bar 2 ft. long. The corners are right-angled and the dimensions are shown in inches. Find (i) the area of the cross-section, (ii) the weight of the bar if the iron weighs 0·32 lb. per cu. in.
Paper 19

1. Write down a rough estimate of the value of (i) £140.8 ÷ 7; (ii) £4 19s. x 201.

2. (i) Simplify \((\frac{3}{4} - \frac{1}{8})(\frac{4}{5} - \frac{3}{8})\) of \(1\frac{1}{8}\).
   (ii) Express 3s. 9d. as a fraction of 8s. 3d.

3. (i) Express in metres the sum of 6 dm. 5 cm. 8 mm.; 9 dm. 8 cm. 4 mm.; 3 dm. 7 cm. 5 mm.; 206 mm.; 46 cm.
   (ii) Evaluate 0.6093 x 17.82.

4. (i) A car uses \(\frac{7}{8}\) gal. of petrol for 180 mi. How much does it use for 160 mi. at the same rate?
   (ii) A chair weighs 21 lb.; how many can be taken by a lorry that is not allowed to carry more than 30 cwt.?

5. Divide £750 between A, B, C so that A gets £30 more than B, and B gets £15 more than C.

6. Find a man's income in 1934 if he spent 11s. 8d. per day and saved £10 9s. 2d. per month.

Paper 20


2. (i) Simplify \(3\frac{1}{2} \times 2\frac{1}{2} \times 1\frac{1}{2} + (6\frac{1}{2} + 4\frac{1}{2})\).
   (ii) How many \(\frac{4}{5}\)-lb. packets can be made up from 60 lb. of tea?

3. (i) Simplify 4.807 - 2.093 - 1.836 + 5.325.
   (ii) Divide 0.1428 by 25.5.

4. A man takes 3\(\frac{1}{2}\) hr. to bicycle a certain distance at 12\(\frac{1}{2}\) mi. per hour; how long will the same journey take in a car which goes 40 mi. per hour faster than the bicycle?

5. A workman is paid 1s. 5d. per hour. He works 9 hr. on 5 days a week and 6 hr. on Saturday. How much does he earn per week? If he works 7 hr. overtime he increases his weekly earnings to £4 6s. 3d.; at what rate per hour is he paid for overtime?

6. The diagram represents the cross-section of a girder, 2\(\frac{1}{2}\) m. long; the corners are right-angled and the dimensions are shown in cm. Find (i) the area of the cross-section, (ii) the volume in cu. dm.
Paper 23

1. (i) Simplify \((1\frac{1}{4} + 3\frac{3}{4})(3\frac{1}{2} - 2\frac{1}{2}) \div (2\frac{1}{4} - 3\frac{1}{4})\).
   (ii) A field of area 14\(\frac{1}{2}\) ac. is divided into allotments each of area \(\frac{1}{3}\) ac. How many allotments are there?

2. (i) Evaluate \(0.16 \times 0.015 \times 0.0625\).
   (ii) Express 13s. 10d. as a decimal of £1, correct to 3 places.

3. The escalator at the Holborn tube station is 156 ft. long and makes the ascent in 65 sec. Find the speed in miles per hour.

4. A headmaster is paid a fixed salary of £800 a year and a capitation fee of £3 10s. on each boy in the school after the first 50. There are 382 boys; how much does he receive altogether?

5. A box without a lid is 1 ft. long, 8\(\frac{1}{2}\) in. wide, 7 in. high, external measurements, and is made of wood \(\frac{1}{4}\) in. thick. Find the volume of the wood used for making the box.

6. Two men, A and B, measure the length of a fence with their walking-sticks. A's stick is 37\(\frac{1}{2}\) in. long and he finds that the fence is between 52 and 53 times as long; B's stick is 34\(\frac{3}{4}\) in. and he finds that the fence is between 57 and 58 times as long. Find in yd., in., the limits between which the length of the fence lies.

Paper 24

1. The number of bunches of bananas imported in 1930 into the United Kingdom was as follows: British West India 5,923,877; Honduras 984,801; Colombia 3,352,002; Costa Rica 2,189,103; other countries 2,539,781. Find the total number imported. If the average weight of a bunch is 34 lb., find the total weight, to the nearest million lb.

2. (i) Simplify \(4\frac{1}{4} \times 8\frac{3}{4} \times 2\frac{1}{2} \div (4\frac{1}{4} + 2\frac{1}{2} - 5\frac{3}{4})\).
   (ii) How many pieces of tape 3\(\frac{1}{4}\) in. long can be cut from a length of 5 yd., and how much remains?

3. (i) Divide 803.76 by 63.04.
   (ii) Express 1 ft. 7 in. as a decimal of 1 yd., correct to 3 places.

4. If 1\(\frac{1}{4}\) tons of coal cost £3 15s., find the cost of 2\(\frac{3}{4}\) tons.
PART II

CHAPTER X

FRACTIONAL PARTS AND PRACTICE

The method for expressing one quantity as a fraction of another quantity of the same kind was explained in Chapter IV, see p. 34; the following exercise is intended merely for revision:

EXERCISE 72 (Oral)

Simplify:

1. \( \frac{1}{3} \) of 1s.  
2. \( \frac{1}{4} \) of £1.  
3. \( \frac{1}{7} \) of £1.  
4. \( \frac{1}{8} \) of 1 ft.
5. \( \frac{1}{5} \) of 1 ton.  
6. \( \frac{1}{6} \) of 1 yd.  
7. \( \frac{1}{5} \) of 1 s.  
8. \( \frac{1}{7} \) of 1 hr.
9. \( \frac{1}{8} \) of 1 cwt.  
10. \( \frac{1}{9} \) of 1 gall.  
11. \( \frac{1}{10} \) of 1 ch.  
12. \( \frac{1}{11} \) of 1 mi.
13. \( \frac{1}{12} \) of £1.  
14. \( \frac{1}{13} \) of 2s. 6d.  
15. \( \frac{1}{14} \) of £1.  
16. \( \frac{1}{15} \) of £1.

Express the first quantity as a fraction of the second:

17. 4d.; 1s.  
18. 5s.; £1.  
19. 3d.; 1s.
20. 9 in.; 1 yd.  
21. 14 lb.; 1 cwt.  
22. 1\frac{1}{2} d.; 1s.
23. 1\frac{1}{2} d.; 6d.  
24. 1\frac{1}{2} in.; 1 ft.  
25. 2s. 6d.; £1.
26. 6s. 8d.; £1.  
27. 440 yd.; 1 mi.  
28. 3s. 4d.; £1.

Example 1. Find the value of \( \frac{3}{4} \) of £8 5s. 8d.

Divide £8 5s. 8d. into 7 equal shares and then find the value of 3 of these shares:

\[ \frac{3}{4} \text{ of } £8 \text{ 5s. 8d.} = £1 \text{ 3s. 8d.} \]

\[ \frac{3}{7} \text{ of } £8 \text{ 5s. 8d.} = (£1 \text{ 3s. 8d.}) \times 3 = £11 \text{ 1s.} \]

EXERCISE 73

Find the value of:

1. \( \frac{1}{3} \) of 1s.  
2. \( \frac{1}{4} \) of £1.  
3. \( \frac{1}{5} \) of £1.  
4. \( \frac{1}{6} \) of £1.
5. \( \frac{1}{7} \) of £2.  
6. \( \frac{1}{8} \) of 10s.  
7. \( \frac{1}{9} \) of 2s. 6d.  
8. \( \frac{1}{10} \) of 10s.
9. \( \frac{3}{8} \) of 1 lb.  
10. \( \frac{5}{6} \) of 1 ton.  
11. \( \frac{2}{3} \) of 1 yd.  
12. \( \frac{3}{8} \) of 1 mi.  
13. \( \frac{3}{5} \) of 1 hr.  
14. \( \frac{2}{3} \) of 15 gall.  
15. \( \frac{2}{3} \) of 1 ft.  
16. \( \frac{2}{3} \) of 1 lb.  
17. \( \frac{1}{4} \) of £2 10s. 6d.  
18. \( \frac{1}{4} \) of £3 8s. 10d.  
19. \( \frac{1}{2} \) of £5 5s.  
20. \( \frac{1}{4} \) of £5 11s. 9d.  
21. \( \frac{1}{2} \) of 5 yd. 2 ft. 8 in.  
22. \( \frac{1}{2} \) of 7 tons 3 cwt. 3 qr.  
23. \( \frac{3}{4} \) of £4 2s. 6d.  
24. \( \frac{3}{4} \) of £8 8s. 5\(\frac{1}{2}\)d.  
25. Subtract \( \frac{3}{4} \) of 5s. 10d. from \( \frac{3}{4} \) of 2s. 8d.  

Simple Practice  

Example 2. Find the cost of 55 note-books at \( \frac{7}{4} \)d. each.  

<table>
<thead>
<tr>
<th>6d. = ( \frac{1}{3} ) of 1s.</th>
<th>1(\frac{1}{4})d. = ( \frac{1}{4} ) of 6d.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost at 1s. each</td>
<td>Cost at 6d. each</td>
</tr>
<tr>
<td>£ 2</td>
<td>15</td>
</tr>
</tbody>
</table>

\( \therefore \) the cost is £1 14s. 4\(\frac{1}{2}\)d.

Note. A line is drawn below £2 15s. to prevent its being added to the total cost.

Example 3. Find the cost of 37 tons of coal at £2 7s. 6d. per ton.  

<table>
<thead>
<tr>
<th>5s. = ( \frac{1}{4} ) of £1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost at £1 per ton</td>
</tr>
<tr>
<td>£ 37</td>
</tr>
</tbody>
</table>

Cost at £2 7\(\frac{1}{2}\)d. per ton \( \frac{3}{4} \) of 1s.  

\( \therefore \) the cost is £87 17s. 6d.

The explanations on the left of the lines drawn down the page in Examples 2, 3 need not be written down, and those to the right of it may be shortened. This also applies to Examples 6–12.

In these examples, the compound quantity is broken up into a set of aliquot parts of the principal unit, that is fractions having 1 as numerators.

Example 4. If the unit is 1s., show how to break up 10d. into the sum of aliquot parts or aliquot parts of aliquot parts.

10d. may be taken as the sum of  
6d. = \( \frac{3}{4} \) of 1s.  
3d. = \( \frac{1}{4} \) of 1d.  
1d. = \( \frac{1}{4} \) of 6d.  

or more shortly  
6d. = \( \frac{3}{4} \) of 1s.  
3d. = \( \frac{1}{4} \) of 1s.  
1d. = \( \frac{1}{4} \) of 1s.

Example 5. If the unit is £1, show how to break up 6s. 3d. into the sum of aliquot parts or aliquot parts of aliquot parts of aliquot parts.

6s. 3d. may be taken as the sum of  
5s. = \( \frac{1}{4} \) of £1 (leaves 1s. 3d.)  or more shortly  
1s. = \( \frac{1}{4} \) of 5s. (leaves 3d.)  
3d. = \( \frac{1}{4} \) of 1s.  

EXERCISE 74 (Oral)  

Break up into the sum of aliquot parts or aliquot parts of aliquot parts of the given unit:  

<table>
<thead>
<tr>
<th>Nos.</th>
<th>1–5</th>
<th>unit 1s.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 9d.</td>
<td>2. 5d.</td>
<td>3. 7(\frac{1}{4})d.</td>
</tr>
<tr>
<td>Nos. 6–21</td>
<td>unit 1s.</td>
<td></td>
</tr>
<tr>
<td>6. 11s.</td>
<td>7. 11s. 6d.</td>
<td>8. 11s. 9d.</td>
</tr>
<tr>
<td>10. 12s. 6d.</td>
<td>11. 16s.</td>
<td>12. 16s. 4d.</td>
</tr>
<tr>
<td>14. 6s. 6d.</td>
<td>15. 6s. 4d.</td>
<td>16. 4s. 8d.</td>
</tr>
<tr>
<td>18. 8s. 4d.</td>
<td>19. 4s. 7(\frac{1}{4})d.</td>
<td>20. 12s. 4(\frac{1}{2})d.</td>
</tr>
<tr>
<td>Nos. 22–25</td>
<td>unit 1 ton.</td>
<td></td>
</tr>
<tr>
<td>22. 14 cwt.</td>
<td>23. 6 cwt. 1 qr.</td>
<td>24. 17 cwt. 2 qr.</td>
</tr>
<tr>
<td>Nos. 26–29</td>
<td>unit 1 yd.</td>
<td></td>
</tr>
<tr>
<td>26. 1 ft. 4 in.</td>
<td>27. 1 ft. 9 in.</td>
<td>28. 10(\frac{1}{2}) in.</td>
</tr>
<tr>
<td>Nos. 30–33</td>
<td>unit 1 cwt.</td>
<td></td>
</tr>
<tr>
<td>30. 2 qr. 16 lb.</td>
<td>31. 3 qr. 11 lb.</td>
<td>32. 1 qr. 5 lb.</td>
</tr>
</tbody>
</table>

EXERCISE 75

Find by Practice the cost of the following articles:—  

| 1. 26 at 9d. each. | 2. 27 at 5d. each. |
| 3. 68 at 4\(\frac{1}{4}\)d. each. | 4. 94 at 7\(\frac{1}{4}\)d. each. |
| 5. 85 at 1s. 8d. each. | 6. 31 at 2s. 7d. each. |
| 7. 43 at £2 5s. each. | 8. 67 at £1 15s. each. |
| 9. 72 at £1 12s. each. | 10. 49 at £2 6s. each. |
| 11. 65 at £5 4s. 8d. each. | 12. 86 at £3 12s. 6d. each. |
| 13. 92 at £4 6s. 8d. each. | 14. 74 at £2 13s. 4d. each. |
| 15. 87 at £7 3s. 9d. each. | 16. 66 at £5 6s. 3d. each. |
FRACTIONAL PARTS AND PRACTICE

Find by Practice the weight of the following:—
17. 35 books, each 9 oz. (in lb., oz.).
18. 47 boxes, each 21 lb. (in qr., lb.).
19. 29 boxes, each 18 lb. (in qr., lb.).
20. 38 jars, each 2 lb. 5 oz. (in lb., oz.).
21. 58 loads, each 1 ton 12 cwt.  [22] 76 loads, each 3 tons 9 cwt.

Work by Practice methods:
23. 1 ft. 9 in. × 85 (in yd., ft., in.).
24. 2 yd. 2 ft. 8 in. × 74 (in yd., ft., in.).
25. 2 hr. 25 min. × 52 (in hr., min.).
26. 3 cwt. 70 lb. × 39 (in cwt., lb.).

Example 6. Find the cost of 31 lb. at 17s. 10d. per lb.

<table>
<thead>
<tr>
<th>£</th>
<th>s.</th>
<th>d.</th>
</tr>
</thead>
<tbody>
<tr>
<td>10s. = 2 of £1</td>
<td>10s.</td>
<td>15</td>
</tr>
<tr>
<td>5s. = 2 of 10s.</td>
<td>5s.</td>
<td>7</td>
</tr>
<tr>
<td>2s. 6d. = 2 of 5s.</td>
<td>2s. 6d.</td>
<td>3</td>
</tr>
<tr>
<td>3d. = 2 of 2s. 6d.</td>
<td>3d.</td>
<td>7</td>
</tr>
<tr>
<td>1d. = 2 of 3d.</td>
<td>1d.</td>
<td>3</td>
</tr>
</tbody>
</table>

Cost at 17s. 10d. per lb. 28 3 0

Note. Draw a line below £31 10s. to show that it must not be added into the answer.

Example 7. Multiply £73 17s. 2d. by 157.

<table>
<thead>
<tr>
<th>£</th>
<th>s.</th>
<th>d.</th>
</tr>
</thead>
<tbody>
<tr>
<td>157 times £1</td>
<td>157 0 0</td>
<td></td>
</tr>
<tr>
<td>10s. = 2 of £1</td>
<td>10s. 0 0</td>
<td></td>
</tr>
<tr>
<td>5s. = 2 of 10s.</td>
<td>5s. 0 0</td>
<td></td>
</tr>
<tr>
<td>2s. 6d. = 2 of 5s.</td>
<td>2s. 6d. 0 0</td>
<td></td>
</tr>
<tr>
<td>3d. = 2 of 2s. 6d.</td>
<td>3d. 0 0</td>
<td></td>
</tr>
<tr>
<td>1d. = 2 of 3d.</td>
<td>1d. 0 0</td>
<td></td>
</tr>
</tbody>
</table>

157 times £73 17s. 2d. 11955 18 5

It is sometimes shorter to use a subtraction method.

Example 8. Find the cost of 185 yards at 11d. per yard.

<table>
<thead>
<tr>
<th>£</th>
<th>s.</th>
<th>d.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost at 1s. per yard</td>
<td>11d.</td>
<td>9 15 5</td>
</tr>
</tbody>
</table>

Subtract. Cost at 11d. per yard 8 9 7

SIMPLE PRACTICE

Example 9. Find the weight of 74 loads, each weighing 2 tons 18 cwt.

| Weight at 1 ton per load | 74 | 0  |
| 2 cwt. = 7 of 1 ton, | 3 | 0  |
| 2 cwt. | 7 | 8  |

Subtract. Weight at 2 tons 18 cwt. per load 214 12

EXERCISE 76

Find by Practice the cost of the following articles:
1. 48 at £2 16s. each.
2. 75 at £1 13s. each.
3. 86 at £3 12s. 9d. each.
4. 37 at £2 6s. 2d. each.
5. 184 at 16s. 4½d. each.
6. 272 at 8s. 4½d. each.
7. 57 at 4£ 9s. 3d. each.
8. 74 at £1 17s. 9½d. each.
9. 3700 at 8s. 7½d. each.
10. 150 at £1 13s. 2½d. each.

Find by Practice the total weight of the following:
11. 83 loads, each 5 cwt. 1 qr. 14 lb.
12. 156 loads, each 1 ton 12 cwt. 2 qr.
13. 190 loads, each 2 tons 16 cwt. 1 qr.
14. 854 loads, each 3 cwt. 1 qr. 20 lb.

Find by Practice the total length of the following:
*15. 345 rods, each 2yd. 2 ft. 5 in. long; answer in ch., yd., etc.
*16. 218 rolls of wire, each 2 ch. 13yd. 1 ft. long; answer in ch., yd., ft.

Find by Practice methods:
17. The cost of 265 tons of coke at 41s. 9d. per ton.
18. The cost of 70,000 mats at 2s. 7½d. each.
19. The cost of 58½ dozen articles at 4s. 6d. per dozen.

Use subtraction methods to find:
20. The cost of 87 yd. at 5½d. per yard.
21. The cost of 168 yd. at 2s. 11½d. per yard.
22. The weight of 63 boxes, each weighing 5 lb. 7 oz. (in lb., oz.).
*23. The weight of 275 boxes, each weighing 1 qr. 24 lb. (in cwt., qr., lb.).
*24. The cost of 1384 yd. at 1s. 10½d. per yard.
Compound Practice. The next example shows the process which is used when two compound quantities are involved.

Example 10. Find the cost of 5 tons 12 cwt. 2 qr. at £4 3s. 4d. per ton.

<table>
<thead>
<tr>
<th>Cost of 1 ton</th>
<th>£</th>
<th>s.</th>
<th>d.</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 tons</td>
<td>4</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>10 cwt. = ½ of 1 ton</td>
<td>20</td>
<td>16</td>
<td>8</td>
</tr>
<tr>
<td>2 cwt. = ½ of 10 cwt.</td>
<td>2</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>2 qr. = ½ of 2 cwt.</td>
<td>2</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>Cost of 5 tons 12 cwt. 2 qr.</td>
<td>23</td>
<td>8</td>
<td>9</td>
</tr>
</tbody>
</table>

In money sums, vulgar fractions should not be used in the pence column, unless only ½d., ¾d., or ¾d. occur. If the answer is required correct to the nearest penny, two places of decimals should be retained in the pence column in the working.

Example 11. Find, correct to the nearest penny, the cost of 3 tons 7 cwt. 3 qr. at £2 11s. 5d. per ton.

<table>
<thead>
<tr>
<th>Cost of 1 ton</th>
<th>£</th>
<th>s.</th>
<th>d.</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 tons</td>
<td>2</td>
<td>11</td>
<td>5</td>
</tr>
<tr>
<td>5 cwt. = ½ of 1 ton</td>
<td>7</td>
<td>14</td>
<td>3</td>
</tr>
<tr>
<td>2 cwt. = ½ of 5 cwt.</td>
<td>2</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2 qr. = ½ of 2 cwt.</td>
<td>1</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>1 qr. = ½ of 2 qr.</td>
<td>1</td>
<td>7</td>
<td>71</td>
</tr>
<tr>
<td>Cost of 3 tons 7 cwt. 3 qr.</td>
<td>8</td>
<td>14</td>
<td>2 08</td>
</tr>
</tbody>
</table>

: the cost is £8 14s. 2d. correct to the nearest penny.

Example 12. Find, correct to the nearest penny, the cost of 129 tons 11 cwt. 1 qr. at £2 6s. 2d. per ton.

<table>
<thead>
<tr>
<th>Cost of 1 ton</th>
<th>£</th>
<th>s.</th>
<th>d.</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 tons</td>
<td>2</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>120 tons</td>
<td>23</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>9 tons</td>
<td>277</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10 cwt.</td>
<td>20</td>
<td>15</td>
<td>6</td>
</tr>
<tr>
<td>1 cwt.</td>
<td>1</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>1 qr.</td>
<td>2</td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>Total cost</td>
<td>299</td>
<td>1</td>
<td>5 6 2</td>
</tr>
</tbody>
</table>

: the cost is £299 1s. 6d. correct to the nearest penny.

COMPOUND PRACTICE

EXERCISE 77

Find by Practice the cost of:

1. 3 tons 6 cwt. at £2 10s. per ton.
2. 5 lb. 11 oz. at £1 14s. per lb.
3. 2 tons 3 cwt. 2 qr. at £4 6s. 8d. per ton.
4. 5 cwt. 2 qr. 21 lb. at £4 10s. per cwt.
5. 7 yd. 1 ft. 8 in. at £2 7s. 3d. per yard.
6. 5 gallon 3 qt. 1 pt. at £1 11s. 4d. per gallon.
7. 2 tons 12 cwt. 2 qr. 8 lb. at 2s. 4d. per cwt.
8. 3 ch. 15 yd. 2 ft. at £48 2s. 6d. per chain.
9. 24 ac. 3 r. 19 p. at £38 per acre.

Find the cost to the nearest penny, the cost of:

10. 6 cwt. 3 qr. 7 lb. at £1 4s. 6d. per cwt.
11. 5 cwt. 2 qr. 21 lb. at 8s. 2½d. per cwt.
12. 4 tons 4 cwt. 1 qr. at £3 18s. per ton.
13. 10 cwt. 12 oz. at £2 7s. 9d. per quarter.
14. 7 yd. 2 ft. 5 in. at 15s. 4½d. per yard.
15. 8 ac. 3 r. 26 p. at £3 18s. 6d. per acre.
16. 4 tons 1 cwt. 3 qr. 17 lb. at £52 8s. 6d. per ton.
17. 15 tons 14 cwt. 60 lb. at £4 13s. 6d. per ton.

Find by Practice the dividend on:

18. £360 at 2s. 3d. in the £. [19] £248 15s. at 1s. 8d. in the £. Find by Practice the tax payable on:
20. £584 at 5s. 9d. in the £. 21. £174 at 13s. 8½d. in the £.

Find, correct to the nearest penny, the cost of:

22. 108 tons 13 cwt. 1 qr. at £2 3s. 2d. per ton.
23. 173 tons 11 cwt. 2 qr. at £3 14s. 8d. per ton.
24. 127 tons 6 cwt. 3 qr. 10 lb. at £2 7s. 8d. per ton.

MISCELLANEOUS EXAMPLES

EXERCISE 78

1. Find the cost of 150 handkerchiefs at 1s. 4d. each.
2. Find the cost of 2½ yd. of material at 4s. 10d. per yard.
3. The duty on tobacco is 13s. 4d. per lb.; find the duty on 4 lb. 12 oz.
4. Find the income tax on £420 at 4s. 6d. in the £.
5. Find the cost of 3 yd. 2 ft. of carpet at 8s. 10½d. per yard.
6. Find the cost of 360 coats at 18s. 11d. each.
7. Use the fact that 1s. per lb. is equivalent to £112 per ton to find the cost per ton which is equivalent to 10½d. per lb.
8. Multiply 13s. 8d. by 2⁵/₈ by a practice method. (8 = 8 + 3)
9. Find to the nearest penny the cost of 650 bulbs at 18s. per 1000.
10. Evaluate by a practice method 0·65 of £3 11s. 8d.
11. An agent receives a commission of £8 15s. on every £100 worth of orders he obtains. What is his commission on orders amounting to £136?
12. Find the cost of 92 yd. at 2s. 11½d. per yard.
13. What dividend is due on £180 at 3s. 10d. in the £?
14. A servant earns 18s. 6d. a week, but 9d. a week is deducted for health insurance. How much money does he receive for 52 weeks?
15. Taking 1 yd. = 0·9144 m., express 1 ft. 5¼ in. in cm., to the nearest mm.
16. A car travels 46 mi. an hour; how far, to the nearest 10 mi., does it go in 28 min. 40 sec.?
17. Find in cwt., lb., oz. the weight of 840 boxes, each of which weighs 1 lb. 11 oz.
18. Find the total cost of 1500 1½d. stamps and 1500 envelopes at 25s. per thousand.
19. £100 a year is equivalent approximately to 5s. 5½d. a day. Use this fact to find, to the nearest penny, what daily expenditure is equivalent to £165 a year.
20. A bankrupt's liabilities (i.e., what he owes) amount to £435, and he is only able to pay 8s. 6d. for every £ he owes. Find the amount of his assets (i.e., what he possesses).
21. How much is produced by a poor-rate of 3s. 2½d. in the £ on a rateable value of £12,850?
22. The duty on cigars is 17s. 11d. per lb. and on cigarettes is 14s. 5d. per lb. Find the total duty on 75 lb. of cigars and 135 lb. of cigarettes.
23. A man who insures his life at the age of 40 pays annually £3 18s. 10½d. for each £100 insurance. What is his annual payment on an insurance for £750?

CHAPTER XI

RATE, RATIO AND PROPORTION

Rate and Ratio. If a man walks 7 miles in 2 hours, we say that he walks at the rate of 3½ miles per hour, 3½ m.p.h. If a man earns 16s. for 3 hours' work, we say that his rate of pay is 5½s. per hour, 5s. 4d. per hour. In these examples, the word rate is used to state how a quantity is altering with the time; it is, however, also used in other ways: If I buy 3 lb. of tea at 5s., I pay at the rate of 1s. 8d. per lb.; if a laundry charges 2s. for washing 12 towels, the rate of charge is 2d. per towel.

In these examples the word rate is used in connection with two quantities of different kinds, it may of course also be used if the quantities are of the same kind: if the price of an article, usually sold for 12s., is reduced in a sale by 3s., the reduction is at the rate of "3d. in the shilling."

The sizes of two quantities of the same kind may be compared in two ways. The ages of two children might be compared by saying one is 9 months older than the other; the temperature of a sick person might be stated as 2 degrees above normal; in each case the comparison is made in the form of a difference. On the other hand, the scale of the model of a ship would be described as 1:100, if 1 foot length of the model represents 100 feet length of the ship; it would be valueless to say that the model is 198 ft. shorter than the ship. Here the comparison is made in the form of a ratio; that is, the fraction which the first quantity is of the second.

Suppose a school contains 150 boys and 200 girls, then the number of boys is 3/5 of the number of girls, and we say that the ratio of the number of boys to the number of girls is 3 to 4, written 3:4; and this ratio can be represented by the fraction 3/4.

Ratios should be expressed as simply as possible; just as the fraction 3/5 is reduced to 3/5, so the ratio 8:36 is equivalent to 2:9.
RATE, RATIO AND PROPORTION

Thus a ratio is unaltered if the two numbers (or quantities) of the ratio are both multiplied, or both divided, by the same number. For example,

\[ \frac{3}{4} : \frac{3}{4} = \frac{9}{12} : \frac{9}{12} = 10 : 9. \]

If the prices of two cars A, B are £450, £600 respectively,

\[
\begin{align*}
\text{price of A} &= \frac{\£450}{3} = £150 \\
\text{price of B} &= £600 = £400 \\
\text{price of A} &= \frac{\£450}{5} = £90
\end{align*}
\]

and we write

price of A : price of B = 3 : 4 and price of B : price of A = 4 : 3.

Conversely, the statement that the ratio of the price of A to the price of B is 3 : 4 means that the price of A is \(\frac{3}{4}\) of the price of B, and that the price of B is \(\frac{4}{3}\) of the price of A.

EXERCISE 79 (Oral)

Write the following rates in a simple form:—

1. 60 miles in 5 hours.

2. 18s. for 3 hours’ work.

3. 4 lb. of coffee for 6s.

4. 15s. for 12 tickets.

5. 2s. 6d. for washing 6 shirts.

6. £30 rent for 4 months.

7. 5 cu. ft. of wood weigh 210 lb.

8. 8 wickets for 92 runs.

9. 12 teachers for 180 pupils.

10. Income £1000, tax £200.

11. House rent £60, tax £18. 15 s. Ordinary price 9s., reduction 3s.

Express each of the following ratios as simply as possible in the form \(a : b\), and state what fraction the first quantity, or number, is of the second:—

13. A length of 8 in. to a length of 12 in.

14. A weight of 5 lb. to a weight of 10 lb.

15. A cost of 9d. per lb. to a cost of 1s. per lb.

16. 12 cm. to 4 cm.

17. 15 ft. to 10 ft.

18. 14 lb. to 18 lb.

19. 16 sq. in. to 36 sq. in.

20. 20 lb. to 5 lb.

21. 15s. to £2.

22. 18 to 36.

23. 32 to 24.

24. 5 to 25.


26. 30 : 90.

27. 0 : 1.

28. 7s. 6d. : £1.

29. 2 ft. 6 in. : 1 yd.

30. 4 cm. : 25 mm.

31. 50 m. : 1 km.

32. 5 gm. : 5 kg.

33. 3 pt. : 3 qt.

34. £1 : £1.

35. \(\frac{1}{3} : \frac{1}{2}\).

36. \(\frac{1}{4} : 2\).

37. 3 sq. ft. : 1 sq. yd.

38. 8 oz. : 1 lb.

39. \(\frac{1}{4}\) hr. : 45 min.

TAXATION AND BANKRUPTCY

40. A speed of 7½ mi. per hour to a speed of 12½ mi. per hour.

41. Wages at 1s. 3d. per hour to wages at 12s. for 8 hr.

42. A price of 2s. 3d. per lb. to a price of 2½d. per oz.

Taxation. The cost of the various national services, such as defence, the upkeep of the Army, Navy and Air Force, general administration carried out by the Civil Service, national debt interest, etc., is met by taxation, levied in accordance with the regulations contained in the Budget which is laid before the House of Commons each April by the Chancellor of the Exchequer.

Revenue is collected in many different forms, such as duties on commodities (customs and excise), special taxes (entertainment tax, motor tax, etc.), stamps on legal documents, death duties, etc., but the largest source is income tax and surtax.

The regulations for income tax change from year to year, but the general principle is that every person whose income exceeds £100 if unmarried, or £150 if married, pays a certain fraction of his income as tax, this fraction being small for small incomes and increasing as the income increases. For example, it might happen that a man whose income is £300 pays £15 tax, that is \(\frac{1}{20}\) of his income, while a man whose income is £1000 pays £150 tax, that is \(\frac{1}{4}\) of his income, and a man with an income of £4000 pays £1000 tax, that is \(\frac{1}{4}\) of his income.

Income tax is usually quoted in a form such as "4s. in the £"; this would mean that on the part of a man's income to which this rate refers, the tax is 4s. for each £ of income.

Those who live in towns enjoy certain advantages such as street-lighting, efficient drainage and sanitation, public parks, free libraries, etc. The cost of these municipal services and other charges such as poor relief, etc., is met by the Rates, that is a tax levied on the householders in the town; there is a similar tax in rural districts.

The principle on which this tax is levied is that a man who lives in a house of "annual value" £60 a year ought to pay twice as much as a man whose house has an annual value of only £30 a year. The annual value is not necessarily the same as the annual rent, but is fixed by periodic assessments under Act of Parliament in which allowance is made for cost of maintenance and other charges and is called the ratable value or assessed value. The tax varies with the locality, from as little as 6s. in the £ up to as much as 2s. in the £, or even more, for every £1 in the rateable value. For example, if a house, whose rent may be £60 a year, is assessed at £50 a year and if a rate of 14s. in the £ is levied, the tax for that year would be (14 x 50) s., that is £35.

Bankruptcy. If a business is being conducted at a loss, a time will some when the capital, that is the money which has been put into the business, has all been spent and the owner of the business is no longer able to pay his debts.
RATE, RATIO AND PROPORTION

If a creditor, that is a person to whom money is owed, is unable to get the money due to him, he can take legal proceedings and eventually, if the debt remains unpaid, the owner of the business will be "adjudicated a bankrupt." When this happens, all the owner's property, called his assets, is taken over by an official, called the Receiver, whose duty it is to pay the creditors as much of the amount due to them as is available.

Certain preferential claims must first be paid in full, if possible, such as rent, taxes and servants' wages; the remainder of the assets is then shared among the "unsecured creditors." Suppose, for example, the value of the remaining assets is £1000, and the liabilities, that is the total amount still owing, are £5000, each unsecured creditor will be paid at the rate of £1 for each £4 due to him, that is will receive ¼ of what he is owed, and the bankrupt is then said to pay at the rate of "$5 in the £." 

Example 1. A man buys a house for £1650 and receives £132 rent a year for it. Find the ratio of the annual rent to the purchase price.

\[
\text{annual rent} = \frac{132}{11} = 12 \\
\text{purchase price} = 1650 \\
\therefore \text{annual rent : purchase price} = 12 : 1650 = 2 : 25.
\]

Example 2. The rateable value of a town is £265,400. Find the money obtained from a rate of 11s. 6d. in the £.

The ratio of the money obtained to the rateable value = \(11\frac{1}{2} : 20 = 23 : 40\);

\[
\therefore \text{the money obtained} = \frac{23}{40} \times 265,400 = \£152,055.
\]

Note. This result can be obtained by using a practice method:

- a rate of 10s. in the £ produces £13,700 (¼ of £265,400);
- a rate of 1s. in the £ produces £13,770;
- a rate of 6d. in the £ produces £6,635; then add.

Example 3. A bankrupt's assets are £432, and he can pay at 13s. 6d. in the £. Find his liabilities.

The ratio of his liabilities to his assets = \(20 : 13\frac{1}{2} = 40 : 27\);

\[
\therefore \text{his liabilities} = \frac{40}{27} \times 432 = \£640.
\]

EXERCISE 80

Express the ratio of the first quantity to the second as simply as possible (i) as a fraction; (ii) in the form \(a : b\) (Nos. 1-8).

1. 960 mi.; 1320 mi. \[2\] 1200; 640. 3. 2s. 4d.; 17s. 6d. [4] 42 lb.; \(\frac{1}{4}\) ton. 5. 2\(\frac{1}{4}\) hr.; 35 min. [6] 11\(\frac{1}{2}\); 1s. 1\(\frac{3}{4}\)

RATIO

*7. 2\(\frac{3}{4}\)d. per oz.; 5s. 6d. per lb. 8. 33 ft. per sec.; 500 yd. per min.

[9] Find the ratio of the length of the Ganges (1500 miles) to that of the Amazon (4000 miles).
[10] The tax on an income of £840 is £126. Find the ratio of the tax to the income.
[11] A school contains 360 girls and 240 boys. Find the ratio of (i) the number of girls to the number of boys; (ii) the number of boys to the number of pupils.
[12] A man earns £750 a year and spends £630 a year. Find the ratio of (i) his income to his expenditure; (ii) his savings to his income.
[13] Two cars cost £110, £375 respectively when new. After 6 months the market value of each has fallen £45. Find the ratio of their values (i) when new; (ii) after 6 months.
[14] The sides of two squares are 6 in., 8 in. long. Find the ratio of (i) their perimeters; (ii) their areas.
[15] Pressed pork is quoted at 6d. per \(\frac{1}{2}\) lb, and roast pork at 3s. 6d. per lb. Find the ratio of their prices.
[16] A man sleeps 7\(\frac{1}{2}\) hr. every day. Find the ratio of the time he is awake to the time he is asleep.
[17] A commercial traveller's commission is £25 4s., if he sells goods to the value of £189. Find the ratio of the commission to the sales.
[18] A hotel charges at the rate of 10s. a day in winter and 5 guineas a week in summer. Find the ratio of summer to winter charges.

*19. In a town, the rates are 11s. 3d. in the £. Find the ratio of the rate paid to the rateable value.
[20] An alloy consists of 27\(\frac{3}{4}\) oz. of copper and 29\(\frac{1}{2}\) oz. of tin. Find the ratio by weight of copper to the alloy.
[21] A car travels 140 mi. in 3\(\frac{1}{4}\) hr., and a train travels 30 mi. in 30 min. Find the ratio of the speed of the car to that of the train.
[22] The average speeds of a train and a car are 45 m.p.h. and 36 m.p.h.; find the ratio of the time taken by the train to that taken by the car for equal distances. Find also the ratio of the speed of the train to that of the car.
[93] What money is produced by a rate of 8d. in the £ on a rateable value of £156,600?

*94. Find the rateable value of a town if £2400 is obtained from a rate of 2d. in the £.

*95. The rateable value of Winchester in 1934 was £226,140; the services administered by the City Council required a rate of 7s. 6d. in the £, those administered by the County Council required a rate of 5s. 10d. in the £. Find the cost of each of these services. The Grant received from the Exchequer was equivalent to a rate of 2s. 10d. in the £; how much was this Grant?

26. Find the tax on an income of £400, if no tax is paid on the first £150, and if tax is paid at 2s. in the £ on the remainder.

27. Find the tax on an income of £900, if no tax is paid on the first £150, and if tax is paid at 2s. 6d. in the £ on the next £175, and at 5s. in the £ on the remainder.

28. A bankrupt paid 2s. 3d. in the £. His liabilities were £7840; what were his assets?

29. A bankrupt paid 1s. 4d. in the £. His assets were £435; what were his liabilities?

*30. A bankrupt's assets are £1628 and his liabilities are £12,762. Find what he can pay in the £, correct to the nearest penny.

If we wish to find which of two given ratios is the greater, it is usually best to express each in the form \( m : 1 \), where \( m \) is calculated to as many places of decimals as is necessary.

Example 4. Each year, A earns £350 and saves £40, B earns £400 and saves £50. For which person is the ratio of savings to income the greater?

The ratio of savings to income is for A, \( \frac{40}{350} = \frac{4}{35} = 0.114 \); for B, \( \frac{50}{400} = \frac{1}{8} = 0.125 \).

\[ \therefore \text{the ratio for B of savings to income is the greater.} \]

Example 5. Express the ratio of £2 13s. 6d. to £5 13s. 6d., in the form \( m : 1 \), giving \( m \) correct to 2 places of decimals.

\[ \frac{\text{£2 13s. 6d.}}{\text{£5 13s. 6d.}} = \frac{227}{227} = 0.471; \]

\[ \therefore \text{the ratio} = 0.47 : 1. \]

Scales of maps are often given as ratios in such forms as 1 : 10,000, 1 : 250,000, etc. The scale 1 : 10,000 means that 1 inch on the map represents 10,000 inches on the ground, or more generally that 1 : 10,000 is the ratio of the distance between any two points on the map to the actual distance between the two places they represent; and the fraction \( \frac{1}{10,000} \) is called the representative fraction, or more shortly the R.F. of the map.

Example 6. The scale of a map is 4 inches to 1 mile. Express this ratio in the form \( 1 : n \).

\[ \frac{4 \text{ inches}}{1 \text{ mile}} = \frac{4 \text{ inches}}{1760 \times 36 \text{ in.}} = \frac{1}{1760 \times 9} = 15840 \]

\[ \therefore \text{the scale of the map is} \ 1 : 15840. \]

Example 7. The scale of a plan is 1 : 120. Find the dimensions of a room which measures 2 in. by \( \frac{1}{8} \) in. on the plan.

1 in. on the plan represents 120 in., = 10 ft., on the ground.

\[ \therefore 2 \text{ in. represents} 20 \text{ ft. and} \frac{1}{8} \text{ in. represents} 15 \text{ ft.} \]

\[ \therefore \text{the} \ 20 \text{ ft. by} 15 \text{ ft.} \]

Example 8. The scale of a map is 2 in. to the mile. Find in acres the area of an estate represented by an area 5-sq. in. on the map.

A square, side 2 in., represents 1 sq. mi., that is, 4 sq. in. represent 640 ac.

\[ \therefore 5^4 \text{ sq. in. represent} \frac{640 \times 5^4}{4} \text{ ac.,} \]

that is \((16 \times 54) \text{ ac. or 864 ac.} \)

EXERCISE 81

Express the following ratios in the form \( m : 1 \):

1. 6 : 4.
2. \( \frac{1}{3} : 2 \frac{1}{2} \).
3. 8 cwt. to 1 ton.
4. \(4 \frac{1}{4} \) in. to 1 yd.
5. £8 12s. to £5.
6. 1 m. to 5 mm.

Find \( m \) for the following pairs of equal ratios:

7. 36 : 24 = \( m : 1 \).
8. 2s. : 2s. 6d. = \( m : 1 \).
9. \( \frac{10}{15} = \frac{m}{6} \).
10. \( \frac{5}{25} = \frac{3}{m} \).
11. \( \frac{3}{4} = \frac{m}{100} \).
Which is the greater of the following pairs of ratios?

15. 3 : 5; 5² : 5². 16. 5 : 2; 5² : 2². *17. \( \frac{1}{2} : \frac{1}{2}; \frac{1}{2} : \frac{1}{2} \).

Express the following ratios in the form \( m : 1 \), giving \( m \) correct to 2 places of decimals:

18. £1 6s.: £3 10s.
19. 1 mi.: 1000 yd.
*20. 3 cwt. 50 lb.: 5 cwt.
*21. £4 17s. 8d.: £5 5s.

22. Express the ratio of the length of the Mont Cenis tunnel (7 3/4 mi.) to that of the Simplon tunnel (12 1/8 mi.) in the form \( n : 100 \).

23. The scale of a plan is 4 in. to 10 ch. Find its R.F.
24. The scale of a map is 10 cm. to 1 km. Find its R.F.
25. The scale of a map is 1 : 100,000. Find in km. the length of a road which is represented by a line 4 7/ cm. long on the map.

26. The scale of a map is 1 : 40,000. Find in inches, correct to 1/100 in., the length on the map of a road 3/4 mi. long.

27. A ground plan of a house is made on the scale 1 in. to 15 ft. Find the R.F. of the plan. Find the length and breadth on the plan of a room 18 ft. by 12 ft. What area on the ground is represented by 1 sq. in. on the plan?

28. On a map, scale 1 in. to the mile, an island has an area of 3 3/8 sq. in. Find its actual area in acres.

29. A train runs at 30 mi. an hour up a gradient of 1 in 110 (i.e., it rises 1 ft. vertically for each 110 ft. travelled). How many feet does it rise in 1 min.?

30. The rates on an assessment of £84 are £39 18s. a year. How much in the £ is the annual rate?

Increase and Decrease in a Given Ratio. If the annual rent of a house is raised from £60 to £80, the ratio of the new rent to the old rent = 80 : 60 = 4 : 3, and we say that the rent has been increased in the ratio 4 : 3. In other words, the new rent is 4/3 times the old rent.

If the annual rent of a house is lowered from £60 to £48, the ratio of the new rent to the old rent = 48 : 60 = 4 : 5, and we say that the rent has been decreased in the ratio 4 : 5. In other words, the new rent is 4/5 times the old rent.

The fraction \( \frac{4}{5} \) by which the old rent £60 must be multiplied to give the new rent £48 is called a multiplying factor.

New Quantity = Multiplying Factor × Old Quantity

The multiplying factor is less than 1 if the new quantity is less than the old quantity; it is greater than 1 if the new quantity is greater than the old quantity.

Example 9. Increase 12s. 3d. in the ratio 10 : 7.

Increased value = \( \frac{12 \frac{3}{4}}{7} \times \frac{10}{7} = \frac{49}{7} \times \frac{10}{7} \)

= \( \frac{35}{7} \times \frac{10}{7} = \frac{175}{7} \).

Example 10. Decrease 2 hours in the ratio 5 : 6.

Decreased time = \( \frac{5}{6} \times 2 \) hr.

= \( \frac{10}{6} \) hr. = 1 hr. 40 min.

Example 11. In what ratio must £75 be increased to become £100?

The ratio, £100 : £75 = 100 : 75 = 4 : 3;

∴ if £75 is increased in the ratio 4 : 3, it becomes £100.

Check: £75 \times \frac{4}{3} = £(25 \times 4) = £100.

Example 12. Find the multiplying factor which alters 90 tons into 63 tons.

The ratio, 63 tons : 90 tons = 63 : 90 = 7 : 10;

∴ 90 tons \times \frac{7}{10} = 63 tons;

∴ the required multiplying factor = \( \frac{7}{10} \).

EXERCISE 82 (CLASS DISCUSSION)

1. Increase 144 in the ratio 7 : 4.
2. Decrease 105 in the ratio 3 : 5.
3. Increase 15s. in the ratio 6 : 5.
4. Decrease 30s. in the ratio 5 : 6.
5. Decrease 2 1/2 yd. in the ratio 7 : 10.
6. Increase 1 lb. 4 oz. in the ratio 4 : 1.
7. In a sale, prices are reduced in the ratio 3:5. Find the sale prices of articles whose ordinary prices are (i) £1; (ii) 7½d.
8. A photograph 3½ in. by 2½ in. is enlarged in the ratio 8:3. Find the dimensions of the enlargement.

[9] A man reduces his weight in the ratio 5:7. What does his weight become if originally it was 16 st.?
10. In what ratio must 24 be increased to become 32?
11. In what ratio must 100 be decreased to become 80?
12. In what ratio must £1 be increased to become £1 10s.?
13. In what ratio must 1½ lb. be decreased to become 1 lb.?
14. What multiplying factor alters 72 into 96?
15. What multiplying factor alters 60 into 48?
16. What multiplying factor increases £100 to £120?
17. What multiplying factor reduces 2s. 6d. to 1s. 6d.?
18. What multiplying factor increases 10s. by 2s. 6d.?
19. What multiplying factor diminishes 1 ft. by 3 in.?
20. If the price of petrol rises from 1s. 6d. to 1s. 8d. per gallon, find the ratio in which the price increases.

[21] In what ratio is the speed of a train reduced when it falls from 45 m.p.h. to 45 m.p.h.?
22. Two sums of money are in the ratio 4:5; the smaller is 6s.; what is the larger?
[23] Two distances are in the ratio 12:7; the larger is 21 mi., what is the smaller?

*24. The ratio of the wholesale to the retail price of a commodity is 9:20. What is the retail price per lb. if the wholesale price is 7s. 6d. per lb.?
25. What multiplying factor makes a number half as large again?
26. What multiplying factor diminishes a number by ¾ of itself?
[27] In what ratio is a number increased if it is increased by ¼ of itself?

*28. A number is diminished by ⅜ of itself. In what ratio has it been decreased?

29. A rectangular rubber sheet 2 in. by 3 in. is stretched so as to measure 3 in. by 4½ in., remaining rectangular. In what ratio has (i) its length, (ii) its breadth, (iii) its area increased?

*30. A man works 8 hr. a day; in what ratio have his earnings changed if his pay is altered from 1s. 6d. per hour to 1s. 7½d. a day?
31. In what ratio has the average speed changed, if the time for a certain journey is reduced from 2 hr. to 1 hr. 40 min.?

[32] When the price of electricity is reduced from 6d. to 5d. per unit, I increase my annual consumption from 300 units to 400 units. In what ratio does my bill for electricity alter?

*33. The price of a table is reduced from £24 to £16 10s. Find the reduced price of a set of chairs originally costing £5 6s., if reduced in the same ratio.
34. A photograph measuring 7½ in. by 5 in. is enlarged so that the larger side becomes 18 in.; what does the shorter side become? In what ratio is the area increased?

*35. The number of germs in a solution increases in the ratio 6:5 every 2 hr. At 4 p.m., 1500 are counted in a drop placed under a microscope. How many will there be in the drop at 8 p.m.? How many would there have been at 2 p.m.?

**Direct Variation.** Suppose a train is running at a constant speed of 48 m.p.h.;

in 5 min. it travels 4 mi., in 10 min. it travels 8 mi.,
in 15 min. it travels 12 mi.; and so on.

Thus the ratio of any two distances is equal to the ratio of the corresponding times taken:

8 mi.: 12 mi. = 2: 3 and
time: 10 min.: 15 min. = 2: 3;
and we say that the distance varies directly as the time.

In 1 min. it travels ⅝ mi.,
in x min. it travels ⅝ x mi.

If we say that it travels y mi. in x min.,

\[ y = \frac{⅝}{x} \]

or

\[ y \times x = \frac{⅝}{x} \]

and we say that y varies directly as x.

The corresponding travel graph is represented by OA in the diagram; it is a straight line through the origin.
Inverse Variation. If a journey of some given length, say 120 mi., has to be made, the greater the speed the smaller is the time taken.

At 10 m.p.h. the time is 12 hr.,
at 20 m.p.h. the time is 6 hr.,
at 30 m.p.h. the time is 4 hr.; and so on.

Thus the ratio of any two times is equal to the reciprocal of the ratio of the corresponding speeds:

6 hr. : 4 hr. = 3 : 2 and 20 m.p.h. : 30 m.p.h. = 2 : 3;
and we say that the time varies inversely as the speed.

At 1 m.p.h. the time is 120 hr.,
at v m.p.h. the time is \( \frac{120}{v} \) hr.;

and if we say that the time is \( t \) hr. when the speed is \( v \) m.p.h.

\[ t = \frac{120}{v} \text{ or } \frac{1}{v} t = 120, \]

and we say that \( t \) varies inversely as \( v \).

If we plot values of \( t \) against values of \( \frac{1}{v} \), we obtain a straight line through the origin.

It is, however, important to recognise the fact that the relation between two connected quantities is often not one either of direct or of inverse proportion. For example:

(i) If the side in one square is 3 times as long as the side of another square, the area of the first is 9 times, not 3 times, the area of the second.

(ii) A shopkeeper who sells buns at 1d. each would be willing to sell 12 dozen for less than 12s.

(iii) It probably costs more to feed 2 boys for 300 days than to feed 200 boys for 3 days.

Example 13. If 4 lb. of almonds cost 9s., find the cost of 7 lb. at the same rate.

If the number of lb. is increased in the ratio 7 : 4, the cost is also increased in the ratio 7 : 4; but 4 lb. cost 9s.,

\[ 7 \text{ lb. cost } 9s. \times \frac{7}{4}, \text{ that is } \frac{63}{4} \text{ s., or } 15 \text{ s. 9d.} \]

Example 14. 6 men take 12 hr. to weed a certain field; how long would 9 men take to do so, if all work at the same rate?

If the number of men is increased in the ratio 9 : 6, the time is decreased in the ratio 6 : 9; but 6 men take 12 hours,

\[ \therefore 9 \text{ men take } 12 \text{ hours } \times \frac{6}{9}, \text{ that is } 8 \text{ hours, or } 8 \text{ hours.} \]

Oral Work. State the multiplying factors required for obtaining the answers to Exercise 83, Nos. 1-10, and simplify them.

Exercise 83

Assume that the rates given in this exercise are uniform

1. 14 lb. of icing sugar cost 4s.; find the cost of 21 lb.
2. 20 tablets of soap cost 9s.; find the cost of 25 tablets.
3. 30 lb. of ground rice cost 7s. 6d.; find the cost of 35 lb.
4. 12 men can mow a field in 10 days; how long will 15 men take?
5. A journey takes 6 hr. if I travel at 30 mi. an hour; how long will it take at 45 mi. an hour?
6. Five equal pipes fill a swimming-bath in 40 min.; how long does it take if only 4 of the pipes are used?
7. \( \frac{4}{3} \) tons of coal cost £3 12s.; find the cost of \( \frac{2}{3} \) tons.
8. A boy bicycles 31\( \frac{1}{2} \) mi. in 3 hr.; how long will he take to go 35 mi. at the same rate?
9. 65 lb. of potatoes cost 2s. 8\( \frac{1}{4} \)d.; find the cost of 135 lb.
10. A hotel charges at the rate of 4 guineas a week; find the charge for 10 days.
11. The rates on a rental of £75 are £3 10s.; find the rates on a rental of £42.
12. Railway fares were increased from 1d. a mile to 1\( \frac{1}{4} \)d. a mile. What is the new fare for a journey which used to cost 8s. 4d.?
13. A garrison has enough food for 24 days. How long will it last if each person’s daily ration is reduced in the ratio 2:3?

14. A train takes 50 min. for a journey if it runs at 48 mi. per hour. At what rate must it run to reduce the time to 40 min.?

15. A man earns 52 shillings for a working week of 48 hr. If he is absent for 6 hr., how much should he receive?

16. The lengths of the rims of 2 wheels of a carriage are 13 ft. 6 in., 10 ft. 6 in. In a certain journey, the larger makes 315 revolutions; how many revolutions does the smaller make?

17. What is the price of 1 cental (100 lb.) of wheat, if 480 lb. of wheat cost 33 shillings?

18. Telegraph wire weighs 440 lb. per mile. Find the length of a line for which 13 cwt. 84 lb. is used.

19. The railway fare for 3 adults and 3 children (half-price) was £2 8s.; what was the cost of the tickets for the 3 adults?

20. On a map whose scale is 5 in. to the mile the distance between two cross-roads is 4 in.; what would be the distance on a map whose scale is 2 in. to the mile?

21. At 1s. 1d. per hour the weekly wage of a number of workmen is £143 10s. 10d. What would be the increase in the wages bill if the rate of pay had been 1s. 2d. per hour?

22. A candle 3 in. long after burning for 40 min. is 7½ in. long. After how many more minutes is its length reduced to 1½ in.?

23. A cog-wheel which has 24 cogs fits into another which has 45 cogs. If the former turns 5 times in 4 sec., how many times does the latter turn in 12 sec.?

24. A speedometer is adjusted to read correctly if the outer edge of the rim of the wheel is 88 in. If owing to over-size tyres being fitted the outer edge is increased to 91 in., find, to the nearest yd., the true length of a journey recorded as 60 mi.

25. A town has enough rations for 10,000 people for 35 days. But after 5 days, 2500 refugees are admitted. If all are now put on half-rations, how much longer will the food last?

**Compound Units.** It is often convenient to invent special units for particular problems. For example, the amount of work required to regulate the traffic in a town each day might be described as 120 policeman-hours; this would mean that if 20 policemen were used, each would be on traffic duty for 6 hours a day; if 24 were used, each would be on traffic duty for 5 hours a day, and so on.

In Example 14, p. 149, the amount of work required to weed the field could be described as (12 × 6) man-hours.

Example 15. A puts 10 cows for 9 days and B puts 12 cows for 5 days into a field owned by X. Find the ratio of the amount A pays X to the amount B pays X, if there is a fixed charge per cow per day.

If the charge per cow per day is called 1 cow-day, A pays (10 × 9) cow-days and B pays (12 × 5) cow-days;

\[ \text{amount A pays} : \text{amount B pays} = 90 : 60 = 3 : 2. \]

Example 16. If 12 men earn £81 in 10 days, how much will 14 men earn in 8 days, if the daily wage is the same for each man?

If the money earned by 1 man in 1 day is called 1 man-day, then:

\[ (12 × 10) \text{ man-days equal } £81, \]
\[ (14 × 8) \text{ man-days equal } \frac{£81}{12} × \frac{14}{10}, \]  
\[ = £75.8. \]
\[ 14 \text{ men in 8 days earn £75.12s.} \]

**Exercise 84**

*Assume that the rates given in this exercise are uniform*

1. If 12 boys earn £9 in 6 days, how much will 8 boys earn in 9 days?

2. If 6 men mow 6 ac. of grass in 6 days, how many acres would 10 men mow in 9 days?

3. If the wages of 4 men for 30 days amount to £39, how many men can be employed for 40 days at a cost of £260?

4. The gas for 10 gas-fires for 18 days costs £7. What is the cost of gas for 12 gas-fires for 5 days?

5. 16 men can make 800 boxes in 9 days; how long will 15 men take to make 1000 boxes?

6. A man earns £7 10s. in 8 days, working 9 hr. a day; how much should he be paid for 20 days, working 6 hr. a day?

7. A mowing machine will cut 10 ac. in 6 hr. if it is driven at 4 mi. an hour; how long will it take to cut 15 ac. if driven at 3 mi. an hour?
8. The cost of carriage of 36 tons for 28 mi. is £15. Find the cost of carriage of 42 tons for 54 mi. at a fixed rate per ton per mile.

9. A carpet 10 ft. long, 9 ft. wide costs £8. Find the cost of a carpet of the same quality 15 ft. long, 4 ft. wide.

[10] 25 men working 8 hr. a day make a road in 63 days; how long would 45 men take, working 7 hr. a day?

11. 20 men can do a piece of work in 18 days; how long will 15 men take to do half as much again?

12. 36 teachers each working for 30 periods a week are required for a certain school; how many teachers would be needed if each teacher took 27 periods a week?

[13] 19 men working 7½ hr. a day can do a piece of work in 21 days; how many hours a day must 45 men work to do it in 7 days?

14. A man pays £5 for keeping 10 horses in a field for 8 weeks; how much will he have to pay for keeping 12 horses in the field for 6 weeks?

*15. If a 6d. loaf weighs 40 oz. when wheat costs 36s. a quarter, what should a 7½d. loaf weigh when wheat costs 32s. a quarter?

*16. If b men earn £c in d days, how much will x men earn in y days, at the same rate?

*17. If b men can mow c acres in d days, how long will x men take to mow y acres, at the same rate?

*18. A certain sum of money was just sufficient to maintain 12 men for 39 days when the index figure was 136. For how many days will the same sum maintain 17 men when the index figure is 156. (Assume that the cost of living is proportional to the index figure.)

*19. A train, travelling at the rate of 37 miles in 50 min., takes 6 hr. for a journey. How long will it take if it travels at the rate of 60 miles in 1½ hr.?

20. A moneylender charges £3 for lending £36 for 8 months; what will he charge for lending £40 for 9 months?

*21. A contractor employs 15 men working 8 hr. a day to do a piece of work in 27 days. At the end of 10 days, work has to be suspended for 5 days, and to finish the work in time he engages more men and all work at 8½ hr. a day. How many more men does he engage?
EXERCISE 85

[Nos. 1–12 are intended for oral work or class-discussion]

1. Divide £60 in the ratio (i) 2 : 3; (ii) 5 : 7; (iii) 4 : 1.
2. Divide 2 ft. in the ratio (i) 3 : 5; (ii) 1 : 5; (iii) 9 : 7.
3. Divide £24 into three shares in the ratios 1 : 2 : 3.
4. Divide 8s. into three shares in the ratios 5 : 2 : 9.
5. Divide 5 lb. into three parts in the ratios 7 : 4 : 9.
6. Divide 1 ch. into three parts in the ratios \(\frac{1}{4} : \frac{1}{2} : \frac{1}{8}\).
7. Divide 5 tons into three parts in the ratios \(\frac{1}{4} : \frac{1}{8} : 1\frac{1}{2}\).
9. Divide 7s. 6d. into four parts in the ratios \(\frac{1}{2} : \frac{1}{3} : \frac{1}{4} : \frac{1}{8}\).

Find as simply as possible three whole numbers proportional to:

10. £2; £1 10s.; £3.
11. \(2\frac{1}{2}\) yd.; 5 ft.; 8 ft. 4 in.
12. \(1\frac{1}{4}\) lb.; 2 lb.; 12 oz.
13. 13s. 4d.; 19s. 4d.; 6s. 8d.
15. 20 tons of coal are shared between three families in the ratios 2 : 5 : 9; how much does each receive?
16. The sides of a triangle are in the ratios 1 : 1\(\frac{1}{2} : 2\), and its perimeter is 1 yd. Find the length of each side.
17. The profits, £750, of a business are divided between 3 men so that their shares are proportional to 3, 4, 8. Find each share.
18. 60 marks are distributed between three questions so that the amounts are proportional to 5, 4, 6. Find the marks for each question.
19. A legacy of £450 is divided among 3 sons in the ratios \(\frac{1}{4} : 2\frac{1}{2} : 3\). How much does each receive?
20. A, B, C provide £250, £500, £750 respectively to buy a business, and their shares of the profits are proportional to the capital they provide. If the profits are £245, what does each receive?
21. Divide 44 shillings into an equal number of shillings, florins, half-crowns.

PROPORTIONAL PARTS

Find the three smallest whole numbers proportional to \(a, b, c\) in Nos. 22–25:

22. \(a : b = 5 : 6; b : c = 9 : 4\).
23. \(a : b = 12 : 5; a : c = 8 : 3\).
24. \(a = 5b; b = 3c\).
25. \(a = 1\frac{1}{2}b; b = \frac{3}{2}c\).
26. Divide £1 15s. into two parts such that one is half as much again as the other.
27. Divide £3 into two parts such that one is \(\frac{1}{3}\) of the other.
28. Divide £66 between A, B, C so that A has twice as much as B, and B has half as much again as C.
29. A load of 95 lb. is distributed between A, B, C, so that A carries \(\frac{1}{2}\) of what B carries, and B carries 4 times as much as C. How much does B carry?
30. A bankrupt owes B £2144, C £2130, D £71, E £315. His assets (what he possesses) are £1200. How much does B receive?
31. A takes 10 days to build a fence which B can build in 8 days. If £4 10s. is paid for the job and if both work together, how much should A receive?
32. Three ingredients costing 3d., 4d., 1d. per lb. are mixed so that their weights are proportional to 2, 3, 7 respectively. Find the cost of 30 lb. of the mixture.
33. A, B, C divide 2 gross of eggs so that B has 5 eggs for every 3 A has, and C has 8 eggs for every 7 B has. How many will each have?
34. A can dig a trench in 10 days which would take B 12 days and C 15 days to dig. If £4 10s. is paid for the job, and if all work together, how much should each receive?
35. Profits amount to £2415 are divided between A, B, C, so that for every £4 A gets, B gets £5, and for every £9 B gets, C gets £16. Find the shares.
36. A puts £900 into a business for 1 year, and B puts into it £400 for 9 months. The profits shared between A and B are £180; how much should each receive?
37. A man borrows £200 for 8 months from B and borrows £500 for 4 months from C, at equal rates of interest, and pays altogether £15 interest. How is the interest shared between B and C?
38. A man borrows £300 for 10 months from B, £500 for 9 months from C, and £700 for 3 months from D, and pays altogether £72 interest. How is this shared between B, C, and D?

*39. Three firms A, B, C undertake a piece of work for which £160 is paid. In carrying out the work, A provides 10 men for 15 days, B 8 men for 20 days, C 15 men for 6 days. If all the men are paid at the same rate, find the amount each firm receives.

*40. One employer has 40 workmen and another 75 workmen, and their weekly wage-bills are in the ratio 3:5. Find the ratio of the average wages per man paid by the two employers.

*41. Divide £330 among 4 persons A, B, C, D so that A may have twice as much as B, B twice as much as C, and A and C together as much as B and D together.

*42. Two partners A, B started with capitals of £6000, £4000 respectively. The profits at the end of each year are divided in proportion to their capitals invested in the business at the beginning of the year. A withdrew his profits at the end of each year, while B left his in the business. The profits for the first 3 years were £1400, £1584, £1639 15s. respectively. What was B's capital at the end of 3 years?

Use of Ratio in Problems

Example 21. If the number is 42, what is the number?
If the number is divided into 7 equal parts, 3 of these parts make up 42. Therefore the ratio of the number to 42 equals 7:3.
∴ the number = 42 × \(\frac{3}{7}\) = 18.

Algebraically, if the number is \(x\), \(\frac{3}{7}x = 42\); ∴ \(x = 42 \times \frac{7}{3}\).

Example 22. A boy, after spending \(\frac{3}{5}\) of his money, has 4s. 8d. left. How much had he at first?

After spending \(\frac{3}{5}\) of his money, \(\frac{2}{5}\) of his money remains;
∴ the ratio of what he had at first to what remains equals 5:2.
∴ he had at first (4s. 8d.) \(\times \frac{5}{2} = 8d. \times 5 = 6s.

Example 23. A picture-dealer deducts 2s. in the £ for cash from the price on the ticket. If the cash price of a picture is £2 5s., find the price on the ticket.

For each 20s. marked on the ticket, the cash price is 18s.;
∴ the ratio of the marked price to the cash price = 20:18 = 10:9;
∴ the marked price = £2 \(\frac{2}{5}\) \(\times \frac{9}{10}\) = £2 10s.

Use of Ratio in Problems

EXERCISE 86

1. If \(\frac{3}{5}\) of a number is 30, find the number.

2. When a man spends \(\frac{2}{3}\) of his income, he saves £360 a year; find his income.

3. When \(\frac{3}{4}\) of a field has been ploughed, 42 ac. still remain to be done; find the area of the field.

4. If 2d. in the shilling is deducted for cash from the marked price of a chair, the cash price is 30s.; find the marked price.

5. After paying tax at the rate of 2s. 6d. in the £, a man has £630 left; what had he at first?

6. To each shilling a boy saves, his father adds sixpence; how much has the boy saved when he possesses 19s. 6d.

7. The duty on a certain commodity is 3s. 4d. in the £. A consignment costs 4 guineas including duty; what is its value free of duty?

8. After selling 85 copies of a book for every 100 copies printed, a publisher finds that 450 copies are left unsold. How many copies were printed?

9. A shopkeeper makes a profit of 4s. in the £ on the cost price of his goods. What does he pay for an article which he sells for 30 shillings?

10. By selling my car for £84, I lose at the rate of 6s. in the £ on the price I paid for it. What did I pay for it?

11. A retailer makes a profit of 7s. 6d. in the £ on the cost price of his goods. What is his profit on an article he sells for £5 10s.?

12. A watch was 2 min. slow at 8 a.m., and 1 min. fast at 6 p.m. on the same day. Find when it was right if it gained time uniformly.

13. In Réaumur's thermometer, freezing-point and boiling-point of water are marked 0, 80 respectively; in Fahrenheit's thermometer they are marked 32, 212. What is the Fahrenheit reading for 12 Réaumur?

14. Coal at 48s. a ton is mixed with coke at 36s. a ton in the ratio 5:3 by weight. Find how much money is saved by using 10 tons of the mixture instead of 11 tons of coal.

15. In an excursion by rail, the ratio of the number of whole tickets to the number of half-tickets was 2:3. Find the number of people who travelled if the equivalent of 84 full tickets was required.
16. An empty bottle weighs 11 oz.; when full of water it weighs 36 oz. and when full of alcohol it weighs 31 oz. Find the ratio of the weights of equal volumes of alcohol and water. Find also the weight of 1 cu. ft. of alcohol, if 1 cu. ft. of water weighs 62.3 lb.

*17. A train X passes three stations A, B, C at 9 a.m., 9.40 a.m., 10.32 a.m. respectively. Another train Y passes A, B at 9.50 a.m., 10.15 a.m. respectively. If both trains travel uniformly, find the time at which Y will pass C. Find also the speed of Y if X is travelling at 24 m.p.h.

18. A man receives a dividend of 10,000 francs which he exchanges at 84 fr. to the £. Income tax is then deducted at 5s. in the £. What is his net receipt to the nearest penny?

*19. A, B, C had each full chests of tea containing respectively 60 lb., 100 lb., 120 lb.; the chests were emptied, and the contents were well mixed and then replaced in the chests. How many pounds of A's tea is probably now in C's chest?

*20. A bank fails and pays its depositors 2s. 3d. in the £; a man who had £2000 in this bank and £550 elsewhere has liabilities of £2500. He declares himself bankrupt; how much in the £ can be paid his creditors, to the nearest penny?

CHAPTER XII

PERCENTAGE

Percentage and Ratio. Suppose that in a consignment of eggs, 1 in every 20 is bad, then it follows that 5 in every 100 are bad, and we say that 5 per cent. of the eggs in the consignment are bad. The ratio of the number of bad eggs to the total number is 1:20 or 5:100; a percentage is simply a ratio in which the second number is 100, and may be represented by a fraction whose denominator is 100.

For example, 7 per cent., written 7%, may denote the ratio 7:100 or may be represented by the fraction $\frac{7}{100}$.

It should be noted that in some cases it is customary to take the second number of the ratio as 1000; birth-rates and death-rates are usually quoted in this form. For example, in 1930 in England and Wales the birth-rate was 16·3 per thousand of the population and the death-rate was 11·4 per thousand of the population. These rates are equivalent to the ratios 16·3:1000 and 11·4:1000 and could be expressed as 1·63% and 1·14% respectively.

Example 1. Express in percentage form the statement: At a certain school, 3 boys in every 4 own bicycles.

\[
\frac{3}{4} \text{ of the total number of boys own bicycles;} \\
\therefore \text{ in every 100 boys, } \left(\frac{3}{4}\right) \text{ of 100 boys own bicycles;} \\
\therefore \text{ in every 100 boys, 75 boys own bicycles.}
\]

The statement may therefore be written:

At a certain school, 75% of the boys own bicycles.

Example 2. If 30% of the pupils in a school are boys, what percentage of the pupils are girls?

In every 100 pupils, there are 30 boys and therefore there are (100 - 30) girls, that is 70 girls;

\[
\therefore 70\% \text{ of the pupils are girls.}
\]

Or we may say, the ratio of the number of boys to the number of pupils is 30:100,

\[
\therefore \text{ the ratio of the number of girls to the number of pupils is 70:100,} \\
\therefore 70\% \text{ of the pupils are girls.}
\]

EXERCISE 87 (Oral)

Express in percentage form the statements in Nos. 1-10:

1. 1 day in every 5 days is wet.
2. A boy obtains 7 marks out of 10.
3. 3 men in every 10 men possess motor vehicles.
4. 3 pupils in every 20 pupils are absent from school.
5. 7 people in every 25 die before the age of 50.
6. A tax is at the rate of 1s. in every £.
7. A boy gets 1 sum in every 3 sums wrong.
8. 1 orange in every 8 oranges in a box is bad.
9. The interest is £1 on every £40.
10. In 1930, the birth-rate in Scotland was 19·6 per thousand of the population.
Write down the answers to the following:

11. If 5% of the pupils in a school are absent, what percentage are present?

12. If a man spends 70% of his income, what percentage does he save?

13. In a railway accident 85% of the passengers were unhurt, what percentage were injured?

14. If 16% of those who enter for a competition get prizes, what percentage get nothing? How many per thousand get nothing?

15. If 28% of the population are men and 30% are women, what percentage are children? How many per thousand are children?

16. A man spends 10% of his income on rent and 55% on household expenses; what percentage remains?

17. A boy eats 35% of a cake and gives away 25% of it; what percentage of the cake is left?

18. On a boat, 24% of the passengers travel 1st class, 32% travel 2nd class, and the rest 3rd class. What percentage travel 3rd class?

Percentages and Fractions. Percentages can be represented by fractions. For example, 40% denotes the ratio 40:100, and is therefore represented by \(\frac{40}{100}\), that is \(\frac{2}{5}\) or 0.4.

Conversely any fraction or decimal can be expressed as a percentage by transforming it so that its denominator is 100. For example, \(\frac{5}{8}\) = \(\frac{75}{100}\); \(\frac{3}{4}\) is equivalent to 75%.

The following table illustrates different ways used for representing fractions in everyday life:

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Decimal</th>
<th>Per cent.</th>
<th>Per thousand</th>
<th>s. in the £</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{2}{3})</td>
<td>0.666...</td>
<td>66%</td>
<td>667</td>
<td>6s. 6d.</td>
</tr>
<tr>
<td>(\frac{2}{3})</td>
<td>0.666...</td>
<td>66%</td>
<td>667</td>
<td>6s. 6d.</td>
</tr>
</tbody>
</table>

Example 3. Express (i) \(\frac{2}{3}\) 100% as percentages.

(i) \(\frac{2}{3}\) = \(\frac{66}{3}\) = 200\% per cent.

(ii) 0.225 = 0.225 \times 100 per cent. = 22.5%.

Example 4. Find the value of \(62\frac{1}{2}\%\) of £4.

\[62\frac{1}{2}\%\] of £4 = \[\frac{62.5}{100}\] of £4 = £(4 \times \frac{62.5}{100})

= \[\frac{4 \times 62.5}{100}\] = £31.25 = £31 12s. 5d.

Example 5. A boy obtains 52 marks out of 80 for a paper. What percentage is this?

The boy obtains \(\frac{52}{80}\) of the total, that is \((\frac{52}{80} \times 100)\) per cent.; \(\therefore\) he obtains 65% of the total.

Example 6. Express £2 12s. 6d. as a percentage of £30.

\[\frac{21}{30}\] of £30 = \[\frac{21}{30}\] of £30 = £21.6d = £21 6s.

but \(\frac{21}{30}\) is equivalent to \(\frac{69}{100}\) per cent. = \(\frac{69}{100}\) per cent.; \(\therefore\) £2 12s. 6d. is \(\frac{69}{100}\) of £30.

Example 7. Find correct to the nearest penny the value of \(6\frac{3}{4}\%\) of £7 16s. 9d.

\(6\frac{3}{4}\%\) of £7 16s. 9d. = \(\frac{27}{100}\) of £7 16s. 9d. = £(0.078375 \times 27)

Value required = \(\frac{27}{100}\) of £7 16s. 9d. = £0.78375 = £0.784.

The value can now be found by ordinary multiplication or by a practice method as follows:

Since the answer is required to the nearest penny, we keep 5 places of decimals in the working.

\(\times\) value = £0.529; to 3 places = £0.529; to 10s. = £0.529; to nearest penny. = 10s. 7d., to nearest penny.

The practice method may also be worked as follows:

\[\begin{array}{cccc}
\$ & 10 & 7 & 16 & 9 \\
\hline
\% & 0 & 5 & 8 & 1 \\
\% & 5 & 7 & 0 & 0 \\
\% & 1 & 6 & 8 & 1 \\
\end{array}\]

As before, the value is 10s. 7d.

\[\frac{64}{100}\] of £7 16s. 9d. = £6 5s. 6d.
Example 9. A man takes out a whole life insurance policy for £1000 on his 25th birthday, by paying a premium of £13 2s. a year. If he dies when he is 64 years old, how much more does the company pay his estate than he has paid in premiums to the company.

In the period, age 25 to age 64\(\frac{1}{2}\), 40 annual premiums are paid.

:. amount paid in premiums = (£13 2s. × 40) = £524;

:. the difference = £1000 - £524 = £476.

Note. The company is able to afford to do this because it has earned interest on the annual payments which the man has been making for 40 years.

Life insurance is beneficial, partly because it is a form of self-imposed saving, but chiefly because it enables a man to make provision for his dependents (his wife or children) in the event of his dying at an early age.

EXERCISE 88

[Nos. 1-30 are suitable for class-discussion]

Express the following percentages as fractions and as decimals:

1. 25%; 50%; 75%; [9] 5%; 15%; 85%.

2. 2%; 4%; 300%.

3. 21%; 112\(\frac{1}{2}\)%; 37\(\frac{1}{2}\)%.

Express the following percentages as fractions and as ratios:

4. 150%; 200%; 275%.

5. 33\(\frac{1}{3}\)%; 68%; 9\(\frac{1}{2}\)%.

Express the following as percentages:

6. \(\frac{1}{2}\) 6\(\frac{1}{2}\); \(\frac{1}{4}\) 4\(\frac{1}{2}\); \(\frac{1}{8}\) 2\(\frac{1}{2}\).

7. \(\frac{1}{4}\) 5\(\frac{1}{4}\); 0\(\frac{1}{2}\); 1\(\frac{1}{2}\).

8. 2\(\frac{1}{2}\); 3\(\frac{1}{2}\); 4\(\frac{1}{2}\); 5\(\frac{1}{2}\).

Express the ratios 1:2; 7:10.

14. The ratios, 3:8; 5:12.

Find the values of the following:

15. 60% of 5s.

16. 180% of 25s.

17. 75% of 1s.

18. 33\(\frac{1}{3}\)% of 2s.

19. 235% of £5.

20. 121% of £40.

21. 166\(\frac{2}{3}\) of 1 yd.

22. 55% of 1 ton.

23. 34% of 2 m.

Express the first quantity in each pair as a percentage of the second quantity:

24. 2s.; 8s.

25. 9s.; 12s.

26. 2d.; 6d.

27. 4 ft.; 1 yd.

28. 3 cwt.; 1 ton.

29. 20 oz.; 1 lb.
54. The window glass in a private house can be insured at the rate of 15s. per £100 of rent. Find the premium for insuring the glass in a house rented at £65 a year.

55. The premium paid to insure the contents of a house against fire, at the rate of 1s. 4d. per cent., is £2 10s. Find the value for which the contents are insured.

56. For National Health insurance a man pays 9d. a week and a woman 7d. a week. How much annual premium does each pay, taking 1 year = 52 weeks?

57. A man takes out a life insurance policy for £1000 on his 35th birthday; the premium is £1 15s. per cent. (i.e. per £100 of amount of policy). How much has he paid altogether in premiums when he is 60½ years old?

58. A man on his 60th birthday is able to buy for £1000 an annuity of £84 16s. a year, paid half-yearly, the first payment being made when he is 60½ years old. How long must he live to receive back more than he has paid?

59. In 1930, there were 15,760 accidental deaths in England and Wales, of which 6404 were due to motor vehicles. What percentage of the deaths were due to motor vehicles? Answer to 2 places of decimals. How many deaths per thousand of accidental deaths were due to motor vehicles?

60. In 1926, 48·7 per cent. of the population of 53,200,000 of the Soviet Union could not read. How many was this, to the nearest hundred thousand?

Percentage Changes. The importance of a change in the size or value of a quantity is often estimated by calculating what percentage the increase or decrease is of the original value. For example, if a car costs £400 when new and if its value is £320 at the end of 1 year, the decrease in value is £80; this is 20 per cent. of the original value, £400, and we say that the car has depreciated in value by 20% after 1 year's use.

Note. If A and B are any two numbers, A is \( \frac{A}{B} \times 100 \) per cent. of B.
PERCENTAGE

An increase of say 30 per cent. means that for each 100 units in the original value there is an increase of 30 units, making the new value 130 units.

\[ \therefore \text{the ratio of the increase to the original value is } 30 : 100; \]
\[ \therefore \text{the increase is } \frac{30}{100} \times \text{the original value.} \]

Also the ratio of the new value to the original value is 130 : 100;

\[ \therefore \text{the new value is } \frac{130}{100} \times \text{the original value.} \]

Similarly the ratio of the new value to the increase is 130 : 30;

\[ \therefore \text{the new value is } \frac{130}{30} \times \text{the increase}; \text{ and so on.} \]

A decrease of say 20 per cent. means that for each 100 units in the original value there is a decrease of 20 units, making the new value 80 units.

\[ \therefore \text{the ratio of the new value to the original value is } 80 : 100; \]
\[ \therefore \text{the new value is } \frac{80}{100} \times \text{the original value; and so on.} \]

The ratios used as multipliers in the examples just given are called multiplying factors; oral practice of the form indicated in the next exercise is intended to secure facility in their use.

EXERCISE 89 (Oral)

By what must a number be multiplied to increase it by:

1. 17%. 2. 83%. 3. 70%. 4. 20%. 5. 139%.

By what must a number be multiplied to decrease it by:

6. 9%. 7. 37%. 8. 61%. 9. 30%. 10. 40%.

11. Increase 300 by 8%. 12. Decrease 400 by 20%.

13. Decrease 80 by 10%. 14. Increase 60 by 30%.

15. If the price of an article is increased by 9%, write down

(i) the ratio of the new price to the old price;
(ii) the ratio of the new price to the change in price;
(iii) the factor by which the new price must be multiplied to give the old price.

16. If the price of an article is decreased by 7%, write down

(i) the ratio of the new price to the old price;
(ii) the ratio of the change in price to the new price;
(iii) the factor by which the change in price must be multiplied to give the old price.

17. Repeat No. 15 for a decrease of 10% in price.

18. Repeat No. 16 for an increase of 20% in price.

19. If the number of pupils in a school increases by 30% in a certain period, write down the factor by which

(i) the number at the beginning must be multiplied to give the number at the end.
(ii) the change in numbers must be multiplied to give the number at the end.

20. Repeat No. 19 for a decrease of 20% for the period.

21. (i) A exceeds B by 13%, write down the ratio of A to B.

(ii) C is 19% less than D, write down the ratio of C to D.

Complete the following:

22. If A exceeds B by 5%, A = B × ...; B = A × ...;
23. If C is 12% less than D, C = ... × D; D = C × ...;
24. If A exceeds B by 6%, A = B × ...; A - B = A × ...;
25. If C is 8% less than D, D - C = C × ...; D = C - D × ...;
26. If A exceed B by 10%, A = (A - B) × ...;
27. If C is 10% less than D, C = (D - C) × ...;
28. If A is 70% of B, A = ... × B; B = A × ...;
29. If A exceeds B by x per cent., A = B × ...;
30. If C is y per cent. less than D, D = C × ...;

Example 10. A man, whose salary is £750 a year, receives an increase of 8 per cent. Find his new salary.

The ratio of the new salary to the old salary is 108 : 100;

\[ \therefore \text{the new annual salary} = \frac{108}{100} \times 750 = £810. \]

Or as follows: the increase = \( \frac{750}{100} \times 8 \) of £750 a year = £60 a year;

\[ \therefore \text{the new annual salary} = 750 + 60 = £810. \]

Example 11. If a man’s salary is raised from £250 a year to £280 a year, find the increase per cent.

The increase is £30 a year; therefore the ratio of the increase to the first salary is 30 : 250, that is 3 : 25.

\[ \therefore \text{the increase per cent.} = \frac{3}{25} \times 100 \text{ per cent.} = 12\%. \]

Example 12. 117 is 36% of a certain number. Find the number.

The ratio of the number to 117 equals 100 : 36;

\[ \therefore \text{the number} = 117 \times \frac{100}{36} = 325. \]

Or algebraically, if \( x \) is the number, \( \frac{100}{36} \times 117 = x \);

\[ x = 117 \times \frac{36}{100} = 325. \]
Example 13. A line, whose true length is known from calculation to be 7.5 cm., is found by drawing and measurement to be 7.2 cm. What is the error per cent.?

The error is 7.5 cm. - 7.2 cm. = 0.3 cm.; therefore the ratio of the error to the true length is 0.3 : 7.5, that is 3 : 75 or 1 : 25; \[\therefore \text{error per cent.} = \frac{1}{25} \times 100 \text{ per cent.} = 4\%\]

Note. A thorough discussion of relative error and error per cent. is reserved for Chapter XVI, see p. 229.

Example 14. After 5% of a bill has been deducted from it, £57 remains to be paid. How much was the bill?

After 5% of the bill has been deducted, 95% of the bill remains. \[\therefore\text{the ratio of the bill to £57 equals 100 : 95; \therefore the bill is £57} \times \frac{100}{95}, \text{that is £60.}\]

Or the working may be finished as follows:—

95% of the bill is £57; \[\therefore 100\% \text{ of the bill is } £57 \times \frac{100}{95}; \therefore \text{the bill is £60.}\]

EXERCISE 90

1. Increase 80 by 35%. [8] Decrease 75 by 40%.
2. Decrease 216 by 37%. [4] Increase 416 by 125%.

Find the number or quantity of which:

5. 25% is 7. \[6] \text{76\% is 57.} \quad 7. \text{37\% is 84.}\]

8. 30% is 1s. \[9. \text{71\% is 12s. 6d.} \quad *10. \text{168\% is 4 oz.}\]

11. What number when increased by 20% becomes 144?
12. What number when decreased by 20% becomes 108?
13. What sum of money when increased by 35% becomes £216?
14. What sum of money when decreased by 35% becomes £156?

15. A man, whose salary is £380 a year, receives an increase of 15%; find his new salary.
16. A car costs £175 when new; after 1 year, its value is £105. By how much per cent. has its value decreased?
17. A man spends £440 a year, and this is 80% of his income. What is his income?

*18. A rectangular enclosure is 80 ft. long, 25 ft. wide. If the length of each side is increased by 20%, find the increase in the area in sq. ft.; find also the percentage increase in the area.

19. 55 per cent. of the pupils in a school are girls. What percentage of the pupils are boys? Find the number of pupils if there are 216 boys.

20. What are full marks for a paper if a boy who gets 112 marks obtains 70 per cent. of the total?

21. A spends 88% of his income. Find his income if he saves £81 a year.

22. If 10% is deducted from a bill, £27 remains to be paid. How much is the bill?

23. 15% of a sum of money is £27 15s.; find the value of 164% of the same sum.

24. The number of pupils in a school increased by 15% during the year. Find the number at the beginning of the year if there were 322 at the end of the year.

25. My bank deposit has increased by 40% during the past year. It is now £504; what was it a year ago?

26. A reel of cotton, which cost 1/4d. in 1914, cost 10d. in 1917. Find the increase per cent. in the price.

27. Wireless licences increased from 3,412,000 in 1930 to 4,330,000 in 1931. Find the increase per cent., to the nearest whole number.

28. Calculation shows that the true length of a line is 3.2 in.; a result obtained from drawing and measurement is 3.4 in.; find the error per cent.

29. Find the error per cent. in taking the area of a field which is 165 yd. long, 120 yd. wide, as 4 ac.

30. 24% by weight of an explosive mixture is saltpetre. Find the weight of a sample in which the other ingredients weigh 9.12 gm.

31. In a sale the price of an armchair was lowered by 30% to 3 guineas. How much was it reduced?

32. If the price of an article is raised by 8%, the increase in the price is 10 shillings. Find the new price.

33. A man buys a house and lets it for £80 a year. Repairs cost him £8 a year. If his net annual receipts (that is, rent less cost of repairs) amount to 5% of what he paid for the house, find the price of the house.

34. The weight of a liquid was 3.75 gm. before heating and 3.25 gm. after heating. Find the loss per cent. of its weight, correct to 2 figures.
Example 15. An article costing 30s. is sold at a profit of 15%. Find the selling price.

The profit is $\frac{1}{6}$ of the cost price.

\[
\text{Profit} = \frac{1}{6} \times 30s. = 5s.
\]

\[
\text{Selling price} = \text{Cost price} + \text{Profit} = 30s. + 5s. = 35s.
\]

Example 16. An article costing £55 is sold for £50, find the loss per cent.

The loss = £55 - £50 = £5; \(\therefore\) the loss is $\frac{5}{55}$ of the cost price.

\[
\text{Loss} = \left(\frac{5}{55} \times 100\right) \text{ per cent.} = 9\frac{1}{11}\%.
\]

Example 17. A dealer gained 40% by selling an article for 21s.; find the cost price.

If the cost price is 100s., the gain is 40s.; \(\therefore\) the selling price is 140s.

\[
\text{Cost price} = \frac{100}{100 + 40} \times 140s. = 100s.
\]
PERCENTAGE

37. S.P. £10; gain £2.
38. S.P. £60; loss £20.
39. S.P. 1s.; loss 3d.
40. S.P. 1s.; gain 3d.
41. C.P. £50; S.P. £58.
42. C.P. £400; S.P. £460.
43. C.P. £30; S.P. £42.
44. C.P. £90; S.P. £100.
45. C.P. 8d.; S.P. 1s.
46. C.P. 4½d.; S.P. 6d.
47. C.P. 2s. 6d.; S.P. 2s.
48. C.P. £1 5s.; S.P. 15s.

EXERCISE 92

Find the S.P. in the following:
1. C.P. £15; gain 5%.
[2] C.P. 6s.; gain 33½%.
3. C.P. £75; loss 8¾%.
4. C.P. 3s. 4d.; loss 17¾%.
5. C.P. 12s. 6d.; loss 13½%.
[6] C.P. £2 1s. 8d.; gain 4½%.

Find the gain or loss per cent. in the following:
7. C.P. £45; gain £9.
[8] C.P. 6s. 8d.; loss 2s.
9. C.P. £1; S.P. 24s.
11. C.P. £4 10s.; S.P. £4.
12. C.P. 16s. 3d.; S.P. 18s. 6d.
13. S.P. 15s.; gain 5s.
*15. S.P. 1s. 9d.; loss 3½d. *16. S.P. £10 15s.; gain 27½s. 6d.

Find the C.P. in the following:
17. S.P. £7; gain 5%.
[18] S.P. 12s.; loss 4%.
19. S.P. 16s. 6d.; loss 12%.
20. S.P. 2s.; gain 6¾%.
*21. S.P. £10 ½d.; loss 31½%.
*22. S.P. 8s. 6d.; gain 13½%.
23. A dealer buys a bicycle for £12 and sells it for £15; find his gain per cent.
24. I bought a house for £1200 and was forced to sell it for £1000; find my loss per cent.
25. A man bought a chair for £6 and sold it on 15 per cent. How much profit did he make?
26. I bought a car for £375 and sold it one month later at a loss of 12 per cent. How much did I lose?
27. By selling a golf club for 30 shillings, a dealer makes a profit of 25%; find what the dealer paid for it.
28. If I sell my house for £380, I shall lose 20%; what did I pay for the house?

GAIN AND LOSS PER CENT.

[89] Potatoes are bought at 7s. per cwt. and sold at 1d. per lb.; find the gain per cent.

[90] Steel screws costing 5s. per gross are retailed at 6½d. per dozen; find the gain per cent.

31. A grocer buys 100 eggs for 7s. 6d.; 4 are broken and the rest are sold at 8 a shilling. Find his gain per cent.

32. If I sell my wireless set for £9, I shall lose 64 per cent.; what did I pay for it?

33. By selling a picture for £6 a dealer makes a profit of 80 per cent.; what did he pay for it?

34. A car is sold for £635 at a gain of 27 per cent. How much is the profit?

*35. A 100-lb. chest of tea is bought for £3 15s. Find the gain per cent. if the tea is sold at 2s. 3d. per lb.

*36. A grocer buys 1500 bananas at 8d. a dozen; he sells 900 of them at two for 3d. and the rest at three for 4d. Find his gain per cent.

37. A man buys eggs at 10 for a shilling and sells them at 8 for a shilling; find his gain per cent.

38. Christmas cards are bought at 15s. per 100 and are sold at 2s. a dozen; find the gain per cent.

39. 1 cwt. of tea costs £7, at what price per lb. must it be sold to make a profit of 20%?

40. Oranges are bought at the rate of 3 for 2d.; how many must be sold for 1s. to gain 80 per cent.?

*41. A tradesman offers an article for £1 6s. 8d., but is willing to deduct 25 per cent. of the bill for cash. If the tradesman paid 16s. 8d. for the article, find his gain per cent. on a cash sale.

*42. The following table shows a week’s business for a tradesman whose total weekly expenses are £3.

<table>
<thead>
<tr>
<th>C.P. of article</th>
<th>2s.</th>
<th>5s.</th>
<th>7s. 6d.</th>
<th>18s.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number sold</td>
<td>33</td>
<td>40</td>
<td>16</td>
<td>10</td>
</tr>
<tr>
<td>Gain per cent.</td>
<td>25</td>
<td>18</td>
<td>60</td>
<td>35</td>
</tr>
</tbody>
</table>

Find the tradesman’s net profit for the week.
MISCELLANEOUS EXAMPLES

EXERCISE 93

1. A man saves $\frac{1}{4}$ of his income. What percentage is this?

2. What is $16\frac{2}{3}\%$ of 1 chain?

3. A man travels $8\%$ of a journey by bus, $87\frac{1}{2}\%$ by train, and walks the rest of the way. For what % of his journey does he walk?

4. What is $14\frac{1}{2}\%$ of £5?

5. Soap, which had a pre-war price of 3d. per tablet, rose to 1s. 2d. per tablet in the war. Find the % increase to the nearest whole number.

6. What percentage is 8s. of £3 6s. 8d.?

7. My insurance premium is 8 guineas a year for my car which is valued at £350; what percentage is the premium of the value?

8. If A exceeds B by $80$ per cent., find the factor by which A must be multiplied to give B.

9. A man measures the length of a road as 1934 yd. It is really 1910 yd.; find his error per cent., to one place of decimals.

10. 150 marks are assigned to each of two papers; if a boy obtains 63 marks for the first, how many must he get for the second to secure $36\%$ on the whole?

11. If the National Debt is £7,700,000,000 and the national income is £350,000,000, what percentage, to 1 place of decimals, is the income of the debt?

12. I can hire a boat at the rate of 5 guineas for 7 days, but have to pay a deposit of $15\%$ of the hire in advance. How much is the deposit if I take the boat for 10 days, at the same rate of hire and deposit?

13. If butter cost 1s. 4d. per lb. yesterday and has risen 25% to-day and will fall 25% to-morrow, what will be the price to-morrow?

14. The price of a hat is reduced from 21s. to 17s. 6d.; find the percentage reduction.

15. If a carpet 12 ft. by 8 ft. is laid in a room, there is a margin 1 ft. wide all the way round. What percentage of the area of the floor is the area of the margin?

16. If the profit on a wireless set, sold at a gain of $32\frac{1}{2}\%$, is £6 10s., find the sale price.

17. A coal merchant buys coal at 32s. per ton, and in addition pays 13s. per ton carriage. Find his gain per cent. if he sells it at 2s. 9d. per cwt.

18. A man, measuring a rectangle 120 yd. by 80 yd., makes each side 15 per cent. too small. How much per cent. will his estimate of the area be too small?

19. If wages are increased 12\% all round, the weekly wage-bill paid by a firm becomes £198. What is the increase per week in the bill?

20. 2 gal. of spirit containing 10% of water are mixed with 5 gal. of spirit containing 8% of water, and 1 gal. of water is added to the mixture. What is now the percentage of water?

21. A London merchant bought wine in France at 30 francs per litre, the price including free delivery in London, and sold it at £2 8s. per gallon, the rate of exchange being 75 francs to the £. Find his gain per cent. [Take 1 gallon = 4-5 litres.]

22. An agent receives 2\frac{1}{4}\% commission on orders under £10 and 5% on other orders. He obtains 12 orders of £8 each and 7 orders of £25 each. Find his average commission percentage, to 1 place of decimals, on the total.

23. A man buys 100 gal. of milk for £8 16s. and then adds 10 gal. of water; 10% of the mixture gets split and he sells the rest at 3d. per pint. Find his gain per cent.

24. A legacy of £4500 is left to 3 persons so that, after duties totalling 20%, have been paid, their shares are proportional to 1, 1\frac{1}{2}, 2. Find the shares.

25. A is 40% older than B; by how much per cent. is B younger than A?

26. Last month petrol was 1s. 8d. per gallon and I used 80 gallon. This month the cost has increased by 15% and I shall use 15% less. What will be the percentage change in my expenditure on petrol?

27. B is 20% heavier than A, and C is 25% heavier than B; by how much per cent. is C heavier than A?

28. A man buys goods catalogued at £350 subject to successive discounts of 25, 10, 2\frac{1}{2} per cent. This means that he is allowed to deduct 25% from the catalogued price, then 10% from this reduced price, and 2\frac{1}{2}% from the last. What does he pay?

29. In 1933 a firm paid £7000 in wages and £2100 for other expenses and made a profit of £1400. In 1934, it paid 75% more in wages and 45% more for other expenses. If the receipts increased by 60 per cent., find the increase per cent. of the profits.
FURTHER AREAS AND VOLUMES

*30. On a journey across London, a taxi averages 20 m.p.h. for 70% of the distance, 25 m.p.h. for 10% of it, and 8 m.p.h. for the remainder. Find the average speed for the whole journey.

*31. By what percentage must a motorist increase his average speed in order to reduce by 20% the time a particular journey takes?

*32. 20 lb. of bronze contained 87% of copper and 13% of tin by weight. With how much copper must it be melted to obtain bronze containing 10% of tin by weight?

*33. Two partners invest £2000 and £1200 respectively in a business and agree that 20% of the annual profit is to be divided equally between them, and the remainder in proportion to the capital invested. The profit for the first year is £260; how will this be shared between them?

CHAPTER XII

FURTHER AREAS AND VOLUMES

AREA OF A RECTANGLE

These examples illustrate the fact that the number of units of area in a rectangle is obtained by multiplying together the numbers of units in the length and in the breadth, a result which was given in Ch. II., see p. 15, for whole numbers.

Express:

5. 1 sq. yd. in sq. ft.; 1 sq. ft. in sq. yd.
6. 1 sq. m. in sq. cm.; 1 sq. cm. in sq. m.
7. 3 sq. yd. in sq. ft.
8. 10 sq. cm. in sq. dm.
9. 50 sq. mm. in sq. cm.
10. 3 sq. m. in sq. cm.
11. 32 ac. in sq. mi.
12. 400 sq. ch. in ac.

Find the areas of the following rectangles:

13. 4 in. by 3 in. in sq. ft.
14. 2 ft. by 3 ft. in sq. in.
15. 1 dm. by 1 cm. in sq. cm.
16. 5 cm. by 2 cm. in sq. dm.
17. 2 ft. by 6 in. in sq. yd.
18. 4 ch. by 5 ch. in ac.

Example 1. Find the area of a rectangle 2 ft. 4 in. long, 1 ft. 3 in. broad.

Length = $2\frac{1}{2}$ ft., breadth = $1\frac{1}{2}$ ft.;
Area = $(2\frac{1}{2} \times 1\frac{1}{2})$ sq. ft. = $3\frac{3}{4}$ sq. ft.

Example 2. How many tiles, each 9 in. by 8 in., are required for the floor of a room, 15 ft. 4 in. long, 13 ft. 6 in. wide?

15 ft. 4 in. = 184 in., 13 ft. 6 in. = 162 in.

The tiles can be arranged in 23 rows, 18 tiles in each row;

Note. In this arrangement, no tiles are broken; but if this possibility can be disregarded, we can work as follows:

Area of floor = $(15\frac{4}{12} \times 13\frac{6}{12})$ sq. ft.;

Area of 1 tile = $(\frac{3}{4} \times \frac{2}{3})$ sq. ft.

Number of tiles = $(15\frac{4}{12} \times 13\frac{6}{12}) \div (\frac{3}{4} \times \frac{2}{3})$

= $23 \times 9 \times 2 = 414$. 

12
Example 3. How many handkerchiefs, each 14 in. square, can be cut from material 30 in. wide, 2 yd. long? What is the area of the material left over?

\[ \frac{30 \text{ in.} \times 14 \text{ in.}}{14 \text{ in.}} = 2 \times \frac{30}{14} = 2 \times \frac{15}{7} = 2 \frac{1}{14} \text{ in.} \]

\[ \therefore \frac{2}{7} \text{ in.} \times \frac{1}{14} \text{ in.} \times 2 \text{ yd.} \times \frac{1}{14} \text{ in.} = \frac{2}{7} \times \frac{1}{14} \times 2 \times \frac{1}{14} \text{ sq. in.} \]

\[ = \frac{2}{7} \times \frac{1}{14} \times \frac{1}{14} \times 2 \text{ sq. in.} = \frac{4}{196} \text{ sq. in.} = \frac{2}{98} \cdot \frac{1}{14} \text{ sq. in.} \]

\[ \therefore \frac{2}{7} \text{ in.} \times \frac{14}{14} \text{ in.} = 2 \frac{1}{14} \text{ in.} \]

\[ \therefore 2 \frac{1}{14} \text{ in.} \times \frac{1}{14} \text{ in.} = \frac{2}{7} \text{ sq. in.} \]

\[ \therefore \frac{2}{7} \text{ sq. in.} \]

\[ \therefore \frac{2}{7} \text{ sq. in.} \]

Area of remaining material = \[ (\frac{72}{14} \times \frac{30}{14} - (14 \times 14 \times 10)) \text{ sq. in.} \]

\[ = (2160 - 1960) \text{ sq. in.} = 200 \text{ sq. in.} \]

Note that \[ \frac{72}{14} \times \frac{30}{14} \text{ is just more than 11, although only 10 handkerchiefs can be obtained.} \]

Example 4. Find to the nearest shilling the cost of a rug, 8 ft. 6 in. long, 4 ft. 8 in. wide, at 5s. 9d. per sq. ft.

\[ \text{Area of rug} = \frac{8}{3} \times \frac{3}{8} \text{ sq. ft., and 1 sq. ft. costs } 1 \text{ shilling; } \]

\[ \therefore \text{cost of rug} = \frac{8}{3} \times \frac{3}{8} \times 1 \text{ shilling. } \]

\[ = \frac{8}{3} \times \frac{3}{8} \times 1 \text{ shilling } = \frac{1}{1} \text{ shilling. } \]

But 69 shillings is nearer to 69 than to 70,

\[ \therefore \text{cost of rug } = \pounds 1 \text{ shilling, to the nearest shilling.} \]

Exercises 95

Find the perimeters and areas of the following rectangles, giving the perimeters as compound quantities and the areas in terms of the unit indicated in brackets:

1. 2 ft. by 1 ft. 3 in. (sq. ft.).
2. 1 ft. 6 in. by 8 in. (sq. ft.).
3. 12 ft. by 8 ft. 9 in. (sq. ft.).
4. 6 ft. 4 cm. by 3 ft. 8 cm. (sq. cm.).
5. 3 ft. 2 m. by 5 dm. (sq. m.).
6. 2 ft. 6 in. square (sq. ft.).
7. 12 ft. 2 in. by 10 ft. 6 in. (sq. ft.).
8. 8 yd. 1 ft. 6 in. by 3 yd. 1 ft. (sq. yd.).

Find the lengths of the following rectangles:

9. Area 48 sq. ft., breadth 4 ft.
10. Area 10 sq. ft., breadth 1 ft. 8 in.
11. Area 5 sq. yd., breadth 6 ft. 9 in.
12. Area 18 sq. cm., breadth 3 ft. 5 cm.

Find in acres the area of a field:

13. 24 ch. long, 15 ch. wide.
14. 165 yd. long, 55 yd. wide.
*33. How many pieces of cardboard, each 5 in. square, can be cut from a sheet, 4 ft. long, 3 ft. wide. What area remains over?

*34. How many dusters, each 16 in. square, can be cut from material, 7 ft. long, 3 ft. wide. What area remains over?

Example 5. Find the cost of the paper for the walls of a room, 15 ft. 6 in. long, 12 ft. 6 in. wide, 9 ft. high, allowing 63 sq. ft. for doors, windows, etc., if the paper costs 3s. 6d. per piece, 12 yd. long, 21 in. wide.

<table>
<thead>
<tr>
<th>Height</th>
<th>Side wall</th>
<th>End wall</th>
<th>Side wall</th>
<th>End wall</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 ft.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The diagram represents the four walls of the room folded out flat.
The perimeter = (15 ft. 6 in. + 12 ft. 6 in.) = 56 ft.;
.: area of walls = (56 x 9) sq. ft. = 504 sq. ft.;
.: area to be papered = (504 - 63) sq. ft. = 441 sq. ft.

But area of each "piece" = (36 x \(\frac{9}{4}\)) sq. ft. = 63 sq. ft.;
.: number of pieces required = 441 / 63 = 7;
.: the paper costs (3s. 6d.) x 7, that is 24s. 6d.

Note. Wall-paper is sold in rolls or pieces, usually 12 yd. long, 21 in. wide, that is of area 63 sq. ft. = 7 sq. yd.
A whole number of "pieces" must be bought; for example, if the area to be papered is 58 sq. yd., although \(\frac{58}{7}\) pieces, = \(\frac{8}{7}\) pieces, are sufficient, 9 pieces must be bought.

Example 6. Axminster carpet 27 in. wide is sold at 7s. 6d. per yard length. Find the cost of the carpet for a piece 16 ft. 4 in. long, 13 ft. 6 in. wide.

The carpet is sold in the form of a long rectangle, 27 in. wide; a strip 3 ft. long costs 7\(\frac{1}{2}\)s.

.: (\(\frac{9}{2}\) x 3) sq. ft. of carpet costs 7\(\frac{1}{2}\)s.
.: 1 sq. ft. of carpet costs (\(\frac{9}{2}\) x \(\frac{3}{4}\))s. = \(\frac{1}{8}\)s.
.: (16\(\frac{1}{2}\) x 13\(\frac{1}{2}\)) sq. ft. of carpet cost \(\frac{9}{2}\) x \(\frac{3}{4}\) x \(\frac{1}{8}\) = \(\frac{27}{8}\).
.: the cost of the carpet = \(\frac{27}{8}\) x 56 = £12 5s.

Note. Since the carpet is sold in a roll 27 in. wide, one side of the made-up carpet must be a multiple of 27 in.; in this example, 13 ft. 6 in. = 13\(\frac{1}{2}\) ft. + \(\frac{2}{3}\) ft. = \(\frac{52}{3}\) x \(\frac{3}{2}\) = 6; and the carpet is made by joining together 6 strips, each 16 ft. 4 in. long.
FURTHER AREAS AND VOLUMES

19. 19 ft. 6 in. by 15 ft. 9 in., if the carpet is 27 in. wide, at 8s. per yard.

20. 18 ft. by 16 ft. 6 in., if the carpet is 22 in. wide, at 4s. 2d. per yard.

21. Find the cost of making a cistern 6\(\frac{3}{4}\) ft. long, 4 ft. broad, 3\(\frac{1}{4}\) ft. deep, without a lid, at 1s. 6d. per sq. ft.

22. Find the area of a wall 7 ft. 6 in. high which encloses a courtyard 24 yd. long, 28 ft. wide.

23. Planks 5 in. wide, 4 ft. high, are placed side by side to form a piling round a plot of ground 35 yd. long, 27\(\frac{1}{2}\) yd. wide. What length of planking is used, and what is its cost at 1s. 6d. per sq. ft?

24. A room is 18 ft. long, 14 ft. broad, 10 ft. high. There is a door 8 ft. by 3 ft. and there are two windows, each 5 ft. 6 in. by 4 ft. Find the cost (i) of distempering the walls at 2d. per sq. ft., (ii) of carpeting the floor at 6s. 6d. per sq. yd.

25. An open tin box, 1 ft. 9 in. long, 9 in. wide, 8 in. high, is fitted with a tin lid which overlaps to a depth of 1 in. all round. What is the total area of tin sheeting used for the box and lid?

26. A rectangular cigarette tin, fitted with a lid, is 6 in. long, 2\(\frac{1}{2}\) in. wide, 2 in. high, and the lid overlaps \(\frac{3}{4}\) in. on all four sides. Find the total area of the tin used.

Example 7. A room 16 ft. long, 12 ft. wide, has a carpet in the middle, leaving a margin 18 in. wide all round which is covered with linoleum at 4s. 6d. per sq. yd. Find the cost of the linoleum.

In the diagram, not drawn to scale, PQRS represents the carpet and ABCD the floor.

The margin is 1\(\frac{1}{2}\) ft. wide; 
\[\therefore PQ = (16 - (1\frac{1}{2} \times 2)) \text{ ft.} = 13 \text{ ft.}\]
\[\therefore PS = (12 - (1\frac{1}{2} \times 2)) \text{ ft.} = 9 \text{ ft.}\]
\[\therefore \text{area of carpet} = (13 \times 9) \text{ sq. ft.}, \text{and area of floor} = (16 \times 12) \text{ sq. ft.}\]
\[\therefore \text{area of linoleum} = (192 - 117) \text{ sq. ft.} = 75 \text{ sq. ft.} = 4\frac{3}{8} \text{ sq. yd.}\]
\[\therefore \text{cost of linoleum} = (4\frac{3}{8} \times \frac{7}{4}) \text{ s.} = (\frac{35}{8}) \times \frac{7}{4} \text{ s.} = 37\frac{1}{8} \text{ s.} = 37 \text{ s. 6d.}\]

Note. It is quicker to use the "subtraction" method than to divide up the border into rectangles.

EXERCISE 97

1. Find the area of a path running all round a lawn 60 ft. long, 40 ft. wide, if the path is (i) 5 ft. wide, (ii) 4 ft. 6 in. wide.

2. Find the area of a frame round the edge of a picture 3 ft. wide, 2 ft. 6 in. high, if the frame is (i) 3 in. broad, (ii) 1\(\frac{1}{2}\) in. broad.

3. A sheet of tin measures 4 dm. 7 cm. by 3 dm. 6 cm. If a strip 2-5 cm. wide is cut off all round, find in sq. cm. the area of the part cut off.

4. A photograph, 6 in. wide, 10 in. high, is mounted on a card so that there is a margin 1\(\frac{1}{4}\) in. wide at top and bottom and 2 in. wide along the sides. Find the area of the part of the card which is not covered.

5. A room, 15 ft. long, 13 ft. wide, has a carpet in the middle of the floor, leaving a margin all round which is stained. What area is stained if the margin is (i) 1 ft. wide, (ii) 1\(\frac{1}{2}\) ft. wide?

6. A photograph 6 in. by 3\(\frac{1}{4}\) in. is mounted on a card and framed. The frame is \(\frac{3}{2}\) in. wide all round and measures 11 in. by 8 in. externally. Find the area of the visible part of the card.

7. Find by the subtraction method the shaded area in the diagram,

(i) if \(a = 2\frac{1}{2}, b = 1\frac{1}{4}, c = 2, d = 1\);

(ii) if \(a = 6\frac{1}{4}, b = 2\frac{1}{4}, c = 4\frac{1}{2}, d = 1\frac{1}{4}\);

the units being inches.

8. The diagram represents a rectangular brick wall pierced with four equal rectangular windows. Find the area of the surface of brickwork, if \(a = 2\frac{1}{4}, b = 1\frac{1}{4}, c = 1\frac{1}{4}, d = 2\frac{1}{4}, e = 21, f = 15\); the units being feet.

9. A walled garden is 25 yd. long, 20 yd. wide, internal measurements. A gravel path 4\(\frac{1}{2}\) ft. wide runs round its edge inside the wall, and a paved path 5 ft. wide runs round the outside edge of the wall which is 6 in. thick. Find the area of each path.
10. A border 2 ft. wide is stained all round the edge of the floor of a room 20 ft. by 16 ft., and the rest is carpeted at 7s. 6d. per sq. yd. Find the cost of the carpet. Find also the cost of staining at 1s. per sq. ft.

11. Linoleum at 3s. per sq. yd. is put down in a room 20 ft. by 18 ft. so as to leave a margin 18 in. all round. Find the cost.

12. A room is 15 ft. long, 12 ft. wide. Find the cost of staining a border 1 ft. 6 in. wide all round the edge of the floor at 1s. per sq. yd.

13. A carpet, 10 ft. 6 in. by 8 ft. 4 in., is laid in a room, 12 ft. 8 in. by 11 ft. 3 in., and the border is stained at 2d. per sq. ft. Find the cost of staining.

14. A border 20 in. wide all round the edge of the floor of a room, 16 ft. by 15 ft., is tiled. How many tiles are required if each tile is 5 in. by 4 in.?

15. A border 2 ft. wide round the edge of the floor of a room, 22 ft. by 16 ft., is stained and the rest is carpeted. Find the ratio of the stained to the carpeted area. If the carpet costs 12s. 6d. per sq. yd. and the staining costs 2d. per sq. ft., find the total cost.

16. A room 16 ft. 6 in. long, 16 ft. wide, is carpeted so as to leave a margin 1 ft. 9 in. wide all round, which is covered with linoleum. If the carpet is made up from a roll 30 in. wide at 10s. 6d. per yard length and if the linoleum costs 6s. per sq. yd., find the cost of (i) the carpet, (ii) the linoleum.

Volume of a Cuboid

EXERCISE 98 (for discussion)

1. How many cubes, edge $\frac{1}{2}$ in., are required for building up 1 cu. in.? What is the volume in cu. in. of a cube, edge $\frac{1}{2}$ in.?

2. How many cubes, edge $\frac{1}{2}$ in., are required for building up a cuboid, 2$\frac{1}{2}$ in. by 2 in. by 1$\frac{1}{2}$ in.? What is the volume in cu. in. of (i) each small cube, (ii) the cuboid? What is the value of $2\frac{1}{2} \times 2 \times 1\frac{1}{2}$?

3. How many cubes, edge 1 cm., are required for building up 1 cu. dm.? What is the volume in cu. dm. of a cube, edge 1 cm.?

4. How many cubes, edge 1 cm., are required for building up a cuboid 1$\frac{1}{2}$ dm. by 1$\frac{1}{2}$ dm. by 1$\frac{1}{2}$ dm.? What is the volume in cu. dm. of (i) each small cube, (ii) the cuboid?

5. How many cubes, each of edge $\frac{1}{4}$ in., can be cut from a cuboid 3$\frac{1}{2}$ in. by 2$\frac{1}{2}$ in. by 1$\frac{1}{2}$ in.? What is the volume of (i) each small cube, (ii) the cuboid? What is the value of $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$?

These examples illustrate the fact that the number of units of volume in a cuboid is obtained by multiplying together the numbers of units in the length, breadth, and height of the cuboid, a result which was obtained in Ch. II, see p. 21, for whole numbers.

Express:

6. 1 cu. ft. in cu. in.; 1 cu. in. in cu. ft.

7. 1 cu. dm. (1 litre), in cu. cm.; 1 c.c. in litres.

8. 3 cu. yd. in cu. ft. 9. 100 c.c. in cu. dm.

10. 10 cu. mm. in c.c. 11. 12 cu. in. in cu. ft.

Find the volumes of the following cuboids:

12. 5 in. by 4 in. by $\frac{1}{2}$ in. 13. 1 ft. by 3$\frac{1}{2}$ in. by 1$\frac{1}{2}$ in.

14. 1 dm. by 5 cm. by 2 cm. 15. 1 m. by 8 dm. by 7$\frac{1}{2}$ cm.

Example 8. Find the number of bricks in a stack, 10 ft. 6 in. long, 6 ft. 9 in. wide, 5 ft. 6 in. high, if each brick is 9 in. by 4$\frac{1}{2}$ in. by 3 in.

Volume of stack = $(10\frac{1}{2} \times 6\frac{3}{4} \times 5\frac{1}{2})$ cu. ft.;

Volume of 1 brick = $(\frac{1}{2} \times \frac{3}{4} \times \frac{1}{2})$ cu. ft.;

Number of bricks = $(10\frac{1}{2} \times 6\frac{3}{4} \times 5\frac{1}{2}) : (\frac{1}{2} \times \frac{3}{4} \times \frac{1}{2})$

$= 21 \times 3 \times 11 \times 8 = 5544$.

Note. The data imply that it is possible to build up a stack of the stated size; the reader should verify that one way of doing so would be to have 22 layers, each containing 8 rows of bricks with 14 bricks in each row.

Example 9. A rectangular tank, with a horizontal base 6 ft. 9 in. long, 4 ft. 8 in. wide, internal measurements, contains 84 cu. ft. of water. Find the depth of the water.

Depth = $\frac{\text{volume}}{\text{area of base}}$;

Area of base = $(6\frac{9}{12} \times 4\frac{8}{12})$ sq. ft. = $(6\frac{3}{4} \times 4\frac{1}{2})$ sq. ft.

Depth of water = $[84 \div (6\frac{3}{4} \times 4\frac{1}{2})]$ ft.

$=(84 \times \frac{4}{15} \times \frac{2}{9})$ ft. $= \frac{5}{8}$ ft. = 2 ft. 8 in.
Example 10. Find the rise in the water-level caused by pumping 30,000 gallons of water into a swimming-bath, 80 ft. long, 45 ft. wide, given that 1 cu. ft. = 63.2 gal.

30,000 gal. = (30,000 \times 63.2) cu. ft. = 1,900,000 cu. ft.

But the area of the surface of the water is (80 \times 45) sq. ft.,

\[ \text{rise in water-level} = \frac{1,900,000 \times 80 \times 45}{25 \times 80 \times 45} \]

\[ = \frac{3}{5} \text{ ft.} = \frac{1}{2} \text{ ft.} 4 \text{ in.} \]

Exercise 99

Find the volumes of the following rectangular blocks:

1. 4 ft. 6 in. by 4 ft. by 2 ft. 9 in.
2. 45 cm. by 24 cm. by 5 mm.
3. 6 ft. 5 in. by 2 in. by 1 ft. 5 in.
4. 2 ft. 8 in. by 2 ft. 3 in. by 1 ft. 2 in.
5. Find the volume of air-space in a room 16 ft. long, 12 ft. wide, 8 ft. 6 in. high.
6. How many cu. ft. of wood are there in a table-top, 13 ft. 6 in. long, 5 ft. 4 in. wide, 1 ft. 6 in. thick?
7. A lock in a canal is 40 yd. long, 7 yd. wide. When the sluices are opened, the depth of water in the lock decreases from 15 ft. to 11 ft. How many cu. ft. of water run out?
8. How many rectangular blocks, each 2 ft. by 1 ft. by 1 in., can be packed in a box, 7 ft. 6 in. by 6 ft. 6 in. by 3 ft. 6 in., internal measurements? How many 4-inch cubes can be packed in this box, with faces parallel to the sides of the box, and how much space is left unoccupied?
9. How many bricks, each 25 cm. by 15 cm. by 8 cm., are required for a wall 32 m. long, 5 m. high, 40 cm. thick?
10. Find the capacity in litres of a tank, 12 dm. by 8 dm. by 5 dm., internal measurements. How many cubes, edge 7 cm., can be packed in the tank, with faces parallel to the sides of the tank, and how much space is left unoccupied?
11. Find the capacity in litres of a tin 2-4 dm. long, 7-5 cm. wide, 12 cm. high.
12. A beam 9 ft. long, 6 in. wide, 4 in. deep, is made of wood which weighs 30 lb. per cu. ft.; find the weight of the beam.
13. Find the weight in lb. of a wooden plank 12 ft. long, 8 in. wide, 2 3\(\frac{1}{2}\) in. thick, if the wood weighs 36 lb. per cu. ft.
14. Find, to the nearest gm., the weight of a rectangular steel plate 7-2 cm. by 6-5 cm. by 8 mm., if the steel weighs 8 gm. per c.c.
15. Find the weight of petrol which a tin 1-5 dm. long, 1-2 dm. wide, 4 dm. high, can hold, if 1 c.c. of petrol weighs 0-7 gm.
16. The volume of a rectangular block, 4 1\(\frac{1}{2}\) in. long, 2 3\(\frac{1}{2}\) in. wide, is 30 cu. in.; what is its height?
17. The volume of the lid of a chest 5 ft. 4 in. long, 2 ft. 3 in. wide is 1 1\(\frac{1}{2}\) cu. ft. Find in inches the thickness of the lid.
18. A tank 10 ft. 6 in. long, 8 ft. wide, contains 35 cu. ft. of water. Find in inches the depth of the water.
19. A rectangular tank 1 1\(\frac{1}{2}\) m. long, 88 cm. wide, contains water to a depth of 65 cm. The water is transferred to an empty tank 2 m. long, 1 m. wide; find the depth of the water.
20. A tank 6 ft. long, 5 ft. wide, 3 ft. high contains water to a depth of 1 ft. 6 in. A metal block, 4 ft. by 3 ft. by 1 ft. 3 in. is put into the tank and totally submerged. Find in inches the amount the water-level rises.
21. If the water in the tank in No. 20 is transferred to an empty tank 4 ft. 6 in. long, 3 ft. 4 in. wide, 5 ft. high, find the depth of the water-level below the top of the tank.

In Nos. 22-26, assume that 1 cu. ft. = 63.2 gal.

22. How many gallons of water will a tank 3 ft. long, 2 ft. wide, 1 ft. 4 in. deep hold?
23. A railway trough is 1 1\(\frac{1}{2}\) ft. wide, 8 in. deep, and half a mile long. How many gallons of water will it hold?
24. A swimming-bath is 100 yd. long, 80 ft. wide. How many gallons must be pumped into it to raise the water-level 1\(\frac{1}{2}\) in.?
25. A tank 5 ft. 2 in. long, 2 ft. 8 in. wide, contains 150 gall. of water; find the depth of the water.
26. Find in tons the weight of water which a tank 8 ft. long, 7 ft. wide, 6 ft. deep, will hold, given that 1 gal. of water weighs 10 lb.

In Nos. 27-31, use the fact that 1 c.c. of water weighs 1 gm.

27. Find in kg. the weight of 1\(\frac{1}{2}\) litres of water.
28. How many litres of water weigh 2 3\(\frac{1}{2}\) kg?
188 FURTHER AREAS AND VOLUMES

[29] Find in kg. the weight of water which a cistern 1.5 m. long, 1.2 m. wide, 64 cm. high, will hold.

[30] The water in a tin weighs 288 kg.; if the tin is 2.4 dm. long, 1.5 dm. wide, find the depth of the water.

31. 560 kg. of water are drawn out of a tank 2.5 m. long, 1.4 m. wide. What distance does the water-level fall?

*32. A tank 16 ft. long, 7 ft. broad, contains 12 1/2 tons of water. Find the depth of water if 1 cu. ft. of water weighs 1000 oz.

*33. A wooden beam, having a square cross-section, is 9 ft. long and weighs 3 1/4 cwt. If the wood weighs 32 lb. per cu. ft., find the thickness of the beam.

*34. Find the value of a rectangular stack of coal, 20 yd. long, 8 ft. wide, 5 1/2 ft. high, if the coal costs £1 18s. per ton, reckoning 44 cu. ft. to the ton.

35. How many loads are required to gravel a path 28 yd. long, 7 1/4 ft. wide, to a depth of 4 in., if 17 1/2 cu. ft. go to the load?

*36. An iron armour plate is 22 ft. long, 15 ft. wide, and weighs 25 tons. If 1 cu. ft. of iron weighs 3 3/4 cwt., find correct to 1/10 in. the thickness of the plate.

Material for making a Box. The method was explained on p. 23.

Example 11. Find the weight of an empty open rectangular pan made of aluminium 0.5 cm. thick, if the base is 20 cm. long, 15 cm. wide, measured internally, and if the pan can hold 13 litres of water, given that 1 c.c. of aluminium weighs 2.5 gm.

1 litre = 1000 c.c., \therefore \text{internal volume of pan = 1500 c.c.}
\therefore \text{internal height of pan = (1500 \div (20 \times 15)) cm. = 5 cm.}

Since the aluminium is 0.5 cm. thick,
\text{external length = (20 + 1) cm. = 21 cm.,}
\text{external breadth = (15 + 1) cm. = 16 cm.,}
\text{(no lid), external height = (5 + 0.5) cm. = 5.5 cm.}

Therefore, by the subtraction method,
\text{volume of aluminium = (21 \times 16 \times 5.5 - 20 \times 15 \times 5) c.c. = (1848 - 1500) c.c. = 348 c.c.}

But 1 c.c. of aluminium weighs 2.5 gm.; \therefore \text{weight of empty pan = (2.5 \times 348) gm. = 870 gm.}

VOLUME OF MATERIAL FOR MAKING A BOX 189

EXERCISE 100

Find the volume of the wood required for making the closed boxes, Nos. 1–6:
1. External dimensions : 10 in. by 9 in. by 8 in.; wood \frac{3}{4} in. thick.
2. External dimensions : 4 in. by 3 in. by 2\frac{1}{2} in.; wood \frac{1}{8} in. thick.
3. External dimensions : 14 cm. by 9.5 cm. by 6 cm.; wood 7.5 mm. thick.
4. Internal dimensions : 4 in. by 3\frac{1}{2} in. by 2 in.; wood \frac{1}{4} in. thick.
5. Internal dimensions : 2 ft. 5 in. by 1 ft. 3 in. by 1 ft. 1 in.; wood \frac{1}{4} in. thick.
6. Internal dimensions : 20 cm. by 12.5 cm. by 9.5 cm.; wood 1.25 cm. thick.

Find the volume of the wood required for making the open boxes, Nos. 7–10:
7. External dimensions : 11 in. long, 6 in. wide, 5 in. high; wood \frac{1}{8} in. thick.
8. External dimensions : 17.5 cm. long, 14 cm. wide, 10 cm. high; wood 7.5 mm. thick.
9. Internal dimensions : 2 ft. 6 in. long, 1 ft. 7 in. wide, 9 in. high; wood \frac{1}{8} in. thick.
10. Internal dimensions : 2 4\frac{1}{2} dm. long, 1.5 dm. wide, 9.5 cm. high; wood 5 mm. thick.

11. Find the weight of a closed box, made of wood \frac{3}{4} in. thick, measuring internally 1 ft. by 8 in. by 5 in., if the wood weighs 48 lb. per cu. ft.
12. An open rectangular tank is made of concrete, the sides and base being 1 ft. thick. Externally, the tank is 13 ft. long, 8 ft. broad, 5 ft. high. Find its weight in tons, if the concrete weighs 140 lb. per cu. ft.
13. The external dimensions of a closed rectangular cistern are 3 ft. 6 in. by 2 ft. 9 in. by 2 ft. 3 in., and the thickness of the material is \frac{3}{8} in. How many gallons, to the nearest gallon, will the cistern hold? [1 cu. ft. = 6\frac{3}{4} gall.]

*14. A wooden door 81 in. high, 32 in. wide is 2 in. thick, except for 4 rectangular panels, each 30 in. by 10 in., which are only \frac{1}{4} in. thick. Find the amount of wood in the door and the weight of the door if the wood weighs 48 lb. per cu. ft.
FURTHER AREAS AND VOLUMES

15. The foundations of the outer walls of a rectangular building are of concrete to a depth of 3 ft. and a width of 15 in., and the external dimensions are 70 ft. by 38 ft. How many tons of concrete, to the nearest ton, were required, if 1 cu. ft. of concrete weighs 150 lb?

16. The external dimensions of a closed chest, made of wood 1\(\frac{1}{2}\) in. thick, are 1 yd. by 2 ft. 6 in. by 2 ft. 3 in. Find the cost of lining the whole of the interior at 6d. per sq. ft.

**Volume of Solid of uniform cross-section**

The **abbreviated statement**

**Volume = area of cross-section x length**

is true for any shape of cross-section, provided only that it is uniform.

**Example 12.** A water-can of uniform cross-section holds 2\(\frac{1}{2}\) gallons. The area of its base is 54 sq. in., internal measurement; find its internal height, given 1 cu. ft. = 63 gal.

\[ 2\frac{1}{2} \text{ gal.} = (2\frac{1}{2} \times 63) \text{ cu. ft.} = \left(\frac{5}{2} \times \frac{63}{12} \right) \text{ cu. ft.} = \frac{9}{2} \text{ cu. ft.}; \]

\[ \therefore \text{volume of can} = 2\frac{1}{2} \times 54 \text{ cu. in.}, \text{ and area of base} = 54 \text{ sq. in.}; \]

\[ \therefore \text{internal height} = \text{volume} \div \text{area of base} \]

\[ = \left(\frac{2\times54}{54}\right) \text{ in.} = \frac{2}{5} \times 1\frac{3}{5} \text{ in.} \]

\[ = 8\\frac{1}{5} \text{ in.} = 12\frac{3}{5} \text{ in.} \]

**Example 13.** The dimensions of the cross-section of a steel girder are shown in inches in the given diagram. If the steel weighs 4-8 oz. per cu. in., find the weight of the girder per foot-run, to the nearest lb.

First find the area of the cross-section.

\[ \sqrt[4]{8\frac{1}{2}} \]

Area of two cross-pieces = \(8\frac{1}{2} \times 1\frac{1}{2} \times 2\) sq. in.

\[ = 25\frac{1}{2} \text{ sq. in.;} \]

width of connecting portion = \(8\frac{1}{2} - 3\frac{1}{2} - 3\frac{1}{2}\) in.

\[ = 1\frac{1}{2} \text{ in.;} \]

\[ \therefore \text{area of connecting portion} = (5 \times 1\frac{1}{2}) \text{ sq. in.;} \]

\[ = 7\frac{1}{2} \text{ sq. in.;} \]

\[ \therefore \text{area of cross-section} = (25\frac{1}{2} + 7\frac{1}{2}) \text{ sq. in.} = 33 \text{ sq. in.}; \]

\[ \therefore \text{volume of portion of girder} \text{ 1 ft. long} = \text{12 in. long}, \]

\[ = \text{area of cross-section} \times \text{length} = (33 \times 12) \text{ cu. in.} \]

VOLUME OF SOLID OF UNIFORM CROSS-SECTION

But 1 cu. in. of steel weighs 4-8 oz.,

\[ \therefore \text{weight of girder per foot-run} = \left(4\frac{4}{5} \times 33 \times 12\right) \text{ oz.} \]

\[ = 18\frac{2}{5} \times 1\frac{3}{4} \text{ lb.} = 118 \frac{1}{8} \text{ lb.} \]

\[ \therefore \text{weight per foot-run} = 119 \text{ lb., to nearest lb.} \]

**Note.** The area of the cross-section can also be found as follows: From a rectangle \(8\frac{1}{2} \times 6\frac{1}{2}\) in. wide, \((1\frac{1}{2} \times 1\frac{1}{2})\) in. high, subtract two rectangles, each \(3\frac{1}{2}\) in. by \(5\) in.

**EXERCISE 101**

Find the volume of a rail of uniform cross-section, given:

1. Area of cross-section = 4.5 sq. in., length = 1 ft. 6 in.
2. Area of cross-section = 12.8 sq. cm., length = 1.25 m.

Find the length of a girder of uniform cross-section, given:

3. Volume = 1500 cu. in., area of cross-section = 7.5 sq. in.
4. Volume = 44 cu. dm., area of cross-section = 12.5 sq. cm.

Find the area of the cross-section, assuming it to be uniform, of a solid, given:

6. Volume = 928 c.c., length = 6.4 m.
7. A vessel of uniform cross-section of area 8.4 sq. dm. contains 5.46 litres of water. What is the depth of the water?
8. What is the base area of a vessel of uniform cross-section of internal height 3.75 dm., if its capacity is 1\(\frac{1}{2}\) litres?

9. The area of the cross-section of a steel rail is 15 sq. cm.; find the weight of the rail per metre run, if 1 c.c. of steel weighs 7.8 gm.

10. The dimensions of the L-shaped cross-section of a bar, 2 ft. long, are shown in inches in the diagram. Find (i) the volume of the bar, (ii) the weight if the material weighs 9.3 lb. per cu. in.

11. The dimensions of the cross-section of a girder 2.5 m. long are shown in cm. in the diagram. Find (i) the volume of the girder, (ii) the weight if the material weighs 7.8 gm. per c.c.
FURTHER AREAS AND VOLUMES

12. 1 in. of rain over an area of 150 sq. yd. is collected in a tank 10 ft. long, 8 ft. wide. What is the rise of water in the tank in inches?

[18] An empty rectangular tank is 5 ft. long, 4 ft. wide, 3 ft. deep. Rain water runs into it from roofs of total horizontal area 100 sq. yd. What depth of rainfall in inches will fill the tank?

[14] The average depth of a pond of area 3 ac. is 7 in. If 1 cu. ft. of water weighs $62\frac{1}{2}$ lb., find the weight of water in the pond, to the nearest 10 tons.

15. A log of wood 10 ft. long has a uniform cross-section of area 180 sq. in. Find its weight if the wood weighs 44 lb. per cu. ft.

16. A can of uniform cross-section contains 1 gal. of water. Find, to the nearest sq. in., the area of the cross-section if the water is 8 in. deep. [1 gal. = 277 cu. in.]

*17. A rectangular gutter is 4$\frac{1}{2}$ in. wide, 2$\frac{1}{2}$ in. deep. If water flows along it at 4 ft. per second, find the number of gallons which pass a given point in 8 min., if the gutter remains full. [1 cu. ft. = 6$\frac{3}{4}$ gal.]

*18. A metal rail 7 yd. long of uniform cross-section weighs 6$\frac{1}{2}$ cwt. If the metal weighs 4$\frac{1}{2}$ oz. per cu. in., find the area of the cross-section of the rail.

*19. The diagram shows the dimensions in feet of the ground plan of a trench of uniform width, 4 ft. If a man can excavate 25 cu. ft. per hour, how long will it take 12 men to dig the trench to a depth of 5$\frac{1}{2}$ ft.?

*20. The cross-section of a pipe is 4 sq. in., and water is pouring out of it at the rate of 6 ft. per second. If the pipe remains full, find the number of gallons discharged by the pipe in 5 min. [1 cu. ft. = 6$\frac{3}{4}$ gal.]

MISCELLANEOUS EXAMPLES

EXERCISE 102

1. The area of a rectangular board is 20 sq. ft. 81 sq. in.; the width is 3 ft. 11 in., find the length.

2. Blankets measuring 56 in. by 78 in. cost 22s. 9d. per pair. Find the cost per sq. yd.

3. Find the weight of a rectangular block of wood, 2 ft. by 9 in. by 1$\frac{1}{2}$ in., if 1 cu. ft. of the wood weighs 45 lb.

4. A hall is 40 ft. 6 in. long, 15 ft. 6 in. high, and its width is $3 \frac{1}{2}$ of its length. Find the total area of the four walls.

5. A border 21 in. wide all round the edge of the floor of a room, 21 ft. 6 in. by 20 ft., is stained at a cost of 2d. per sq. ft. Find the cost.

6. 2250 gal. of water are drawn out of a tank, 18 ft. long, 15 ft. wide; find how much the water-level falls. [1 cu. ft. = $6\frac{3}{4}$ gal.]

7. A fertiliser costs 7s. 6d. for a 28-lb. bag. It is used at the rate of 2 oz. per sq. yd. Find the cost for a lawn, 42 yd. long, 80 ft. wide.

8. A swimming-bath 30 yd. long, 40 ft. wide, is being filled by a pipe which delivers 60 cu. ft. of water a minute. How long will it take for the water-level to rise 1 in.?

9. A metal sheet, 8 in. long, 6 in. wide, weighs 9 lb. 6 oz. If 1 cu. ft. of the metal weighs 450 lb., find the thickness of the sheet.

10. A metal sheet, 2 ft. 2 in. long, 1 ft. 4 in. wide, 1$\frac{1}{2}$ in. thick, weighs 15 lb. Find the weight of 1 cu. ft. of the metal, to the nearest lb.

11. The cross-section of a pipe is 42 sq. cm., and water is pouring out of it at the rate of 1$\frac{1}{2}$ m. per second. If the pipe remains full, find the number of litres discharged per minute.

12. A rectangular grass field, 260 yd. long, 180 yd. wide, is surrounded by a paved walk 2$\frac{1}{2}$ yd. wide, and a roadway 15 yd. wide runs round outside the pavement. Find the area of (i) the pavement, (ii) the roadway.

13. An open cardboard box, 1 ft. long, 8 in. wide, 5 in. high, is fitted with a cardboard lid which covers the top and one of the larger sides and both of the smaller sides. Find the total area of cardboard required for the box and lid.

14. A lawn, 30 yd. long, 40 ft. wide, is surrounded by a path 5 ft. wide. How many cu. ft. of gravel are required to cover the path to a depth of 3 in.?

15. The external dimensions of a closed rectangular cistern are 5-5 dm. by 3-6 dm., by 2-5 dm., and the material is 5 mm. thick. How many litres will the cistern hold?

16. Find in lb. the weight of a box, without a lid, 2 ft. 4 in. long, 2 ft. 1$\frac{1}{2}$ in. wide, 1 ft. 4$\frac{1}{2}$ in. high, external measurements, made of wood 3$\frac{1}{2}$ in. thick, if 1 cu. ft. of the wood weighs 48 lb.
17. A roll of wire 375 m. long weighs 2.88 kg. Find the area of the cross-section of the wire if the material weighs 9.6 gm. per c.c.

18. A block of copper, 2 in. by 1\(\frac{3}{4}\) in. by 6 in. weighs 63 lb. It is drawn out into wire of cross-sectional area \(\frac{3}{8}\) sq. in. Find the weight of a length of 35 ft. of the wire.

CHAPTER XIV

CIRCLES AND CYLINDERS

CIRCUMFERENCE OF A CIRCLE

Greek letter \(\pi\), and a convenient approximation for \(\pi\) is \(\frac{22}{7}\), since \(22\approx 3.1428\)

We therefore write

\[
\text{circumference} = \pi \times \text{diameter}
\]

But if the radius of a circle is \(r\) units, its diameter is \(2r\) units,

\[
\therefore \text{the circumference of a circle, radius } r \text{ units, is } 2\pi r \text{ units, where } \pi = \frac{22}{7} \text{ or, to a closer approximation, } \pi = 3.1416.
\]

Approximate results should be expressed as decimals, not as vulgar fractions.

Example 3. Find the circumference of a circle of radius 4 cm.

\[
\text{Circumference} = \pi \times \text{radius} = 3.14 \times 4 = 12.56 \text{ cm.}
\]

Example 4. Find the radius of a circle whose circumference is 10 in.

If the radius is \(r\) in., the circumference is \(2\pi r\) in.;

\[
\therefore 2\pi r = 10; \quad r = \frac{10}{2\pi} = \frac{5}{\pi}; \quad \frac{5}{\pi} = \frac{5 \times 7}{22} = \frac{35}{22} = 1.59
\]

\[
\therefore \text{the radius is } 1\frac{5}{22} \text{ in., approximately.}
\]

Note. Do not give the answer as \(1\frac{5}{22}\) in., because vulgar fractions are generally used only for exact results.

Do not substitute for \(\pi\) its approximate numerical value before it is necessary to do so.

Example 5. The diameter of the wheel of a car is \(2\frac{1}{2}\) ft. Find the number of revolutions made by the wheel per minute when the car is travelling at 40 mi. an hour.

In 60 min. the car travels \((40 \times 1760 \times 3)\) ft.,

\[
\therefore \text{in 1 min. the car travels } \frac{40 \times 1760 \times 3}{60} \text{ ft.}
\]

Since the circumference of the wheel, diameter \(2\frac{1}{2}\) ft., is \((\pi \times \frac{5}{2})\) ft.,

the car travels \(\frac{5\pi}{2}\) ft. for each revolution of the wheel.

\[
\therefore \text{the number of revolutions made in 1 min.}
\]

\[
= \frac{40 \times 1760 \times 3}{2 \pi} = \frac{2 \times 1760 \times 3}{\pi} = \frac{2 \times 1760 \times 3}{\pi}
\]

\[
= \frac{2 \times 1760}{\pi} = 448, \quad \therefore \text{the wheel makes 448 revs. per min., approximately.}
\]
EXERCISE 103

[Do not give any of the answers to more than 3 figures]

Find the lengths of the circumferences of the circles, Nos. 1-9:

[Take \( \pi = 3\frac{1}{2} \) in Nos. 1-3, and take \( \pi = 3\frac{1}{14} \) in Nos. 4-9; \( 3\frac{1}{2} \) is a slightly closer approximation to \( \pi \) than 3-14.]

1. Radius, 14 in. 2. Diameter, 5-6 cm. 3. Radius, 6-3 in.
4. Radius, 3 in. 5. Diameter, 4-5 in. 6. Diameter, 20 in.

Find the radii of the circles, Nos. 10-15:

[Take \( \pi = 3\frac{1}{2} \) in Nos. 10-12, and take \( \pi = 3\frac{1}{14} \) in Nos. 13-15.]

10. Circumference, 11 in. 11. Circumference, 8-8 cm.
12. Circumference, 1 ch. 13. Circumference, 100 yd.
14. Circumference, 6-4 dm. 15. Circumference, 8 in.

The minute-hand of a clock is 8 in. long. What distance does the tip of this hand move in 1 hr.? [Take \( \pi = 3\frac{1}{14} \).

17. A \( \frac{1}{2} \)-mi. running-track is a circle. Find its radius in yards. [Take \( \pi = 3\frac{1}{8} \).

[For the remainder of this exercise, take \( \pi \) to be 3-14 or \( 3\frac{1}{8} \), whichever is the more convenient. Give the answers as decimals, not vulgar fractions.]

18. The diameter of a semicircular protractor is 3-5 in.; find its perimeter.

19. AB is a diameter of a circular pond of radius 100 yd. How much farther does a man go who walks from A to B round the edge than a man who rows straight across the pond?

*20. A boy finds the value of \( \pi \) experimentally by wrapping a piece of thread 5 times round a cylinder of diameter 5 in. He finds that the length of the thread when unwrapped is 78-65 in. What value for \( \pi \) should he obtain?

21. A bicycle wheel is 28 in. in diameter. How many revolutions does it make per mile?

22. A bicycle wheel, diameter 28 in., is making 25 revolutions in 10 sec. At what speed in miles per hour is the bicycle travelling?

*29. A dog-cart is being driven at 10 mi. an hour. If each wheel is 35 in. in diameter, find the number of revolutions made by each wheel per minute.

*30. A wheel of diameter 7 in. is rotating at 3000 revolutions per minute. Find the speed of a point on the rim in feet per second.

*31. A bucket is raised from the bottom of a well 55 ft. deep by a rope wound on an axle of diameter \( \frac{1}{4} \) ft. How many turns of the axle are required to bring the bucket up to the top? [Neglect the thickness of the rope.]

26. The radii of the inner and outer edges of a circular running-track are 70 yd. and 75 yd. What is the difference between the lengths of the two edges? Would it be the same if the two radii were 100 yd. and 105 yd.?

*32. The inner edge of a circular running-track is \( \frac{1}{2} \) mi. long. If the track is 10 yd. wide, find the length of the outer edge of the track.

*33. A quadrant (i.e. quarter of a circle) of radius 8 in. is cut away from each corner of a rectangle 3 ft. long, 2\(\frac{1}{2} \) ft. wide. Find the perimeter of the remaining figure.

28. An arc AB of a circle, of radius 10 in., subtends an angle 144° at the centre O of the circle. Find the length of the arc AB. [arc = \( \frac{\theta}{360} \) of circumference.]

*34. An arc AB of a circle subtends an angle 144° at the centre O of the circle. If the length of the arc AB is 44 cm., find the radius of the circle.

31. A rectangular table-top has its corners rounded off into quarter-circles of 5 in. radii. By how much, to the tenth of an inch, is the perimeter of the table-top reduced?

35. Four equal tins of circular section, radius 2\(\frac{1}{2} \) in., stand touching two by two. Find, to the nearest inch, the shortest length of string required for tying them together, allowing 3 in. for the knot.

36. A piece of wire is in the form of an arc of a circle of radius 10 in., subtending an angle 150° at the centre of the circle. It is bent into the form of a complete circle; find the radius of this circle.

*37. A piece of wire 1 yd. long is bent into the form of a semicircular arc and its diameter. Find the radius.
CIRCLES AND CYLINDERS

Area of a Circle

Example 6. A circle of radius 3 in. is drawn on a piece of squared paper, ruled in inches and tenths of an inch. It is found by counting that there are about 2828 small squares inside the circle. In counting, disregard those small squares for which less than half the square lies inside the circle, and count as whole squares those for which more than half the square is inside the circle. Use this result to calculate the ratio of the area of the circle to the area of the square on the radius.

The area of 100 small squares = 1 sq. in.; \( \therefore \) the area of 2828 small squares = 28-28 sq. in.

Since the radius is 3 in., area of square on radius = 9 sq. in.; \( \therefore \frac{\text{area of circle}}{\text{area of square on radius}} = \frac{28.28}{9} \approx 3.14. \)

The reader should repeat this experiment with another circle, say of radius 2 in.; he will then obtain approximately the same value for this ratio; and these results suggest that the value of this ratio is actually the number denoted by \( \pi \). Thus

\[ \frac{\text{area of circle}}{\text{area of square on radius}} = \pi \quad \text{or} \quad \text{area of circle} = (\text{square on radius}) \times \pi. \]

But if the radius of a circle is \( r \) units, the square on the radius contains \( r^2 \) units of area;

\( \therefore \) the area of a circle, radius \( r \) units, is \( \pi r^2 \) units of area.

The reader should use this fact to show that the area of a circle of diameter \( d \) in. is \( \frac{1}{4} \pi d^2 \) sq. in.

If \( \pi \) is taken as \( 3 \frac{1}{3} \) or as 3-14, results must not be given to more than 3 figures.

Example 7. Find the area of a circle whose diameter is 9 cm.

\[ 9 \times \pi \div 2 = 3 \times \pi = 3\cdot14 \]

Since the diameter is 9 cm., the radius = 4-5 cm.

\[ 2 \times 2 \times 4 \cdot 5 = 3 \cdot 14 \times 20 \times 25 = 8100 \approx 63 \cdot 6 \text{ sq. cm.} \]

Example 8. Find the diameter of a circle whose area is 154 sq. in.

If the radius of the circle is \( r \) in., the area = \( \pi r^2 \) sq. in.;

\[ \therefore \pi r^2 = 154; \quad \therefore r^2 = \frac{154}{\pi} = 49; \quad \therefore r \approx 7. \]

\( \therefore \) the diameter of the circle is 14 in., approximately.

Area of a Circle

The figure bounded by two concentric circles is called an annulus. If the radii of the outer and inner circles are \( R \) in. and \( r \) in., area between circles = \( \pi R^2 - \pi r^2 \) sq. in.

\[ = \pi (R^2 - r^2) \text{ sq. in.} \]

and it is sometimes easier to express this in factors:

\[ \text{area of annulus} = \pi (R + r) (R - r) \text{ sq. in.} \]

For example, if the radii are 8-6 in. and 7-9 in.,

\[ \text{area of annulus} = \pi (8 \cdot 6 + 7 \cdot 9) (8 \cdot 6 - 7 \cdot 9) \text{ sq. in.} \]

\[ = (52 \cdot 9 \cdot 16 \cdot 5 \cdot 0 \cdot 7) \text{ sq. in.} \]

\[ = (22 \times 16 \cdot 5 \times 0 \cdot 1) \text{ sq. in.}, \text{ that is 36} \cdot 3 \text{ sq. in.} \]

EXERCISE 104

[Do not give any of the answers to more than 3 figures]

Find the areas of the circles, Nos. 1–6:

\[ \text{Take } \pi = 3 \frac{1}{3} \text{ in Nos. 1–3, and take } \pi = 3 \cdot 14 \text{ in Nos. 4–6:} \]

1. Radius, 7 cm. \( [9] \) Radius, 1 ft. 9 in. \( \text{3. Diameter, 2} \cdot 8 \text{ in.} \]

4. Radius, 10 in. \( [5] \) Diameter, 11 cm. \( \text{6. Diameter, 5 ft.} \]

Find the radii of the circles whose areas are given in Nos. 7, 8, taking \( \pi = 3 \frac{1}{3} : \)

7. (i) 616 sq. in.; (ii) 38-5 sq. cm. \( \text{8. Diameter, 5} \cdot 5 \text{ in.} \)

*8. Find the diameter of a circle of area 5544 sq. yd.

[Take \( \pi = 3 \cdot 14 \).]

[For the remainder of this exercise, take \( \pi \) to be \( 3 \cdot 14 \) or \( \frac{22}{7} \), whichever is the more convenient, except where otherwise stated. Give the answers as decimals, not vulgar fractions.]

[10] Find the area of the top of a penny, diameter 1-3 in.

[11] Find the area of a semicircle of diameter 3 ft. 6 in.

[12] A circle of diameter 3 in. is cut out of a sheet of paper 3 in. square; what is the area of the remainder?

[13] A circle of radius 4 in. is drawn on squared paper ruled in inches and tenths of an inch, and the number of small squares inside the circle is counted. What should be the result?

[14] A donkey's head is tied by a rope 20 yd. long to a post in a fenced field 50 yd. square. What area of the ground can the donkey cover if the post is (i) at a corner of the field, (ii) at the middle point of one side of the field?]
CIRCLES AND CYLINDERS

15. Find the circumference of a circle whose area is 3850 sq. yd.
16. 440 yd. of fencing are available for enclosing part of a field. Find the area of the enclosure (i) if square, (ii) if circular.
17. How many plants can be put in a circular flower-bed whose circumference is 5 ft. 6 in., allowing 20 sq. in. for each plant?
18. Find the area of the ring between two concentric circles of radii 3 in. and 4 in.
19. From a metal sheet, 4 in. square, four quadrants of a circle, each of radius 2 in., are cut away at the corners. Sketch the shape of what remains, and find its area.
20. A circular pond of diameter 28 ft. is surrounded by a path 7 ft. wide. Find the area of the path.
21. A circular metal plate of radius 10 in. weighs 5 lb.; find the weight of a plate of the same material and thickness of radius 1 ft. [There is no need to substitute for \( \pi \).]
22. Find the radius of a circle whose area is equal to the sum of the areas of two circles of radii 3 in., 4 in. respectively. [There is no need to substitute for \( \pi \).]
23. A rectangular lawn 15 yd. by 10 yd. is surrounded by flower-beds: a man can, without stepping off the lawn, water the ground up to a distance of 2 yd. from the edge. What is the total area of the flower-beds he can so water?
24. OA, OB are two radii of a circle, centre O, radius 4 in.; if the angle between OA and OB is 144°, find the area of the part of the circle between OA and OB. [This area is called a sector of the circle of angle 144°. What fraction is the area of the sector of the area of the circle?]
25. Find the area of a sector of a circle of radius 6 cm., if the angle of the sector is 108°. [Use the method indicated in No. 24.]

Area of Surface of a Circular Cylinder

A solid, whose cross-section is uniform and circular, such as a curtain rod or a pencil of circular section, is called a circular cylinder, and the radius of the cross-section is called the radius of the cylinder, and the straight line which passes through the centre of every cross-section is called the axis of the cylinder. The length of the axis is often called the length of the cylinder, as for a pencil, but

AREA OF SURFACE OF A CIRCULAR CYLINDER

This depends to some extent on the kind of cylindrical object. Thus for a jug of circular cross-section, the length of the axis is called the height of the cylinder, while for a circular disc, like a penny, the length of the axis is called the thickness of the cylinder, and for a garden roller the length of the axis is called the whish of the cylinder. The name used is merely a matter of common sense.

A rectangular sheet of paper can be rolled up into the shape of a circular cylinder with open ends. Therefore

\[ \text{the area of the curved surface of a cylinder} = \text{perimeter of base} \times \text{height of cylinder}. \]

If the radius of a cylinder is \( r \) units and if the height of the cylinder is \( h \) units, the perimeter of the base is \( 2\pi r \) units,

\[ \text{area of curved surface of cylinder} = 2\pi rh \text{ units of area}. \]

In order to find the area of the total surface of a solid circular cylinder, it is necessary also to calculate the area of each end.

The area of each end is the area of a circle of radius \( r \) units, and is therefore \( \pi r^2 \) units of area.

\[ \text{total area of surface} = (2\pi rh + 2\pi r^2) \text{ units of area} = 2\pi r(h + r) \text{ units of area}. \]

Example 9. Find the total area of the surface of a solid cylinder, base-radius 3 in., height 4 in.

Perimeter of base \( = (2\pi \times 3) \text{ in.} = 6\pi \text{ in.} \)

\[ \text{area of curved surface} = (6\pi \times 4) \text{ sq. in.} = 24\pi \text{ sq. in.} \]

and area of base \( = (\pi \times 3^2) \text{ sq. in.} = 9\pi \text{ sq. in.} \)

\[ \text{total area of surface} = (24\pi + 9\pi \times 2) \text{ sq. in.} = 42\pi \text{ sq. in.} \]

\[ \approx 42 	imes 3.142 \text{ sq. in.} \]

\[ \text{total area of surface is 132 sq. in., approximately.} \]

Notice that in the above working the substitution for \( \pi \) is done as late as possible.

EXERCISE 105

[Nos. 1, 2 are intended for Oral work]

1. A rectangular sheet of paper 11 in. wide, 5 in. high, is rolled into the form of an open hollow cylinder, 5 in. high. What is the greatest possible diameter of the cylinder (take \( \pi = \frac{44}{7} \))? What is the area of its curved surface? What is the area of the smallest piece of paper which will cover one end of it?
202 CIRCLES AND CYLINDERS

2. The radius of an open hollow cylinder is 3.5 cm. and the height of the cylinder is 6 cm. A cut is made straight down the surface so that the cylinder can be unwrapped to form a rectangle. Find the breadth, height, and area of the rectangle.

Find the areas of the curved surfaces of the cylinders, Nos. 3-6:

4. Diameter 2.8 cm., width 8 cm.
5. Diameter 1 yd., thickness \( \frac{1}{4} \) in.

Find the total areas of the surfaces of the solid cylinders, Nos. 7-10:

9. Diameter 1 ft., width 1 yd. 10. Diameter 5 cm., length 12 cm.

Find the total area of the external surface of a cylinder closed at one end and open at the other end, with the given external measurements, Nos. 11, 12:


Find the radius of each cylinder, Nos. 13, 14:

13. Area of curved surface 110 sq. in., height 5 in.
14. Area of curved surface 79.2 sq. cm., height 3 cm.

Find the height of each cylinder, Nos. 15-17:

15. Area of curved surface 132 sq. cm., radius 6 cm.
16. Area of curved surface 132 sq. cm., radius 6 cm. [17] Total exterior area of surface of hollow cylinder closed at one end 198 sq. cm., exterior diameter 6 cm.

18. Find the area of thin tin sheeting required for making a tin cylinder, radius 4 cm., height 6 cm., with a slip-on lid which overlaps 5 mm.

19. The funnel of a ship is 12 ft. in external diameter and 60 ft. high. Find the number of pounds of paint required to cover it externally, if 1 lb. of paint covers 72 sq. ft.

20. A garden roller is 2 ft. 6 in. in diameter and is 3 ft. 6 in. wide. What area does it cover in 40 revolutions?

21. The diagram shows the section of a closed tin by a plane through the common axis of the two cylinders which compose it; dimensions in inches. Find the area of thin tin-sheeting required for making it.

VOLUME OF CIRCULAR CYLINDER 203

23. A cylindrical tank, 7 ft. in diameter, contains water to a depth of 4 ft. Find the total area of the wetted surface.

24. The internal section of a tunnel is a rectangle 24 ft. wide, surrounded by a semicircle, and the greatest height is 20 ft. Find the total area of the sides and roof of the tunnel, if it is 50 yd. long.

24. A fence contains 36 cylindrical posts, each 4 in. in diameter and 2 ft. 10 in. high. The sides and tops of the posts are painted. Find the total cost at 3d. per sq. ft.

Volume of Circular Cylinder. The volume of any solid of uniform cross-section, see p. 190, is given by the rule, area of cross-section \( \times \) length.

If a circular cylinder is of radius \( r \) units and of height \( h \) units, the area of its cross-section is \( \pi r^2 \) units of area,

\[ \text{volume of cylinder} = \pi r^2 h \text{ units of volume}. \]

Also if a hollow tube is \( l \) in. long, and if its outer and inner radii are \( R \) in., \( r \) in.,

\[ \text{volume of material composing tube} = \pi (R^2 - r^2) l \text{ cu. in.} = \pi l (R + r)(R - r) \text{ cu. in.} \]

Example 10. Find the number of gallons of water in a cylindrical tank, 8 ft. in diameter, if the water is 3 ft. deep. [1 cu. ft. = 6\( \frac{1}{4} \) gall.]

Since the radius is 4 ft., and the depth of water is 3 ft.,

Volume of water \( = (\pi \times 4^2 \times 3) \text{ cu. ft.} = (\pi \times 16 \times 3 \times \frac{1}{8}) \text{ gal.} \]

\( = 300 \pi \text{ gal.} \approx (300 \times 3.14) \text{ gal.} \]

\( \approx 942 \text{ gal.} \)

Example 11. A cylindrical tank, 3 ft. in diameter, contains 150 gal. of water. Find the depth of the water. [1 cu. ft. = 6\( \frac{1}{4} \) gal.]

150 gal. occupy \( (150 \times \frac{1}{6}) \) cu. ft., that is 24 cu. ft.

Radius of base \( = \frac{3}{2} \) ft., \( \therefore \) area of base \( = \pi \times \left( \frac{3}{2} \right)^2 \text{ sq. ft.} \)

\( = \text{depth of water} = \text{volume of water} \div \text{area of base} \)

\( = \left( 24 \times \frac{1}{6} \right) \text{ ft.} = \frac{12}{3} \text{ ft.} \)

\( \approx 4 \times \frac{1}{8} \text{ ft.} = 8 \pi \text{ ft.} \)

\( \approx 24 \times \frac{1}{8} \times \frac{1}{8} \text{ ft.} \)

\( = \frac{1}{3} \text{ ft.} \approx 39 \text{ ft.} \)
Example 12. Find the volume of metal in a hollow pipe, 2 ft. long, of internal diameter 10 in., made of metal $\frac{1}{4}$ in. thick.

Internal radius = 5 in., \( \therefore \) external radius = (5 + $\frac{1}{4}$) in. = 5$\frac{1}{4}$ in.

The cross-section is the area between two concentric circles, radii 5$\frac{1}{4}$ in. and 5 in.;

\[ \therefore \text{area of cross-section} = \pi \left( \left(\frac{5\frac{1}{4}}{2}\right)^2 - 5^2 \right) \text{ sq. in.} \text{, see p. 199}, \]

\[ = \left(\pi \times \frac{21}{16} \times \frac{1}{2}\right) \text{ sq. in.} \]

But the length of the pipe is 24 in.,

\[ \therefore \text{volume of metal} = \left(\frac{21}{16} \times \frac{1}{2} \times 24\right) \text{ cu. in.} \text{, that is 396 cu. in.} \]

EXERCISE 106

[Take 1 cu. ft. = 6$\frac{1}{4}$ gall.; do not give results to more than 3 figures]

Find the volumes of the circular cylinders, Nos. 1-4:

1. Radius 3$\frac{1}{2}$ in., height 6 in.
2. Diameter 2:1 cm., length 2 dm.
3. Diameter 1:4 m., thickness 1 cm.
4. Radius, 0:35 in., length 2 ft.

Find the heights of the circular cylinders, Nos. 5, 6:

5. Volume 66 cu. in., radius 2 in.
6. Volume 4 litres, radius 5 cm.

Find the diameters of the circular cylinders, Nos. 7, 8:

8. Volume 385 cu. cm., height 1 dm.
9. A telegraph pole is 21 ft. high and 8 in. in diameter. Find its weight if the wood weighs 36 lb. per cu. ft.
10. Find the weight of 10 m. of silver wire, diameter 4 mm., if 1 c.c. of silver weighs 10-5 gm.
11. A cylindrical tank is 4 ft. in diameter and 5$\frac{1}{2}$ ft. high, internal measurements. How many gallons will it hold?
12. A cylindrical tank, 6 ft. in diameter, contains 550 gall. of water. Find the depth of the water.
13. A circular lawn, diameter 40 m., is given a top-dressing 1 cm. thick. Find, to the nearest kg., the weight of the amount required, if 1 cu. m. of dressing weighs 2-7 kg.

VOLUME OF A CIRCULAR CYLINDER

Find the volume of metal in the hollow pipes, Nos. 14-16:

14. Internal radius 3 cm., metal 1 cm. thick, length 6 cm.
15. External diameter 11 in., metal $\frac{1}{4}$ in. thick, length 1 ft.
16. External diameter 6 in., metal $\frac{1}{4}$ in. thick, length 100 yd.

17. How many cylindrical glasses, diameter 3 in., height 5 in., can be filled from a cylindrical vessel, diameter 1 ft., height 2 ft. 6 in., full of milk. [There is no need to substitute for $\pi$.]

18. A cylindrical jar, diameter 6 in., depth 8 in., is full of water. If this water is poured into an empty cylindrical jar of diameter 4 in., find the depth of the water. [There is no need to substitute for $\pi$.]

19. Find the weight per metre of wire of sectional area 0:13 sq. cm., if 1 cu. dm. of the material weighs 7-7 kg.

20. A solid cylinder 5 in. in diameter, 6 in. long, is packed in a box which measures internally 5 in. by 5 in. by 6 in., and the empty space is filled with sawdust. Find the volume of sawdust required.

21. Find the volume of the closed tin whose dimensions are given in Ex. 105, No. 21.

22. A groove of semicircular section, radius 3 in., is made across one face of a wooden cube of edge 1 ft., parallel to an edge. Find the volume of the solid which remains.

23. Water is pouring into a cylindrical tank of diameter 20 ft. at the rate of 4000 gall. per minute. Find, to the nearest inch, the rise of water-level per minute.

24. Find the number of gallons discharged in an hour from a pipe 6 in. in diameter through which water is flowing at $\frac{3}{4}$ ft. per second, if the pipe remains full.

25. A bath holding 5$\frac{1}{4}$ cu. ft. is filled in 5 min. by water from a pipe of diameter $\frac{1}{2}$ in. At what speed in ft. per sec. is the water flowing in the pipe, if the pipe remains full?

26. The curved surface of a cylinder is formed from a sheet of paper 6 in. square, and the ends of the cylinder are closed by circular caps. Find the greatest possible volume of the cylinder.
MISCELLANEOUS EXAMPLES

EXERCISE 107

[Do not give results to more than 3 figures]

1. The diameter of the clock-face on a town-hall is 9 ft. Find the area of the clock-face, to the nearest sq. ft.

2. The hour-hand of a clock is 4 in. long; find the distance the tip of this hand travels in 45 min.

3. The circumference of a circle is 1 mile long. Find the area of the circle, to the nearest acre.

4. The wheel of a vehicle is 35 in. in diameter. Find the number of revolutions made by the wheel for each mile the vehicle travels.

5. ABCD is a rectangular plate, AB = 6 in., BC = 3 in.; two quadrants, centres A, B, each of radius 3 in. are cut away from the plate. Sketch the shape of what remains, and find its area.

6. The perimeter of a semicircular window is 9 ft.; find (i) the diameter, (ii) the area of the window.

7. A uniform metal circular plate of radius 6 in. weighs 12 lb. Three circular holes, each of radius 2 in., are pierced in it; find the weight of the remainder.

8. The girth of a solid circular cylinder is 6 in., and its length is 11 in.; find its volume.

9. The wheel of a bicycle, 28 in. in diameter, is making two revolutions per second. At what speed in miles per hour is the bicycle travelling?

10. The base of a metal plate, 3 in. thick, is a rectangle 6 in. by 5 in., from which quadrants each of radius 2 in. have been removed at each of the four corners. Find the weight of the plate if the metal weighs 5 oz. per cu. in.

11. Find the radius of a circle whose area is equal to the sum of the areas of three circles of radii 3 in., 4 in., 1 ft., respectively.

12. A cylindrical well, internal diameter 10 ft., contains 14,000 gallons of water. Find the depth of the water, to the nearest foot.

13. An archway is formed by two vertical walls, 5 ft. high, 8 ft. apart, surmounted by a roof of semicircular section. The length of the archway is 20 ft.; find the area of its internal surface.

14. The area of a sector of a circle of radius 7 cm. is 55 sq. cm.; find the angle of the sector.

15. A path 2 yd. wide surrounds a circular pond of diameter 40 yd. How many cu. yd. of gravel are required to gravel the path to a depth of 3 in.?

16. A goat is tethered by a chain 10 yd. long to a ring which can slide along a low straight rail 15 yd. long, in the middle of a field. What is the area of ground the goat can cover?

17. A bar 2 ft. long of square cross-section, side 3 in., is recast so as to have a circular cross-section of diameter 3 in. What does its length become, to the nearest inch?

18. A swimming-bath 22 yd. long, 10 yd. wide, 7 1/4 ft. deep is filled by water issuing from a pipe of diameter 6 in. at 4 ft. per second. How many minutes does it take to fill the bath?

19. Water flows through a cylindrical pipe of diameter 10 in. at the rate of 10 ft. per second, the area of the cross-section of the stream in the pipe being 1/4 of that of the pipe. Find the number of gallons discharged per minute.

20. Four circular holes, each of diameter 3 1/2 in., are punched through a flat circular disc of diameter 25 in. Find what percentage of the disc has been removed. Find also the diameter of a circular disc of the same thickness whose volume is equal to the portion which remains.

21. The diameter of a hollow cylindrical roller, closed at both ends, is 21 in., and its width is 3 ft., external measurements. If the sides and ends are 3/8 in. thick, and if the material weighs 5 oz. per cu. in., find the weight of the roller in lb., to the nearest lb.

22. The radius of a parallel of latitude of the Earth in latitude 60° is 2000 mi. What is the difference of longitude of two places 1000 mi. apart, each in latitude 60°?

CHAPTER XV

SIMPLE INTEREST AND DISCOUNT

If a man occupies a house which does not belong to him, he pays the owner money, called the rent of the house, for being allowed to make use of it. If you put money into the Post Office Savings Bank, you are lending money to the Government, and if you deposit £1 for 1 year, the Government will pay you 6d. for being allowed to make use of your money, and this payment is called interest. Just as the house for which a man pays rent remains the
Simple Interest and Discount

Property of the landlord, so the £1 you have put in the bank remains your property; one year's rent of a house and one year's interest on money deposited in the bank are merely fair payments for being allowed to make use of what has been borrowed for the year.

Any money lent or borrowed is called the Principal, and the charge made for its use is called the interest and depends on how much has been borrowed and the length of time for which it is borrowed.

If the charge for borrowing £100 for 1 year is £4, we say that interest is reckoned at the rate of 4 per cent. per annum, or merely at 4 per cent., as the words per annum are implied if not actually stated.

Interest is usually paid at fixed intervals, yearly, half-yearly, or quarterly, just as rent is so paid; and in this case the principal is said to be lent at simple interest. If the simple interest for any given time is added to the principal, the sum is called the amount at simple interest for that time.

Thus if a man borrows £400 for 3 years at 5 per cent. per annum, the simple interest for 1 year is £10, of £400, that is £20, and therefore for 3 years is £60; and the amount at simple interest for 3 years is £460.

Compound Interest

In transactions where the interest as it falls due is not paid to the lender but is added on to, i.e. compounded with, the principal, the money is said to be lent at compound interest, and the total sum owed after any given time is called the amount at compound interest for that time; the difference between the amount and the original principal is called the compound interest.

Exercise 108 (Oral)

Find the simple interest on, and the amount at simple interest of:

1. £100 for 1 yr. at 3%.
2. £100 for 1 yr. at 4\(\frac{1}{2}\)%.
3. £100 for 3 yr. at 4%.
4. £100 for 4 yr. at 5%.
5. £100 for 2 yr. at 3\(\frac{1}{2}\)%.
6. £100 for 6 yr. at 2\(\frac{1}{2}\)%.
7. £100 for 3 yr. at 8%.
8. £100 for 3 mo. at 6%.
9. £200 for 1 yr. at 4%.
10. £300 for 1 yr. at 7%.

Calculation of Simple Interest

Method I. Direct Proportion

Example 1. Find the simple interest on £285 for 2\(\frac{1}{4}\) years at 3 per cent. Find also the amount after 2\(\frac{3}{4}\) years.

The interest on £285 for 1 year is \(\frac{7}{8}\) of £285,

\[\text{the interest on £285 for } 2\frac{3}{4}\text{ years is } \left(\frac{7}{8}\right) \times 2\frac{3}{4}\times \frac{7}{8}\].

\[\therefore \text{interest} = \£285 \times \left(\frac{7}{8}\right) \times \left(\frac{7}{8}\right) = \£211\frac{3}{4}\ = \£21\ 7s.\ 6d.\]

\[\therefore \text{the amount} = \£(285 + 217\ 7s.\ 6d.) = \£306\ 7s.\ 6d.\]

If the numbers involved have no simple factors, the interest is calculated more easily by using a practice method than by working with awkward fractions.
Example 2. Find to the nearest penny the simple interest on £239 4s. 5d. for 2\frac{3}{4} years at 4\%.

\text{Rough Estimate:} \text{ Interest on £200 for 3 yr. at 4\% is £24.}

\text{Interest on £100 for 2\frac{3}{4} years at 4\% is } \frac{1}{100} \times (4\times 2\frac{3}{4}) = \frac{1}{100} \times (92) = \frac{92}{100} = 0.92.

\therefore \text{required interest} = \frac{1}{100} \times (239 \times 0.92) = \frac{220}{100} = 2.20\text{.}

\$2.20 \approx £2.20\text{.}

\therefore \text{ required interest} = \frac{128}{100} \times £239 \times 0.92 = £229.22\text{, to 4 places.}

\text{Note. It is shorter, though less easy, to regard 12\frac{1}{2} as } \frac{1}{2} \text{ of } 12; \text{ then deduct } \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}\% \text{, that is } \frac{1}{4} \text{ of } 1\%.

Method II. Simple Interest Formula

The method of Example 1 on p. 209, which should first be worked, may be used to obtain a general formula:

\text{To find the simple interest on } £P \text{ for } T \text{ years at } R\% \text{ per annum.}

\text{The interest on } £P \text{ for } 1 \text{ year is } \frac{R}{100} \times £P = \frac{PR}{100}.

\therefore \text{the interest on } £P \text{ for } T \text{ years is } \frac{PR}{10} \times T = \frac{PR \times T}{100}.

\text{Hence we have the following formula:—}

\text{If the simple interest on } £P \text{ for } T \text{ years at } R\% \text{ per annum is } £I, \text{ then}

\[ I = \frac{PR \times T}{100} \]

Example 3. Find the simple interest on £213 6s. 8d. for 15 months at 3\% per annum.

\text{Principal} = £213 \frac{3}{4} = £213.90; \text{ time} = 15 \text{ months} = \frac{1}{4} \text{ year;}

\therefore \text{ in the formula, } P = 213.90; \quad T = \frac{1}{4}; \quad R = 3;

\therefore \text{the interest } £I \text{ is given by } I = \frac{213.90 \times \frac{1}{4} \times \frac{3}{4}}{100} = 8.

\therefore \text{the simple interest is } £8.

\text{Note. In the formula, } T \text{ is the number of years; we must therefore express 15 months as } \frac{1}{4} \text{ year.}

Example 4. Find to the nearest penny the simple interest on £239 4s. 5d. for 2\frac{3}{4} years at 4\%.

\text{4s. 5d. = 4.166... = £0.2208, to 4 places.}

\therefore \text{ with the notation of the formula,}

\[ P = 239.2208; \quad R = 0.04; \quad T = \frac{3}{4}; \]

\[ I = 239.2208 \times \frac{3}{4} \times \frac{3}{4} = 239.2208 \times \frac{9}{16} = 80.25 \times \frac{3}{4} = 60.1875 \approx £60.19\text{, to nearest penny.}

\text{Note. To multiply by 99, multiply by 100 - 1;}

\[ 239.2208 \times 99 = 239.2208 \times 100 - 239.2208 = 23922.8 - 239.2208 = 23683.579 = \frac{23683579}{8} \approx £29.60357\text{, to nearest penny.}

\text{The working may also be completed by using a practice method as in Example 2 on p. 210.}

In calculating the interest for a period between two given dates, assume that the year contains 365 days. Do not count the day when the money is deposited, but count the day when it is removed. For example, to find the number of days for the period May 28 to August 11,

(i) Subtract 28 from 31 (31 days in May) = 5 \text{ days.}

(ii) Add the days of any complete months = 30 \text{ days, i.e., June} = 30 \text{ days.}

(iii) Add the last date = 31 \text{ days, i.e., July} = 31 \text{ days}.\text{ and Aug} = 11 \text{ days.}

Therefore, the period = 75 days.

Example 5. Find to the nearest penny the simple interest on £450 at 6\% per annum from July 4 to August 16.

\text{Number of days,} \quad \text{July} = 27 \text{ days, Aug} = 16 \text{ days, i.e.,} \quad \text{Aug} = 41 \text{ days.}

\therefore \text{ with the notation of the formula,}

\[ P = 450; \quad R = 0.06; \quad T = \frac{41}{365}; \]

\[ I = 450 \times \frac{41}{365} \times 0.06 \approx £450 \times \frac{41}{365} \times 0.06 \approx £450 \times \frac{41}{365} \times 0.06 = £9.392 \approx £9.39\text{, to nearest penny.}
Find the simple interest on, and the amount at simple interest of:

1. £350 for 3 yr. at 6%.
2. £420 for 5 yr. at 2 1/2%.
3. £184 for 2 yr. at 5%.
4. £845 for 3 1/2 yr. at 6%.
5. £765 for 2 1/2 yr. at 5 1/2%.
6. £62 10s. for 2 1/2 yr. at 3%.
7. £375 for 2 yr. 8 mo. at 4 1/2%.

Find to the nearest penny the simple interest on:

9. £48 for 2 yr. at 4%.
10. £272 for 2 1/2 yr. at 3 1/2%.
11. £528 for 1 1/2 yr. at 3 1/2%.
12. £416 for 16 mo. at 4 1/2%.
13. £632 for 14 mo. at 5 1/2%.
14. £168 10s. for 3 yr. at 4%.
15. £342 16s. for 2 1/2 yr. at 3 1/2%.
16. £47 11s. 4d. for 4 yr. at 2 1/4%.
17. £169 3s. 5d. for 2 yr. at 4 1/2%.
18. £604 13s. 10d. for 1 1/2 yr. at 6%.
19. £860 5s. 7d. for 3 yr. at 4 1/4%.
20. £946 14s. 2d. for 100 days at 4 1/2%.
21. £724 from April 5 to June 10 at 4 1/2%.
22. £847 16s. 4d. from August 17 to November 4 at 5 1/2%.

Inverse Problems on Simple Interest

Example 6. At what rate per cent. per annum will £300 yield £60 interest in 5 years?

The interest on £300 for 5 years is £60.
. . . . the interest on £300 for 1 year is £12.
. . . . the interest on £100 for 1 year is £4.
. . . . the rate is 4% p.a.

Note. p.a. is often used as an abbreviation for per annum.

Example 7. In what time will £400 amount to £472 at 6% p.a. simple interest?

The interest = £472 - £400 = £72.

The interest on £400 at 6% is £24 for 1 year.
. . . . the interest on £400 at 6% is £72 for 2 1/2 years, = 3 years.

Example 8. What sum of money will yield £54 interest in 3 years at 6% p.a.?

The interest on £100 for 3 years at 6% is £18.

£18 interest is yielded by £100.
. . . . £54 interest is yielded by £\(100 \times \frac{54}{18}\), = £300.

In Nos. 1–4, find the rate per cent. p.a. if:

1. £600 yields £72 interest in 4 yr.
2. £400 amounts to £428 in 2 yr.
3. £700 yields £105 interest in 2 1/4 yr.
4. £250 amounts to £265 in 1 1/2 yr.

In Nos. 5–8, find in what time:

5. £300 will yield £60 interest at 5% p.a.
6. £800 will amount to £896 at 4% p.a.
7. £500 will yield £105 interest at 3 1/2% p.a.
8. £150 will amount to £186 at 6% p.a.

In Nos. 9–12, find what sum of money will yield:

9. £48 interest in 4 yr. at 3 1/2% p.a.
10. £70 interest in 2 1/4 yr. at 4 1/2% p.a.
11. £126 interest in 3 1/2 yr. at 6% p.a.
12. £72 interest in 9 yr. at 2 1/4% p.a.

13. Find the rate per cent. p.a. if the interest on £100 for x yr. is £x.
14. Find the rate per cent. p.a. if the interest on £P for T yr. is £I.
15. In what time will £100 yield £x interest at y per cent. p.a.?
16. In what time will £P yield £I interest at R per cent. p.a.?
17. What sum of money will yield £x interest in 1 yr. at y per cent. p.a.?
18. What sum of money will yield £I interest in T years at R per cent. p.a.?

Example 9. If the simple interest on £560 for 4 years is £78 8s., find the rate per cent. per annum.

With the notation of the formula, \(P \times R \times T = I\); \(R = \frac{100 \times I}{P \times T}\).

\(R = \frac{100 \times \frac{78}{12}}{560 \times 4} = \frac{100 \times 652}{560 \times 4} = \frac{391}{56} = \frac{1}{2}\) = 0.5

. . . . the rate of interest is 50% p.a.
Example 10. In what time will £640 amount to £684 at 2½% p.a. simple interest?

With the notation of the formula \( P = 640, I = 684 - 640 = 44, R = 2\frac{1}{2}\), and it is required to find the value of \( T \).

\[
P \times R \times T = 100 \times \frac{I}{P \times R}; \quad T = \frac{100 \times I}{P \times R}
\]

\[
T = \frac{100 \times 44}{640 \times 2\frac{1}{2}} = \frac{4400}{1600} = 2\frac{3}{8};
\]

\( \therefore \) the time is 2\( \frac{3}{8} \) years.

Example 11. What sum of money will yield £81 interest in 2\( \frac{1}{2} \) years at 4\( \frac{1}{2} \)% p.a. simple interest.

With the notation of the formula, \( I = 81, T = 2\frac{1}{2}, R = 4\frac{1}{2}\), and it is required to find the value of \( P \).

\[
P \times R \times T = 100 \times \frac{I}{P \times R}; \quad P = \frac{100 \times I}{R \times T}
\]

\[
P = \frac{100 \times 81}{2\frac{1}{2} \times 2 \times 2} = \frac{8100}{10} = 810;
\]

\( \therefore \) the principal is £810.

EXERCISE 111

Find the unknown quantities in Nos. 1–16:

<table>
<thead>
<tr>
<th>Principal</th>
<th>Interest</th>
<th>Amount</th>
<th>Time</th>
<th>Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. £120</td>
<td>£18</td>
<td>£696</td>
<td>3 yrs</td>
<td>...</td>
</tr>
<tr>
<td>2. £640</td>
<td>...</td>
<td>£696</td>
<td>2( \frac{1}{2} ) yrs</td>
<td>...</td>
</tr>
<tr>
<td>3. £240</td>
<td>...</td>
<td>£267</td>
<td>...</td>
<td>4( \frac{1}{2} )</td>
</tr>
<tr>
<td>4. £960</td>
<td>£198</td>
<td>...</td>
<td>...</td>
<td>5( \frac{1}{2} )</td>
</tr>
<tr>
<td>5. ...</td>
<td>£48</td>
<td>...</td>
<td>1( \frac{1}{2} ) yrs</td>
<td>5( \frac{1}{2} )</td>
</tr>
<tr>
<td>6. ...</td>
<td>£42</td>
<td>...</td>
<td>1( \frac{3}{4} ) yrs</td>
<td>4( \frac{1}{2} )</td>
</tr>
<tr>
<td>7. £360</td>
<td>...</td>
<td>...</td>
<td>2( \frac{1}{2} ) yrs</td>
<td>4( \frac{1}{2} )</td>
</tr>
<tr>
<td>8. £10</td>
<td>...</td>
<td>...</td>
<td>5 months</td>
<td>4( \frac{1}{2} )</td>
</tr>
<tr>
<td>9. £1560</td>
<td>£245 14s.</td>
<td>...</td>
<td>3( \frac{1}{2} ) yrs</td>
<td>...</td>
</tr>
<tr>
<td>10. £205</td>
<td>...</td>
<td>£207 1s.</td>
<td>146 days</td>
<td>...</td>
</tr>
</tbody>
</table>
True Present Worth and Discount. For a given rate per cent, and given time, the amount is proportional to the principal.

For example, at 4% p.a. for 1½ years we have:

<table>
<thead>
<tr>
<th>Principal</th>
<th>Interest</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>£100</td>
<td>2 × 1/2</td>
<td>£106</td>
</tr>
<tr>
<td>£200</td>
<td>4 × 1/2</td>
<td>£212</td>
</tr>
<tr>
<td>£300</td>
<td>6 × 1/2</td>
<td>£318</td>
</tr>
</tbody>
</table>

and so on.

Therefore, for a given rate per cent. and given time, if the amount is given the principal can be found by proportion.

Example 12. Find what sum will amount to £332 in 9 months at 5% p.a.

The interest on £100 for 9 months at 5% is £5 × 3/4, and therefore the amount is £100 × (1 + 3/4) = £137.50.

The amount is £41.6 if the principal is £100;

∴ the amount is £332 if the principal is £100 × (3/4 × 332);

∴ the principal is £320.

Example 12 shows that £320 amounts in 9 months at 5% to £332; it follows that at this rate of interest, a man who is owed £320 now is in as good a position as a man who will be due to receive £332 in 9 months' time. For this reason, £320 is called the true present worth of £332 due in 9 months' time, reckoning simple interest at 5% p.a.

Further, if a man is due to pay £332 in 9 months' time, it would be fair to allow him to clear off the debt by paying £320 now, that is £12 less than the nominal amount of the debt or bill, and this reduction of £12 is called the true discount, allowed for an immediate payment; it is the interest on the true present worth for the time which will elapse before the debt is due to be paid.

Example 13. Find correct to the nearest penny the true present worth of a bill for £95 due in 7 months' time, allowing interest at 4½% p.a. Find also the true discount.

Interest on £100 for 7 months at 4½% p.a. = £100 × (7 × 1/6) = £8.75;

∴ In 7 months, £100 amounts to £100 + £8.75 = £108.75;

∴ the present worth of £108.75 is £100;

∴ the present worth of £95 is £100 × (108.75 × 95).

Hence, we find by reduction that

the present worth is £92.11s. 5d., to the nearest penny;

∴ the discount = £95 - £92.11s. 5d. = £2 8s. 7d.

Practical Discount. It is a custom of shopkeepers in certain cases to deduct a certain percentage from the marked prices of their goods for a cash payment. Thus many tailors allow a discount of 10% off the quoted price of a suit to those who pay cash on delivery. For example, the price of a suit is quoted as £14 by a tailor to a customer whom he does not expect will pay him for a year or so after the suit has been made, but he is willing to deduct 10% of £14, that is £1 8s., from the price if the customer pays at once.

The cash price of the suit is therefore £12 12s., and the amount deducted, here £1 8s., is called the cash discount.

In shops, cash discounts are always calculated as fractions or percentages of the marked prices; a discount of 1d. in the shilling is equivalent to a deduction of 1/12 of the marked price, that is 8⅔% of the marked price.

When a retailer buys goods from a wholesale merchant, the prices in the trade catalogue issued by the wholesaler are usually subject to a trade discount, which is a percentage of the catalogue prices; this percentage varies from time to time according to the costs of raw materials, manufacture, taxation, and so on.
Example 14. A watch is marked £18, but a discount of 2s. 6d. in the £ is allowed for cash. What is the cash discount?

The rate of discount = \frac{2.6}{100} = \frac{1}{4}

\therefore the cash discount on £18 is \frac{1}{4} of £18, that is £2 5s.

Example 15. In a wholesale catalogue, a wireless set is quoted at £65, but the retailer is allowed a discount of 30%. What price is paid by the retailer?

The discount = \frac{30}{100} of £65 = £19 10s.

\therefore the retailer pays £65 - £19 10s., that is £45 10s.

Alternatively, as follows:

Since a discount of 30% is allowed, the retailer pays 70% of the catalogued price;

\therefore the retailer pays \frac{7}{10} of £65, that is £45 10s.

EXERCISE 113

1. What percentage represents a discount of (i) 1s. in the £; (ii) 2d. in the s?

2. Find the discount on a bill of £1, if the rate of discount is (i) 10%; (ii) 7\frac{1}{2}.

3. A tailor gives a discount of 2s. in the £ on a suit quoted at 15 guineas. What is the cash price?

4. A manufacturer's catalogued price of a car is £240, but he allows the retailer 30% discount. What does the retailer pay for the car?

5. Find the cost of 15 gross of pencils at 1\frac{3}{4}d. each, if a discount of (i) \frac{2}{5}, (ii) \frac{1}{4}d. in the s., is allowed.

6. Find the cost of 1 cwt. of tea at 2s. 6d. per lb., if a discount of 10% is allowed.

7. A shopkeeper marks a piano 50 guineas, but accepts £50 8s. for a cash payment. Express the discount as a percentage.

8. A carpet is quoted at £24 in a wholesaler's catalogue, but the retailer only pays £20 for it. Express the trade discount allowed (i) as a percentage, (ii) at so much in the £.

The next section may be omitted at a first reading.

Banker's Discount. If a merchant A buys goods from another merchant B, payment is often made by what is called a Bill of Exchange, which is a document drawn up by B, called the Drawer of the bill, stating the sum of money to be paid and the date on which payment is to be made. The document is then signed by A, who is called the Acceptee of the bill.

When the bill has been signed and stamped, it forms a legal promise on the part of A to pay the agreed sum at the stated date. But it may happen that B requires the money before the date on which it is due, and in this case he takes the document to a Bank or a Bill-broker who will not pay him the full face-value (or nominal value) of the bill, but a smaller amount depending on the length of time before the bill matures (i.e. is due for payment by A), provided of course that A is a reliable creditor. In actual practice, the bill-broker calculates the simple interest on the nominal value
of the bill for the time which will elapse before the payment is due, and deducts this from the nominal value of the bill.

Since a bill of exchange does not mature legally till 3 days after the stated date, the bill-broker, when calculating the interest, adds 3 days to the length of time the bill has still to run nominally, these 3 days being called days of grace.

**Example 16.** Find how much a bill-broker gives for a bill of £400, drawn on June 1 for 2 months, which he discounts on July 6 at 5%.

Since 3 days' grace are allowed, the bill matures on August 4; therefore when it is discounted on July 6, there remain 29 days before payment will be made.

Interest on £400 for 29 days at 5% is £(4 × 5 × 30/365);

\[\therefore\text{the discount} = \£1 \frac{1}{9} = \£1 \text{ 11s. 9d.}, \text{to nearest penny.}\]

\[\therefore\text{ the bill-broker gives (£400 - £1 11s. 9d.) = £398 8s. 3d.};\]

\[\therefore\text{ the "discounted value" of the bill is £398 8s. 3d.}\]

**Note.** The sum £1 11s. 9d., deducted by the bill-broker, is called banker's discount or commercial discount. It should be noted that it is slightly larger than the true discount, because it is calculated as a percentage of the face-value of the bill, whereas the true discount is the same percentage of the true present worth of the bill.

**EXERCISE 114**

Find correct to the nearest penny the discounted values of the following bills, allowing 3 days' grace:

1. £100, drawn on June 1 for 1 mo. and discounted on June 6, at 5%.

2. £1000, drawn on May 10 for 3 mo. and discounted on May 12, at 4%.

3. £2500, drawn on August 20 for 4 mo. and discounted on December 1, at 3%.

4. £750, drawn on March 10 for 20 days and discounted on March 13, at 4½%.

5. £1800, drawn on July 12 for 56 days and discounted on August 20, at 3½%.

6. £5500, drawn on July 1 for 1 mo. and discounted on July 1, at 4%.

**BANKER'S DISCOUNT**

Find correct to the nearest penny the discount on the following bills, allowing 3 days' grace:

7. £300, drawn on May 15 for 3 mo. and discounted on July 20 at 5%.

8. £1250, drawn on March 1 for 6 mo. and discounted on April 20, at 4%.

9. £4500, drawn on March 20 for 14 days and discounted on April 1, at 3½%.

10. £138, drawn on September 22 for 60 days and discounted on October 5, at 4½%.

*11. A bill for £680, drawn on March 1 for 3 mo., is discounted on March 20 for £674 6s. 9d. Find the rate of discount, allowing 3 days' grace.

12. A bill for £436, drawn on June 24 for 90 days, is discounted on July 13 for £431 2s. 9d. Find the rate of discount, allowing 3 days' grace.

*13. [The third-tenth-tenth Rule.] Prove that the interest on £P for n days at R per cent. per annum is £P \times 2n/73000, and that this is approximately equal to

\[\£(P \times 2nR) \times (1 + \frac{3}{8} + \frac{5}{60} + \frac{6}{60}) \times 10^5\]

Use this rule to find correct to the nearest penny the simple interest on £175 for 37 days at 4½% p.a.

Use the "third-tenth-tenth rule," see No. 13, to find to the nearest penny:

*14. The simple interest on £230 for 45 days at 3½% p.a.

*15. The simple interest on £583 10s. for 24 days at 4½% p.a.

*16. The banker's discount on a bill for £385 drawn on September 15 for 1 mo., if discounted on September 20 at 4%, allowing 3 days' grace.

*17. The banker's discount on a bill for £630 drawn on May 10 and nominally due on August 12, if discounted on May 14 at 3%, allowing 3 days' grace.
MISCELLANEOUS EXAMPLES

EXERCISE 115

1. Find the simple interest on £260 for 15 mo. at 7½% p.a.

2. Find the rate of interest if £150 amounts to £157 in 16 mo., at simple interest.

3. A man's annual interest on his Savings Bank deposit is 27s. 6d. If the rate of interest is 2½% p.a., find the size of his deposit.

4. At what rate % p.a. will the yearly interest on £625 be the same as the yearly interest on £550 at 8% p.a.?

5. The graphs OA, OB, OC in the diagram represent the amount of £100 for various periods. What rates of interest are represented by the graphs.

6. A discount of 17½% is allowed on an article catalogued at £3 15s. What is its cash price?

7. What sum of money amounts to £627 in 2½ yr. at 4% p.a. simple interest?

8. A man borrows £10 at 1% per month simple interest and repays £2 after 4 mo. How much does he still owe?

9. After what time will £7 amount to 7 guineas at 4% p.a. simple interest?

10. A club of 264 members has to meet a bill of £100 in 7 months' time. Each member subscribes 7s. 6d., and the money accumulates at 5% p.a. simple interest. What balance remains when the bill has been paid?

11. A shopkeeper gives a discount of 12½% on the marked price of a chair for cash. If the cash price is £2 9s., find the marked price.

12. A man lends on mortgage £600 at 5½% p.a. and £900 at 4½% p.a. What is the average rate of interest on the total sum lent?

13. A man earns £2500 a year and invests each year 18 per cent. of it at 5% p.a. simple interest. How much interest does he receive in the first 3 yr. from the time he began to invest money in this way?

14. A man borrows £40 from a moneylender and discharges his debt at the end of 3 mo. by repaying £42 8s. At what rate per cent. p.a. is interest charged on the loan?

15. In what time does a sum of money increase by 4½%, if invested at 6½% simple interest?

16. Find correct to the nearest penny the interest on a loan of £243 5s. at 5½% p.a. which runs from April 5 to June 3.

17. A sum of money amounts to £840 after 1 yr. and to £882 after 2 yr. at simple interest. Find the amount after 2 more years and the rate per cent. p.a. at which interest is paid.

18. During the War the Government borrowed money at 5½% p.a. simple interest. In 1927 it offered to call each £100 borrowed £142, and to pay 3½% p.a. on that sum. What is the change in income of a man who originally lent £5000?

19. A man lends £1000, partly at 3½% p.a. and the rest at 4½% p.a. He receives each year £39 7s. 6d. in interest; how much did he lend at 4½%?

20. The true discount on a bill for £780 due in 8 months' time is £30. At what rate per cent. p.a. is interest reckoned?

21. The catalogue price of a table is £5, but the retailer is allowed a trade discount of 35%. At what price does the retailer sell it if he makes a profit of 40%?

22. If it would cost £300,000,000 to electrify the railways in Great Britain, and if this would result in saving 7½ million tons of coal a year, find the price of the coal per ton, if the amount saved would pay interest at 4% on the capital outlay.

23. A sum of money amounts at simple interest to £688 in 3 yr. at 2½% p.a. To what amount would it amount in 2½ yr. at 3½% p.a. simple interest?

24. At the beginning of the year I have £24 7s. 6d. in the Savings Bank. During the year I make deposits of £1 10s. on March 15, 18s. on July 4, £5 10s. on December 23, and withdraw £2 on September 21. Interest is allowed at the rate of 4d. per £ per month on every complete pound on deposit for the whole month and is credited at the end of the year. How much have I got in the bank at the end of the year, including the interest credited?
CHAPTER XVI

APPROXIMATIONS

"The important thing about approximate answers is that they should be right as far as they go."

As was pointed out in Chapter VII, see p. 71, any result deduced from approximate data can only be approximate and must not contain more figures than are justified by the degree of accuracy of the data. No practical measurements, such as those of length, area, weight, etc., can be exact, and the degree of accuracy of results so obtained should be stated: for example, it may be given that the length of a bench is 4.03 m., correct to the nearest cm., and this means that the true length is between 4.035 m. and 4.025 m.; this could also be indicated by writing the length as (4.03 ± 0.005) m.; another very important way of expressing the same fact is to say that the length is 4.03 m., correct to 3 significant figures.

 Contracted Addition and Subtraction. In general, two more places of decimals should be retained in the working than are required in the answer; no account need then be taken of figures outside this range.

Example 1. Find, correct to 1 place of decimals, the sum of

\[ 186.34725, 8.91567, 100.5723, 47.8319. \]

Since the answer is required correct to 1 place of decimals, retain 3 places in the working.

Draw a line down the page, marking off 3 places of decimals, and ignore all figures to the right of it.

The sum is 343.7, correct to 1 place of decimals.

\[ \text{Plus} \quad \text{Minus} \]
\[ 186.34725 \quad 0.0000 \]
\[ 8.91567 \quad 0.0000 \]
\[ 100.5723 \quad 0.0000 \]
\[ 47.83191 \quad 0.0000 \]

\[ \therefore \text{the value is 0.154, correct to 3 places of decimals.} \]

Note. To obtain the value, correct to 4 places of decimals, we should start by retaining 6 places in the working; but if this is done it will be found that the result is 0.154151, and this is a border-line case where it is possible that, with a closer approximation, the result may prove to be nearer 0.1541 than 0.1542. In such exceptional cases as this, it becomes necessary to retain 7 places of decimals in the working.

EXERCISE 116

Find, correct to the number of places of decimals indicated in brackets, the values of:

1. \[ 7.605812 + 0.913645 + 1.286712 + 5.071658 \] (2).
2. \[ 18.97904 + 6.512783 + 12.90712 + 0.896705 \] (2).
3. \[ 0.7430727 + 2.8165723 + 1.6929457 \] (3).
4. \[ 18.7246812 - 7.8156073 \] (2).
5. \[ 3.960754 - 1.0986254 \] (3).
7. \[ 8.07254 - 2.89173 + 0.90954 - 3.50816 \] (2).
8. \[ 7.14916 + 2.8375 - 6.37488 + 1.96723 \] (2).

The difference is 11,300,000, to the nearest 100,000.
9. The population of the United Kingdom was made up as follows:

<table>
<thead>
<tr>
<th>Year</th>
<th>England</th>
<th>Wales</th>
<th>Scotland</th>
</tr>
</thead>
<tbody>
<tr>
<td>1921</td>
<td>35,520,225</td>
<td>2,656,474</td>
<td>4,882,497</td>
</tr>
<tr>
<td>1931</td>
<td>37,354,917</td>
<td>2,593,014</td>
<td>4,842,554</td>
</tr>
</tbody>
</table>

Find (i) the total population for 1921 and for 1931, correct to 4 significant figures, (ii) the increase in the total population for the period 1921–1931, correct to 3 significant figures.

[10] Net receipts from income tax were made up as follows:

<table>
<thead>
<tr>
<th>Year</th>
<th>England and Wales</th>
<th>Scotland</th>
<th>Ireland</th>
</tr>
</thead>
<tbody>
<tr>
<td>1929</td>
<td>£218,851,564</td>
<td>£17,282,382</td>
<td>£1,739,106</td>
</tr>
<tr>
<td>1950</td>
<td>£235,523,656</td>
<td>£18,041,937</td>
<td>£1,743,731</td>
</tr>
</tbody>
</table>

Find (i) the total receipts for 1929 and for 1930, correct to 4 significant figures, (ii) the increase in the total receipts, correct to 3 significant figures.

[11] Imperial Airways issued the following traffic figures for the 5 years, 1927 to 1931:

<table>
<thead>
<tr>
<th>Year</th>
<th>Letters</th>
<th>Passengers</th>
<th>Miles flown</th>
</tr>
</thead>
<tbody>
<tr>
<td>1927</td>
<td>2,334,850</td>
<td>26,469</td>
<td>867,075</td>
</tr>
<tr>
<td>1928</td>
<td>2,911,345</td>
<td>34,502</td>
<td>1,032,842</td>
</tr>
<tr>
<td>1929</td>
<td>3,941,070</td>
<td>29,338</td>
<td>1,345,217</td>
</tr>
<tr>
<td>1930</td>
<td>5,104,365</td>
<td>30,993</td>
<td>1,295,648</td>
</tr>
<tr>
<td>1931</td>
<td>6,348,720</td>
<td>34,162</td>
<td>1,721,962</td>
</tr>
</tbody>
</table>

Find, correct to 3 significant figures, the total of each item for the 5 years.

12. The value in £ of goods imported into and exported from Great Britain by air for the 5 years 1927 to 1931 was as follows:

<table>
<thead>
<tr>
<th>Year</th>
<th>Imports</th>
<th>Exports</th>
</tr>
</thead>
<tbody>
<tr>
<td>1927</td>
<td>1,251,921</td>
<td>1,439,386</td>
</tr>
<tr>
<td>1928</td>
<td>2,003,551</td>
<td>981,139</td>
</tr>
<tr>
<td>1929</td>
<td>1,990,350</td>
<td>1,003,219</td>
</tr>
<tr>
<td>1930</td>
<td>1,656,682</td>
<td>834,533</td>
</tr>
<tr>
<td>1931</td>
<td>2,022,532</td>
<td>773,958</td>
</tr>
</tbody>
</table>

Find, correct to the nearest £10,000, the total value for the 5 years of (i) imports, (ii) exports, (iii) excess of imports over exports.

Find, correct to 4 significant figures, the values of:

13. \[ 1 + 1 + 1 + 1 + 1 + 1 + \ldots \] \[ 1 + 1 + 1 + 1 + 1 + 1 + \ldots \]

14. \[ \frac{1}{3} - \frac{2}{3} + \frac{1}{3} - \frac{3}{3} + \ldots \]

15. \[ \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{2} + \ldots \]

16. \[ \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{3} + \frac{1}{3} - \frac{1}{3} + \ldots \]

17. \[ \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{4} + \ldots \]

18. \[ \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \ldots \]

CONTRACTED MULTIPLICATION

In general, two more significant figures should be retained in the working than are required in the product. The position of the decimal point should be obtained by a rough estimate, just as is necessary if a slide rule is used.

Example 4. Find, correct to 2 places of decimals, the value of

\[ 814.7325 \times 0.00426308 \]

Rough Estimate: 800 \times 0.004 \approx 3.2 (count the decimal places).

Since the product is about 3, and since 2 places of decimals are required, we must find the value of the product, correct to 3 significant figures, and must therefore retain 5 significant figures in the working.

814,7325
\[ \approx 4263 \text{ (08)} \]

(a) First multiply by 4. Since only 5 figures are needed in the working, put a comma after the 7 in 8147325 and multiply 8147 by 4; this gives 5 figures, namely, 32588, but we carry 1 from 3 \times 4 so that this becomes 32589.

(b) Cross off the digit 7 in the top line and multiply 814 by 2, but carry 1 from 7 \times 2.

(c) Cross off the digit 4 in the top line and multiply 81 by 2, but carry 4 from 2 \times 6.

(d) Cross off the digit 1 in the top line and multiply 8 by 3.

(e) The sum is 34730, but the rough estimate gave 32-7.

\[ \therefore \text{the product is 3473, correct to 2 places of decimals.} \]

EXERCISE 117

Evaluate correct to 2 significant figures:
1. 27.64 \times 13.23.  2. 1.695 \times 0.3486.  3. 8716 \times 9248.

Evaluate correct to 3 significant figures:
4. 3.7214 \times 2.5168.  5. 70916 \times 41837.

Evaluate correct to 2 places of decimals:
6. 0.39326 \times 2.3174.  7. (2.163)^2.

Evaluate correct to 3 significant figures:
8. 2.71564 \times 3.18247.  9. 72.6358 \times 64.7183.
Evaluate correct to the number of places of decimals indicated in brackets:

[10] $3.62471 \times 0.256372 \text{ (2).}$
[11] $0.716244 \times 0.638173 \text{ (3).}$
[12] $271.083 \times 0.362185 \text{ (2).}$
[13] $3.14159 \times 176.432 \text{ (1).}$

Evaluate correct to 4 significant figures:

[14] $809.43627 \times 35.1284. \text{ 15. (0.0571826)$^3$.}$

18. The average postal receipts in 1931 were £133,469 per day. Find the total receipts for the year, correct to 4 figures.

17. The population of London in 1931 was 8,202,818, and the death-rate for the year was 11,872 per thousand. Find the number of deaths in London in 1931, correct to 3 figures.

Evaluate:

*18. $1.7394 \times 2.1243 \times 3.8143$ correct to 2 decimal places.

*19. $36.7134 \times 4.8624 \times 2.3154$ correct to 3 significant figures.

*20. $(2.17246)^3 \times 38.517$ correct to the nearest integer.

Contracted Division. In general, two more significant figures should be retained in the working than are required in the quotient.

Example 5. Find, correct to 1 place of decimals, the value of

$0.876243 \div 0.0342174.$

Rough Estimate: $0.876243 \div 0.0342174 \approx 26 \approx 437.624 \text{ (a).}$

25.60

Since the quotient is about 30, and since it is required correct to 1 place of decimals, we must find the quotient, correct to 3 significant figures, and must therefore retain 5 significant figures in the working.

25.60

(a) Write down the divisor and dividend to 5 figures; the first digit in the quotient is 2.

(b) The first remainder is 19190; cross off the right-hand digit 7 of the divisor, and divide 19190 by 3421; the next digit in the quotient is 5; since $7 \times 5 = 35$, 3 must be carried in multiplying 3421 by 5; 5 times 1 is 5, and 3 makes 8, then continue as usual.

(c) Cross off the next digit 1 of the divisor, and divide 2082 by 342; the next digit in the quotient is 6; since $1 \times 6 = 6$, 1 must be carried in multiplying 342 by 6.

(d) Since 3 significant figures are required in the answer, the working ceases as soon as 4 figures have been obtained in the quotient.

The quotient is 25.6, correct to 1 place of decimals.

EXERCISE 118

Evaluate correct to 2 significant figures:

1. $81.17 \div 32.54.$
2. $4.273 \div 68.13.$
3. $596.7 \div 0.08134.$

Evaluate correct to 3 significant figures:

4. $17.364 \div 2.9175.$
5. $66.324 \div 0.051728.$

Evaluate correct to 2 places of decimals:

6. $5.268 \div 8.147.$
7. $8.364 \div 5.923.$

Evaluate correct to 4 significant figures:

8. $73.2698 \div 278.514.$
9. $48379.46 \div 0.0726384.$
10. $890736.4 \div 76.0824.$
11. $359672.8 \div 513.8247.$

Evaluate correct to 3 places of decimals:

12. $8.172945 \div 59.6304.$
13. $0.372691 \div 0.624738.$

14. The area of Hampshire is 1458.3 sq. miles, and its population in 1931 was 472,022. Find the average per sq. mile, to the nearest whole number.

15. In 1931, exports by air from Great Britain amounted to £7,961,877 and imports by air to £14,041,065. Express the exports as a percentage of the imports, correct to 4 figures.

Evaluate correct to 2 significant figures:

*18. $32.326 \times 2.4683.$
*17. $19.6273 \div 0.48936.$

Errors. If a length is stated to be 2.4 cm., correct to the nearest mm., then the true length is between 2.35 cm. and 2.45 cm.; therefore the error in taking the length as 2.4 cm. does not exceed 0.05 cm., and this is called the maximum absolute error.

Thus if a number is given correct to a stated number of figures, the maximum absolute error is represented by a 5 in the next place.
The standard of accuracy of a measurement is not, however, judged by absolute error. For example, an error of 1 in. in the size of a 15-in. collar is so big as to make the collar almost useless, but an error of 1 in. in marking out a 100-yd. running-track is of no importance, although the absolute error is 1 in. in each case. What matters in estimating the importance of an error is the ratio of the absolute error to the true value, and this is called the relative error. Thus, in the above example, the relative error in the size of the collar is \( \frac{1}{15} \) and in the length of the track is \( \frac{1}{100} \); the second relative error is much smaller than the first, but it is easier to compare them if they are expressed as percentages:

\[
\text{relative error} = \frac{1}{15} = 7\% \quad \text{and} \quad \frac{1}{100} = 0.01\%.
\]

If the relative error is expressed as a percentage, it is called the percentage error or the error per cent.

In calculating percentage errors, it is usually sufficient to work to one significant figure, because this indicates sufficiently clearly the degree of accuracy of measurement.

When the true value is unknown, as is often the case, the relative error is taken as the ratio of the maximum absolute error to the measured value, and if the relative error is less than \( \frac{1}{10} \), the result to 1 significant figure will not in general be affected by doing so.

**Example 6.** The length of a line is 7.2 cm., correct to the nearest mm. Find (i) the maximum absolute error, (ii) the relative error and error per cent., to 1 significant figure.

(i) The true length lies between 7.15 cm. and 7.25 cm.; 
∴ the maximum absolute error is 0.05 cm.

(ii) The relative error \( \approx \frac{0.05}{7.2} \approx 0.007; \)
∴ the error per cent. \( \approx 0.7\%\).

**Exercise 119**

Find to 1 significant figure the maximum absolute error, the relative error and error per cent. in Nos. 1-7:

1. A length of 4.5 cm., correct to the nearest mm.
2. A weight of 0.328 kg., correct to the nearest gm.
3. A profit of 11s. 3d., correct to nearest penny.
4. A time of 10.4 sec., correct to nearest \( \frac{1}{2} \) sec.

---

**Errors**

6. A length is known to be 2480 yd., correct to 3 figures.
7. A weight is known to be 1 ton, within 20 lb.

Find to 1 significant figure the error per cent. in Nos. 8-12:
8. Calculated length 5.32 cm., by measurement 5.4 cm.
9. Calculated volume 86.62 c.c., by measurement 86.5 c.c.
10. \( \pi \) is taken as 3.4 instead of 3.14159.
11. \( \sqrt{3} \) is taken as 1.3 instead of 1.7321.
12. 1 m. is taken as 39 in. instead of 39.2809.
13. A line is measured as 7.5 cm.; if the maximum error per cent. is 4\%, between what limits (to 2 figures) does the true length lie?
14. A sum of money is given in £ correct to 3 places of decimals; show that the error is less than half a farthing.

*15. A speed is given as 118 ft. per sec. correct to the nearest ft. per sec. What is the maximum error in calculating the distance travelled in 20 min.?

*16. A merchant buys tea wholesale at the rate of 1s. 10\( \frac{1}{2} \)d. per lb., correct to the nearest \( \frac{1}{2} \)d. What is the maximum error in calculating the wholesale price per cwt.?

**Addition**

It is important to remember that it is useless to give more significant figures in a result than are justified by the data, taking account of the uses that have been made of the data.

**Example 7.** Find the sum of the numbers, 8.633, 1.728, 2.413, 5.712,

if each is given correct to 4 significant figures.

\[
8.633 \pm 0.0005
1.728 \pm 0.0005
2.413 \pm 0.0005
5.712 \pm 0.0005
\]

\[
8.486 \pm 0.002
\]

∴ the sum lies between 18.488 and 18.484 and may be expressed as either 18.5, correct to 3 figures, or 18.468 ± 0.002.
Approximations

Example 8. Subtract 49.37 from 92.84 if each number is given correct to 4 significant figures.

<table>
<thead>
<tr>
<th>Greatest</th>
<th>Least</th>
</tr>
</thead>
<tbody>
<tr>
<td>92.845</td>
<td>49.365</td>
</tr>
<tr>
<td>difference</td>
<td>difference</td>
</tr>
<tr>
<td>43.48</td>
<td>43.46</td>
</tr>
</tbody>
</table>

The difference is 43.48, correct to 3 significant figures, or may be written 43.47 ± 0.01.

Exercise 120

In Nos. 1-12, each number is given correct to the number of significant figures shown in brackets. Find the value of each expression (i) to as many significant figures as the data justify, (ii) in the form A ± x:

1. \(237 + 834 + 179 + 408\) (3).
2. \(1036 + 4744 + 2607 + 2791 + 1186\) (4).
3. \(714.5 + 580.3 + 542.4 + 718.6 + 200.2\) (4).
4. \(1030 + 2804 + 0.5712 + 0.7214\) (4).
5. \(307 + 264 + 116 + 179 + 120\) (3).
6. \(872 - 467\) (3).
7. \(4.618 - 1.096\) (4).
8. \(149 - 63.2\) (3).
9. \(3524 - 0.8172\) (4).
10. \(562 - 183 - 209 + 486 + 257\) (3).
11. \(7326 - 1082 + 8109 - 1030\) (4).
12. \(3156 - 8072 - 3147 - 1120 + 4108\) (4).

State the limits between which the values of the expressions in Nos. 13-17 lie, and write the values in the form \(A ± x\):

13. The perimeter of a triangle whose sides are measured as 6.2 cm, 5.7 cm, 4.3 cm, to the nearest mm.
14. The sum of 7s, 4d., 3s, 9d., 5s, 2d., 2s, 8d., 6s, 11d., if each amount is correct to the nearest penny.
15. The difference of 5 lb, 7\(\frac{1}{4}\) oz. and 3 lb, 12\(\frac{1}{4}\) oz., if each weight is correct to the nearest \(\frac{1}{4}\) oz.
16. The sum of 1s, 6\(\frac{1}{4}\)d., 2s, 4\(\frac{1}{2}\)d., 8\(\frac{3}{4}\)d., 1s, 5\(\frac{1}{2}\)d., if each amount is correct to the nearest farthing.
17. The difference of 2\(\frac{1}{4}\) mi. and 2\(\frac{3}{4}\) mi., if each length is correct to the nearest 20 yd.

Addition and Subtraction

18. The value of \(\pi\) is 3.14159265...; express correct to 4 significant figures the value of \(\frac{1}{3} - \pi\).

19. A flask when empty weighs 63.45 gm, and when full of water weighs 201.07 gm., each weight being correct to the nearest centigram. Between what limits does the weight of the water lie? Find the volume of the flask to as many significant figures as the data justify.

20. The weight of 1 c.c. of copper is given as \(8.86 ± 0.05\) gm. Between what limits does the weight of 20 c.c. of copper lie?

21. The weights of three blocks are given as \(5.86 ± 0.04\) gm., \((4.76 ± 0.04)\) gm., \((3.86 ± 0.02)\) gm. Find a similar expression for their total weight, and express it to as many significant figures as the data justify.

22. The volume of a block is given as \(1763 ± 0.06\) c.c.; a portion of it is removed and the volume of the remainder is found to be \(1254 ± 0.05\) c.c. Find a similar expression for the volume of the portion removed and express it to as many significant figures as the data justify.

23. The length of a thread wound 10 times round a cylinder is measured independently by 7 boys as follows: \(85.6\) cm., \(85.7\) cm., \(85.5\) cm., \(85.8\) cm., \(85.5\) cm., \(85.9\) cm., \(85.8\) cm. What length should be taken for the circumference of the cylinder?

24. On a certain road leading to Winchester the distances on the signposts are given correct to the nearest \(\frac{1}{4}\) m. A motorist passes a signpost "Winchester 7\(\frac{1}{4}\) miles," and 6 min. later another "Winchester 3\(\frac{3}{4}\) miles." Between what limits, in miles per hour, does his average speed lie?

Multiplication. Suppose that the length and breadth of a rectangle are 6.3 cm., 5.4 cm., correct to the nearest mm., and that it is required to find the area to as great a degree of accuracy as the data justify.

The area according to the actual measurements is \((6.3 × 5.4)\) sq. cm., that is 34.02 sq. cm.;

But actually we can say is that the area does not exceed \((6.35 × 5.45)\) sq. cm., that is 34.6075 sq. cm., and is not less than \((6.25 × 5.35)\) sq. cm., that is 33.4375 sq. cm.

Therefore the area is 30 sq. cm., correct to one significant figure, but cannot be given correct to two figures.

Since the area lies between 33.4 sq. cm. and 34.6 sq. cm. (nearly), the area may be given as \((34 ± 0.6)\) sq. cm. approximately.
Example 9. Find, as accurately as the data justify, the volume of a rectangular block 3·27 in. by 2·63 in. by 1·95 in., if the error in each measurement does not exceed 0·01 in.

The volume cannot exceed

\[(3·28 \times 2·64 \times 1·96) \text{ cu. in., that is } 16·972032 \text{ cu. in.}\]

and cannot be less than

\[(3·26 \times 2·62 \times 1·94) \text{ cu. in., that is } 16·56928 \text{ cu. in.}\]

Therefore the volume is 17 cu. in., correct to two significant figures. Since the volume lies between 17·0 cu. in. and 16·5 cu. in., the volume may be given as \((16·75 \pm 0·25)\) cu. in.

Note. If the measurements had been given correct to 3 significant figures, the error in each measurement would not have exceeded 0·005 in.

Division. If the dividend and divisor are both approximate, the largest value of the quotient is obtained by taking the largest value of the dividend and the smallest value of the divisor; and the smallest value of the quotient is obtained by taking the smallest value of the dividend and the largest value of the divisor.

Example 10. The area of a rectangular field is 24,500 sq. yd. and its length is 185 yd., both measurements being correct to 3 significant figures. Find, as accurately as the data justify, the breadth of the field.

The actual area is between 24,450 sq. yd. and 24,450 sq. yd., and the actual length is between 185·5 yd. and 184·5 yd.

To obtain the greatest possible breadth, we take the greatest area and least length:

\[
\text{greatest breadth} = \frac{24450}{184·5} \approx 133·1 \text{ yd.}
\]

Similarly, taking the least area and greatest length,

\[
\text{least breadth} = \frac{24450}{185·5} \approx 131·8 \text{ yd.}
\]

Therefore, the breadth is 130 yd., correct to two significant figures. Since the average of 133·1 and 131·8 is about 132·5, the breadth may be given as \((132·5 \pm 0·7)\) yd.

Note. The breadth obtained from the actual measurements is \(24480\) yd., \(\approx 132·4\) yd. Thus the “measured” breadth represents closely the mean value indicated by \((132·5 \pm 0·7)\) yd.; the object of the working set out in the example is to find out how large an error there may be in taking the “measured” breadth as the actual breadth.

RELIABILITY OF RESULTS

Reliability of Results. The examples given above illustrate the fact that if two numbers are each given correct to \(n\) significant figures, the value of their product and their quotient can generally be obtained correct to \((n-1)\) significant figures, and, if taken to \(n\) figures, the result is a close approximation to the mean value, that is the average of the greatest and least possible values. But border-line cases occur where \((n-1)\) figures are not reliable.

EXERCISE 121

In Nos. 1-8, each number is given correct to the number of significant figures shown in brackets. Find the value of each expression (i) to as many significant figures as the data justify, (ii) in the form \(A \pm \varepsilon\):

\[
\begin{align*}
1. \quad 5 \times 4 (1). & \quad 2. \quad 0·65 \times 0·24 (2). & \quad 3. \quad 3·7 \times 2·4 (2). \\
4. \quad 0·453 \times 1·26 (3). & \quad 5. \quad 0·84 \div 2·1 (2). & \quad 6. \quad 4·7 \div 1·8 (2). \\
7. \quad 0·632 \div 2·46 (3). & \quad 8. \quad 151 \div 472 (3). 
\end{align*}
\]

In Nos. 9-17, give the answers correct to as many significant figures as the data justify. Write them also in the form \(A \pm \varepsilon\):

9. The length and breadth of a rectangle are measured as 5·6 cm., 4·8 cm., correct to the nearest mm. Find the area.

10. The length and breadth of a rectangular field are measured as 264 ft., 182 ft., correct to the nearest ft. Find the area.

11. The circumference of a circle of radius \(r\) in. is \(2\pi\) in., where \(\pi\) lies between 3·141 and 3·142. Find, to the nearest \(\frac{1}{10}\) in., the circumference of a circle of actual radius 4·65 in.

12. The time of 15 swings of a pendulum is measured as 12·6 sec. to the nearest \(\frac{1}{2}\) sec. Find the time of 1 swing.

13. The area of a rectangle is 34·6 sq. in. and its length is 7·25 in., both measurements being correct to 3 significant figures. Find the breadth.

14. The volume of a rectangular block is 68·5 c.c., and the base area is 14·5 sq. cm., both measurements being correct to 3 significant figures. Find the height.

15. The number of deaths in a large town for a certain period was 568, and it was stated that this was at the rate of 1·20 (to 3 figures) per thousand. Find the population of the town.
16. The weight of 7.5 c.c. of a metal is 52.5 gm., each quantity being correct to one place of decimals. Find the weight of 1 c.c. of the metal.

*17. In July 1932, there were 12,800,000 insured persons in Great Britain, of whom 23.2 per cent. were unemployed. If the data are correct to 3 significant figures, find the number of unemployed.

*18. In a table of cricket averages a batsman is stated to have an average of 31.68, correct to 2 places of decimals, for 32 completed innings. Show that this statement must be incorrect.

*19. It was stated in the Press that Sir Malcolm Campbell took 11.83 sec. for a measured mile and that his average speed was therefore 304.311 miles per hour. Find his average speed to as many figures as the data justify if the time was measured correct to (i) 1/10 sec., (ii) 1/100 sec., assuming that the distance was measured correctly.

**MISCELLANEOUS EXAMPLES**

**EXERCISE 122**

1. A distance is given as 3.84 mi. correct to 3 significant figures. Find the greatest possible error in yards.

2. To 5 significant figures, \( \sqrt{2} = 1.4142 \), \( \sqrt{3} = 1.7321 \). Find as accurately as the data justify the values of (i) \( \sqrt{3} + \sqrt{2} \); (ii) \( 5\sqrt{3} - 6\sqrt{2} \).

3. Calculate \( 1 - \frac{1}{10} + \frac{1}{10} - \frac{1}{10} + \frac{1}{10} - \frac{1}{10} \); correct to 6 places of decimals.

4. The greatest known ocean depth is 32090 ft. in the Pacific; express this in miles to 3 figures.

5. In an examination there were 1607 candidates of whom 73.53% (to 4 figures) passed. How many passed?

6. Find to the nearest inch the difference between 5 mi. and 8 km. [1 mi. = 1.60934 km.]

7. It is known that 1 c.c. of a liquid weighs 0.84 gm. correct to 2 figures. What is the greatest possible error in calculating the weight of 2 litres of the liquid?

8. In 1932 the London rainfall was 25.39 in., and the total duration of the rain was 469 hr. Express the rate in inches per hour to 2 figures.

9. Given 1 yd. = 91.44 cm., express 1 m. in feet to 3 significant figures.

10. Given 1 gal. = 4.546 litres, express 1 litre in pints to 3 significant figures.

11. In 1931 the population of Great Britain was 44,790,485 and the revenue was £770,963,000. Find the revenue per head in £ to 3 significant figures.

12. To how many significant figures is it correct to take the product of 0.347 and 2.8918 as 1?

13. 323125 c.c. of gold weigh 621.0834 gm.; find the weight of 1 c.c. of gold to 4 figures.

14. At the election in 1931 the total electorate was 29,952,361, and 21,656,373 votes were recorded. What percentage of the electorate, to 3 figures, voted?

15. The Scouts of the British Empire increased from 756,883 in 1930 to 808,307 in 1931. Find the increase per cent., to 3 figures.

16. If 40 ml. per hr. is equivalent to 34-74 knots, find the number of feet in a nautical mile, to the nearest 10 ft. [1 knot = 1 nautical mile per hour.]

*17. The output of coal in 3 mo. was 57,065,000 tons and the average profit was £4.225, per ton. Find the total to the nearest £10,000.

18. Find to 1 figure the error per cent. in taking 32 m. as equivalent to 35 yd. [1 m. = 3.9701 ft.]

19. In 1932 a motor-boat travelled a measured mile at an average speed of 124.91 m. per hour. Find in seconds the time taken, to 3 figures.

*20. Out of 10,000 people born, 6652 live to be 35 yr. old, 4418 live to be 60, 2279 live to be 70. Find to 3 figures what percentage (i) of persons of age 35 may expect to survive to 60, (ii) of persons of age 60 may expect to die before 70.

*21. If 1 litre = 1.7616 pt., express 1 lb. in grams to the nearest gram, given 1 gal. of water weighs 10 lb. and 1 litre of water weighs 1 kg.

*22. The record for the present Derby course (1 mi. 4 furlongs) is 2 min. 34.4 sec.; express this speed in miles per hour to 3 figures.

*23. 5 cu. ft. of wood, weighing 46 lb. per cu. ft., and 102 lb. of metal, weighing 556 lb. per cu. ft., are used in making an article. Find to the nearest lb. the average weight per cu. ft. of the article.

*24. The rateable value of a district is £133,224. Find to the nearest farthing how much in the £ must be levied to raise £2500?
MISCELLANEOUS EXTENSIONS

CHAPTER XVII

Averages

Example 1. At the 8 performances of a play in a week the numbers present were 1473, 1641, 1709, 1688, 1483, 1429, 1846, 1955 respectively. Find the average number present per performance.

Since there were more than 1400 present at every performance we need only calculate the average attendance above 1400: the numbers exceed 1400 by

73, 241, 309, 288, 83, 29, 446, 555,

that is, by 2024 in all;

\[ \therefore \text{the average exceeds } 1400 \text{ by } \frac{2024}{8}, \text{ that is } 253, \]

\[ \therefore \text{average } = 1400 + 253 = 1653. \]

Note. As a rough check, it should be noted that the average of any set of numbers must lie between the smallest and largest number in the set.

Example 2. A man walks from his house to a town 6 mi. away at 4 mi. per hour and bicycles back again at 12 mi. per hour. Find his average speed for the double journey.

He walks 6 mi. at 4 m.p.h. in 1.5 hr.;

he bicycles 6 mi. at 12 m.p.h. in 0.5 hr.

\[ \therefore \text{he travels altogether } 12 \text{ mi. in } 2 \text{ hr.}, \]

\[ \therefore \text{his average speed for the double journey is } 6 \text{ m.p.h.} \]

Note. The average speed is not the same as the average of 4 m.p.h. and 12 m.p.h., because he spends more time travelling at 4 m.p.h. than at 12 m.p.h.

Example 3. If 9 lb. of tea at 1s. 8d. per lb., 12 lb. of tea at 1s. 10d. per lb. and 15 lb. of tea at 2s. 4d. per lb. are mixed together, find the value per lb. of the mixture.

9 lb., 12 lb., 15 lb. are proportional to 3, 4, 5.

2 lb. of tea at 1s. 8d. per lb. are worth 5s.;

4 lb. of tea at 1s. 10d. per lb. are worth 7s. 4d.;

5 lb. of tea at 2s. 4d. per lb. are worth 11s. 8d.

\[ \therefore \text{the value of } (3 + 4 + 5) \text{ lb. of the mixture is } 24s. \]

that is, 12 lb. of the mixture are worth 24s.;

\[ \therefore 1 \text{ lb. of the mixture is worth } 2s. \]

Note. The value of the mixture per lb. is not the average of 1s. 8d., 1s. 10d., 2s. 4d., because the amounts of the three qualities, which are mixed together, are not equal.

AVERAGES

Exercise 123

Find the average of:

1. 3, 5, 7, 9, 11, 13, 15. 2. 752, 763, 759, 754, 772.
5. 3s. 6d., 4s. 8d., 2s. 10d., 7s. 3d., 5s. 4d., 1s. 5d.
6. 2 t. 15 cwt. 1 qr., 1 t. 7 cwt. 2 qr., 4 t. 12 cwt. 3 qr.

Find the cost per lb. of the following mixtures:

7. 6 lb. at 1s. 6d. per lb., 9 lb. at 2s. 4d. per lb.
8. 12 lb. at 7d. per lb., 8 lb. at 8½d. per lb., 10 lb. at 9d. per lb.
9. 35 lb. at 3s. 5d. per lb., 55 lb. at 4s. 2d. per lb.
10. Find the average cost per book of 8 books, 5 of which cost 3s. each and the rest 5s. each.

What is the average cost per book of 800 books, 500 of which cost 3s. each and the rest 5s. each?

11. In 1932, Larwood took 162 wickets for 2084 runs; find the average number of runs per wicket, to the nearest whole number.

12. A firm buys 824 tons of coal at £2 7s. 6d. per ton and 776 tons at £2 4s. per ton. Find, to the nearest halfpenny, by how much the average price per ton exceeds £2 4s.; hence find, to the nearest halfpenny, the average price per ton.

13. A cyclist rides 35 mi. at 10 mi. per hour and a further 30 mi. at 12 mi. an hour. Find the total time his journey takes and his average speed for the journey.

14. A motorist drives for 2 hr. at 32 m.p.h. and for half an hour at 42 m.p.h.; find his average speed for the whole journey.

15. A cricketer's average was 17 for 5 completed innings. He obtained 16, 3, 28, 12 runs in his first four innings; what was his score in his last innings?

16. The average age of 5 men is 46, and the average age of 4 of them is 43; what is the age of the fifth man?

17. A firm's monthly sales from July to November were respectively £8775, £7869, £7907, £8564, £8753. Find the value of the sales in December if the monthly average for the half-year was £8472.

18. A team of eight entered for a shooting competition. The best marksman scored 85 points; if he had scored 92 points, the average score for the team would have been 84. How many points altogether did the team score?
19. The average annual profits of a company for its first four years were £350 a year. For the first two years the losses were £420 and £230 respectively, and for the third year the profits were £750. Find the profits for the fourth year.

20. If 30 lb. of tea costing 1s. 8d. per lb. are mixed with 50 lb. of tea costing 2s. 6d. per lb., find the cost price of the mixture per lb., and the price per lb. at which it must be sold to gain 20 per cent. 

[21] If 35 tons of coal costing 41s. per ton are mixed with 55 tons of coal costing 50s. per ton, find the price per ton at which the mixture must be sold to gain 33 1/3 per cent.

*22. A swimmer, who takes 30 strokes per minute, can swim 1 mi. in 38 min. 20 sec. What is the average distance he travels per stroke, to the nearest inch?

*23. The average age of $m$ boys is $b$ years and of $n$ girls is $c$ years. Find the average age of all together.

*24. The average age of $x$ boys and $y$ girls is $p$ years. If the average age of the boys is $r$ years, find the average age of the girls.

*25. The average noon temperature for Monday, Tuesday, and Wednesday was 53°, and for Tuesday, Wednesday, and Thursday was 56°. If the noon temperature on Thursday was 60°, find the noon temperature on Monday.

Inverse Problems on Mixtures

Example 4. In what ratio must coffee at 1s. 5d. per lb. be mixed with coal costing 60s. per ton, so that the mixture can be sold for 60s. per ton at a gain of 20 per cent?

If the cost price is 100s., the profit is 20s., and therefore the selling price is 120s.;

\[
\therefore \text{the cost price} = \frac{100}{120} \text{ of the selling price;}
\]

\[
\text{but the selling price of 1 ton} = 60s.,
\]

\[
\therefore \text{the cost price of 1 ton} = \frac{100}{120} \text{ of 60s.} = 50s.
\]

Therefore the coal is mixed so that the cost price of 1 ton of the mixture is 50s.

1 ton at 44s. in a mixture at 50s. gives 6s. profit;

1 ton at 60s. in a mixture at 50s. gives 10s. loss.

\[
\therefore \text{ratio of coal at 44s. per ton to coal at 60s. per ton}
\]

equals 10 : 6, that is 5 : 3.

EXERCISE 124

In what ratio must the following commodities be mixed to produce a mixture of the stated value, Nos. 1–6—

1. Teas at 2s. per lb. and 2s. 8d. per lb.; mixture 2s. 2d. per lb.

[2] Chocolates at 3s. 4d. per lb. and 5s. per lb.; mixture 4s. per lb.

3. Sugars at 3d. per lb. and 3½d. per lb.; mixture 3½d. per lb.

[4] Cocoas at 9d. per lb. and 1s. 1¾d. per lb.; mixture 11d. per lb.

5. Coals at 28s. per ton and £2 per ton; mixture 35s. 6d. per ton.

[6] Currants at 3s. 3½d. per lb. and 3s. 9¾d. per lb.; mixture 6d. per lb.

7. How many lb. of tea at 2s. 6d. per lb. must be mixed with 100 lb. at 1s. 4d. per lb. to make the mixture worth 1s. 8d. per lb.

[8] A crate contains 56 dozen golf balls, some are sold at 15s. a dozen and the rest at 24s. a dozen. The average price is 1s. 7d. per ball; how many are there of each kind?

*6. In what ratio by volume must a liquid weighing 0·6 gm. per c.c. be mixed with a liquid weighing 1·1 gm. per c.c. so that the mixture may weigh 0·75 gm. per c.c.?

10. In what ratio must tobacco costing 10d. per oz. be mixed with tobacco costing 1s. 4d. per oz. so that a profit of 25% is made by selling the mixture at 1s. 3d. per oz.
[11] In what ratio must tea costing 2s. 7½d. per lb. be mixed with tea costing 3s. 3d. per lb. so that a profit of 20% is made by selling the mixture at 3s. 6d. per lb.

12. In what ratio must coffee costing 1s. 7d. per lb. be mixed with chicory costing 4d. per lb. so that a profit of 50% is made by selling the mixture at 2s. per lb.?

[13] Oranges costing 5s. 8d. per 100 are mixed with oranges costing 7s. 2d. per 100 so that a profit of 12½% is made by selling them at 10 for 9d. In what ratio are they mixed?

14. In what ratio must coal costing 3s. 9d. per ton be mixed with coal costing 48s. 9d. per ton so that a profit of 30% is made by selling the mixture at 58s. 6d. per ton?

*15. A liquid consists of 85% pure spirit and the rest water. In what ratio must it be mixed with water so that the mixture contains 75% pure spirit?

*16. A liquid X contains 80% pure spirit, and a liquid Y contains 55% pure spirit. In what ratio must X and Y be mixed so that the mixture contains 70% pure spirit?

Rate of Working

Example 6. A can do a piece of work in 3 days, B can do it in 9 days, and C in 4⅓ days. How long will they take if all work together?

In 1 day, A can do ⅓ of the work and B can do ⅙ of the work.
Since C takes 9 days to do it, he does ⅙ of the work in 1 day.
∴ A, B, C together do (⅓ + ⅙ + ⅙) of the work in 1 day;
∴ A, B, C together do ⅔ of the work in 1 day;
∴ A, B, C together do the whole work in 2 days;
∴ they take 2 days.

Example 7. One tap fills a bath in 12 min. and another tap fills it in 15 min. The waste-pipe can empty the bath in 10 min. In what time will the bath be filled if both taps are turned on and if the waste-pipe has been left open accidentally?

In 1 min. the first tap fills ⅕ of the bath, the second tap fills ⅕ of the bath, and the waste-pipe empties ⅕ of the bath. Therefore if both taps are turned on, with the waste-pipe open,

\[(\frac{1}{5} + \frac{1}{5} - \frac{1}{5}) = \frac{1}{5}\]

of the bath is filled in 1 min.;
but \[\frac{1}{5} + \frac{1}{5} - \frac{1}{5} = \frac{2}{5}\] or \[\frac{2}{5}\] of the bath is filled in 1 min.;
∴ the bath will be filled in 20 min.

RATE OF WORKING

EXERCISE 125

1. A can do a piece of work in 30 days, and B can do it in 6 days. How long will A and B take, working together?

2. A bath can be filled by 1 tap in 10 min. and by another tap in 15 min. How long does it take to fill the bath if both taps are turned on?

3. A can do a piece of work in 12 days which A and B, working together, can do in 8 days. How long would B take, working alone?

4. Two taps, running together, can fill a bath in 4 min., which is filled by one of the taps by itself in 7 min. How long would it take if the other tap is running by itself?

5. A cistern can be emptied by 3 pipes in 3 hr.; one alone would take 6 hr., another alone 9 hr.; how long would it take with the third pipe alone?

6. A tap fills a bath in 15 min. and the waste-pipe empties it in 12 min. If the bath is full and the tap is turned on, and the waste-pipe is open, how long will it be before the bath is empty?

7. A sum of money is sufficient to pay A's wages for 21 days or B's wages for 28 days. For how long will it suffice to pay the wages of A and B together?

8. A, B, C, working together, take 30 min. to address a pile of envelopes. A and B together would take 40 min.; and A and C together would take 45 min. How long would each take, working alone?

9. A and B can do a piece of work together in 15 days. They both start, but after 6 days B gets ill and A then takes 30 more days to finish it by himself. How long would each take, working alone?

10. A cistern can be filled by 3 taps, A, B, C, when turned on separately in 24 min., 10 min., 27 min. respectively. If all are turned on together for 4⅓ min. and if B, C are then turned off, how much longer is it before the cistern is full?

11. A can plough a field in 60 hr., B can do it in 48 hr. and C in 50 hr. How long will C take to finish it by himself if he starts when A and B have each done 12 hr. work?

12. A, B can do a piece of work in 15 days and 18 days respectively, working separately, working together. A, B work together for 3 days, then B leaves, and after another 3 days C joins, and the work is finished in 4 days more. How long would C take to do the whole thing by himself?
Miscellaneous Extensions

[13] A, B, C can do a piece of work in 11 days, 20 days, 55 days respectively, working alone. How soon can the work be done if A is assisted by B and C on alternate days?

*14. A has 2/70 envelopes to address. If B helps him, they do it in 1 hr. If B and C did them together, they would take 45 min. A starts alone, and after 2 hr. there are 50 left. How long would it take C to finish these alone?

15. £3 are paid for a task which A can do in 3 days, B in 4 days, and C in 6 days. If all work together, how much money should each receive?

*16. 5 guineas are paid for a task which A, B, C, working together, can do in 4 days, and A, B, working together, can do in 6 days, and A alone in 10 days. If all work together, how much money should each receive?

*17. £25 are paid for a task which A can do in 32 days, B in 20 days, C and D together in 12 days, and D in 24 days. If A, B, C, D work together, how much money should each receive?

*18. A sum of money is sufficient to pay the wages of A and B for 20 days, or the wages of B and C for 24 days, or the wages of A and C for 40 days. For how long will it suffice to pay B’s wages?

Harder Fractions

Example 8. Simplify \( \frac{3\frac{1}{5} - \frac{3}{5}}{\frac{4}{3} + \frac{1}{5} - \frac{5}{21} \frac{11}{2}} \).

Numerator = \( 3\frac{1}{5} - \frac{3}{5} = 3\frac{2}{5} \times \frac{5}{5} = 2 \frac{5}{5} \);

Denominator = \( (\frac{4}{3} + \frac{1}{5}) - (\frac{5}{21} \frac{11}{2}) = 21 \frac{1}{21} - 21 \frac{11}{2} = 21 \frac{1}{2} - 21 = 21 \frac{1}{2} - 21 \frac{1}{2} = \frac{1}{2} \);

:. Fraction = \( 2 \frac{5}{2} + \frac{1}{5} = \frac{9}{5} \times \frac{9}{9} = \frac{81}{9} = \frac{9}{9} = \frac{1}{2} \).

Example 9. Simplify \( \frac{5}{3 + \frac{4}{5}} \).

\[ \frac{1}{2} = \frac{1 \times 5}{2 \times 5} = \frac{4}{5} \];

:. Expression = \( 4 - \frac{5}{3 + \frac{4}{5}} = 4 - \frac{5}{\frac{18}{5}} = 4 - \frac{5}{\frac{3}{2} \times \frac{5}{5}} = 4 - \frac{5}{3} - \frac{5}{8} = \frac{1}{8} \).

Exercises 126

Simplify:

1. \( \frac{3\frac{2}{5} + 4\frac{1}{3} + 5\frac{1}{4}}{\frac{3}{8} + \frac{1}{2} - \frac{5}{6}} \)
2. \( \frac{2\frac{1}{3} - \frac{3}{4} - \frac{1}{6}}{\frac{2}{3} - \frac{1}{2}} \)
3. \( \frac{\frac{1}{2} + \frac{1}{3}}{\frac{3}{4} + \frac{1}{5}} \)
4. \( \frac{1\frac{1}{3} - \frac{4}{5}}{\frac{2}{3} - \frac{1}{6}} \)
5. \( \frac{3\frac{1}{2} + 2\frac{1}{4}}{\frac{4}{5} \times \frac{7}{10} - \frac{1}{3}} \)
6. \( \frac{5\frac{1}{3} \times \frac{6}{7} - \frac{1}{2}}{\frac{1}{3} + \frac{1}{4} - \frac{1}{2}} \)

Exercises involving Fractions

Example 10. I spend \( \frac{1}{4} \) of my money on lunch and \( \frac{1}{6} \) of the remainder on tea. If 10s. is left, what had I at first?

After spending \( \frac{1}{4} \) of my money, \( \frac{3}{4} \) of my money remains.

After spending \( \frac{1}{6} \) of this remainder, \( \frac{5}{6} \) of this remainder is left, that is \( \frac{5}{6} \) of the original amount is left.

But \( \frac{5}{6} \) of \( \frac{3}{4} \) = \( \frac{5}{6} \) of \( \frac{3}{4} \) of the original amount is 10s. ;

:. \( \frac{5}{6} \) of the original amount is 2s.

:. the original amount is 16s.

Check: Lunch costs \( \frac{1}{4} \) of 16s., = 4s., leaving 12s.; tea costs \( \frac{1}{6} \) of 12s., = 2s., leaving 10s.
EXERCISE 127

1. If of a journey takes 36 min., how long will the rest of it take at the same rate?
2. If I take 1 more minutes to walk of a mile than of a mile, how long do I take to walk a mile?
3. by weight of a certain alloy is copper. Find in lb. the weight of a lump of the alloy which contains 1 cwt. of copper.
4. If it costs £28 to fill of a cellar with coal, what will it cost to fill of the cellar?
5. A, B, C own a business between them. A owns of it, and B's share is twice C's share. What fraction of the business does B own?
6. A man loses of his capital one year and of what remains the next year. What fraction of his original capital is still left?
7. One-tenth of a post is painted red, of what remains is painted white, and the rest black. What fraction of the post is black?
8. A man spends of his income on rent and of the remainder in other ways. This leaves £135. What is his income?
9. A money prize is divided between A, B, C; A receives of the prize, B receives of the remainder. If C's share is £s, find the value of the prize.
10. of the water in a tank, which is full, is used on one day, and of the remainder on the next day. If 20 gall. of water are then left, how much does the tank hold?
11. A man left of his estate to his wife, of the rest to his son, and the balance, £3200, to charity. What did the son receive?
12. One-quarter of a field is too rough to use; two-fifths of the remainder is kept for cricket, and this leaves acres for other games. What is the area of the field?
13. One-eighth of a plank is damaged; a man cuts off three-quarters of the sound part, and this leaves ft. still to be used. What length of the plank was damaged?
14. A car loses of its value in its first year, and in the second year loses of its value at the beginning of that year. At the end of the second year it is worth £40; what was its value when new?
15. A and B own a business between them and the ratio of A's share to B's share is 11 : 9. If B sells of his share to A for £720, find the value of the business, and the percentage of the business then owned by A.

EXERCISE 128

1. A bowler takes 109 wickets for 1564 runs. Find the average number of runs per wicket, to 1 place of decimals.
2. A swimming-bath can be filled by two pipes in 15 hr., and by one of them alone in 24 hr. How long will it take if the second pipe is used by itself?
3. A train averages 42 mi. per hour for a journey of 140 mi. What increase must be made in the average speed to reduce the time by 18 min.?
4. A man goes of a journey by train, and then of the remainder by bus. This leaves him a quarter of a mile still to go. What is the length of the journey?
5. In what ratio must eggs costing a shilling for 10 be mixed with eggs costing 1s. 4d. a dozen to obtain a mixture costing 2s. 1d. per score?
6. A batsman has an average of 28.3 runs for 31 innings, 4 times not out. What must he score in his next innings, if he gets out, to raise his average to 30?
7. A boy spent of his money and then had left 1s. 6d. more than what he had spent. How much had he at first?
8. 1 c.c. of a liquid A weighs 0.6 gm. and 1 c.c. of a liquid B weighs 0.75 gm. If equal weights of A and B are mixed together, find the weight of 1 c.c. of the mixture, correct to 1 place.
9. A can do a piece of work in 8 days, which takes B 10 days and takes C 15 days. All start together, but after 2 days A stays away. How long will B and C take to finish the job?
10. The expenses of a party of 7 men for 14 days are £50, and of another party of 10 men for 7 days are £64. Find the average cost per man per day for all the men.
11. A liquid X contains 38% alcohol and a liquid Y contains 71% alcohol. In what ratio must X and Y be mixed to obtain a liquid containing 50% alcohol?
12. One tap fills of a bath in the same time that another tap fills of the bath. If both are turned on, the bath is filled in 12 min.; how long will each tap take by itself?
13. What is the average annual profit for 5 years' working if for the first 2 years the losses are £142, £75, and for the next 3 years the profits are £96, £217, £508?
14. Tea at 1s. 10d. per lb. is mixed with tea at 2s. 6d. per lb. in the ratio 5 : 3. In what ratio must this mixture be mixed with tea at 2s. 11d. per lb. to give a mixture worth 2s. 5d. per lb?
THREE-MINUTE AND FOUR-MINUTE ORAL PRACTICE

ORAL PRACTICE 17-24 (Ch. IV-XII)

[Nothing must be written down except the answer]

Oral Practice 17 (3 min.)
1. 2\frac{1}{2} - \frac{3}{4}.  2. 1 - (0.63 + 0.36).  3. 0.01 ÷ 0.4.
4. 2\frac{3}{4}d. × 24, in s. d.  5. Ratio of 7s. 6d. to 10s.
6. What % is 12 oz. of 1 lb.?  7. 7\frac{1}{2}% as a fraction.
8. Decrease 2s. 6d. in the ratio 3:5.
9. Cost of 21 lb. of sugar if 14 lb. cost 3s.
10. A salary of £350 a year is increased by 10%; what does it become?

Oral Practice 18 (3 min.)
1. \frac{1}{3} × \frac{1}{4} + \frac{1}{8}.  2. 0.2 \times \frac{3}{4} = 0.6.  3. (0.03)^2.
4. \frac{1}{4} of 2s.  5. Ratio of 10 oz. to 2\frac{1}{4} lb.  6. 50% as a percentage.
7. 65% of 1 ton, in cwt.  8. Divide 2 ft. in the ratio 3:7.
9. \frac{1}{9} of a number is 36; what is the number?
10. C.P. 4s.; S.P. 5s.; what is the gain per cent.?

Oral Practice 19 (3 min.)
1. \frac{2}{3} × 1\frac{1}{2}.  2. 0.0709 × 1.2.  3. 0.1 ÷ 0.001.  4. \frac{1}{4} of £3 16s. 5d.
5. Ratio of 1320 yd. to 1 ml.  6. 80% of 15s.
7. What % is 36 of 80?
8. In what ratio must £80 be increased to become £100?
9. A man spends 83% of his income; what fraction of his income does he save?
10. C.P. £2; gain 12\frac{1}{2}%; what is the S.P.?

Oral Practice 20 (3 min.)
1. Add: 2:73  2. 11 × 9.19.  3. 0.23 + 23.  4. \frac{1}{69} of 1 ton, in cwt., qr.
5. Ratio of 150 m. to 1 km.

Oral Practice 21 (4 min.)
1. \frac{1}{3}\frac{1}{8} × 3.  2. (0-2)^4.  3. 156 ÷ 1.2.  4. 11\frac{3}{4}d. × 36, in s. d.
5. Increase 75 by 20%.  6. 66\frac{2}{3}% of 2 yd., in yd., ft.
7. What % is 2\frac{1}{2} cwt. of 1 ton?
8. The scale of a map is 1 : 100,000; what length in km. is represented by 12-6 cm. on the map?
9. 4s. is divided into 3 shares in the ratios 7:3:2; what is the largest share?
10. S.P. 2s.; gain 20%; what is the C.P.?

Oral Practice 22 (4 min.)
1. Add: 1 617  2. 1 ÷ 2\frac{3}{8}.  3. 0.028 ÷ 14.  4. 5:42.
5. 1554  6. 0.125.  7. 3:504  8. 0.125.
9. Decrease 72 by 16\frac{2}{3}%.  10. Petrol costs 1s. 3d. per gallon; what does the price become if it rises 10%?

Oral Practice 23 (4 min.)
1. 0.75 × 14.  2. 1 0.01 ÷ 0.11.  3. Subtract: 3 001.
4. £2 4s. 35.  5. 12\frac{1}{2}% of 12s.  6. What % is 14 of 35?
7. Decrease £1 by 15%.  8. Find 3 integers proportional to 1s. 9d., 1s., 2s. 3d.
9. Two weights are in the ratio, 8 : 5; the larger is 1 lb., what is the smaller in oz.?
10. S.P. 6s.; loss 2s.; what is the loss per cent.?
TESTS IN COMPUTATION

Oral Practice 24 (4 min.)
1. $(\frac{1}{2} + \frac{3}{4})$.  
2. $0.129 \times 0.12$.  
3. 4s. 11d. $\times 7$, in s. d.  
4. $3 \div 0.01$.  
5. 17% of 50.  
6. Ratio of 1d. per oz. to 1s. per lb.  
7. Express $3$ Kg. $\div 200$ in grams.
8. What is the error per cent. in saying that a road is 4 mi. long, if its real length is 5 mi.?
9. The scale of a map is 2 in. to the mile; how many acres are represented by 1 sq. in. on the map?
10. The perimeter of a triangle is 12 cm., and the ratios of the sides are $5:6:9$; find the length of the shortest side.

TESTS IN COMPUTATION

TESTS 9-10 (Ch. IV-XII)

Test 9
1. $(2\frac{1}{2} - 3\frac{3}{4} + 5\frac{1}{2}) \div (2\frac{1}{2} - 3 + 2\frac{1}{2})$.
2. (i) $0.3098 \times 0.01015$; (ii) $1.72 \times 0.1075$.
3. Cost of 73 articles at £2 14s. 7d. each.
5. (i) What is 43% of £3 15s.? (ii) What percentage is $11\frac{3}{5}$ of 36?
6. An article costs 13s. 4d. and is sold at a gain of $22\frac{1}{2}$%; find the selling price.

Test 10
1. Express 13 cwt. 1 qr. 21 lb. as a fraction of 15 cwt.
2. (i) $0.01 \times 0.01 \times 0.11$; (ii) $9.9176 \div 0.01771$.
3. Cost of 12,000 articles at 7s. 8d. each.
4. Cost of 3 tons 12 cwt. of coal at £2 3s. 4d. per ton.
5. (i) Express $\frac{3}{8}$ as a percentage.  
(ii) Find the number of which 81 is $67\frac{1}{3}$ per cent.
6. By how much %, to 1 place of decimals, does 8435 exceed 7942?

Test 11
1. Arrange in descending order of magnitude:
   5 17 84 0.073 1
   3 24 125 0.13 1\frac{1}{8}
2. (i) $(0.04)^3 \times (0.15)^2$; (ii) $1237236 \div 0.4017$.

3. Cost of 670 articles at 17s. 8d. each.
4. Taking 1 km. $= 0.6214$ mi., express 835 m. in yards, to the nearest yard.
5. Divide £100 into three shares proportional to 8, 5, 3.
6. (i) What percentage is 16s. 4d. of £1?
   (ii) If $7\frac{3}{4}$% is deducted from a bill, £6 18s. 9d. remains to be paid. How much was the bill?

Test 12
1. $2\frac{1}{4} + \frac{1}{4}(1\frac{3}{8} - \frac{7}{4}) = (4\frac{3}{4} - 7\frac{3}{8} + 4\frac{1}{2})$.
2. (i) $10101 \times 0.01011$; (ii) $2644528 \div 0.004195$.
3. Cost of 2 tons 11 cwt. 2 qr. at £5 13s. 4d. per ton.
4. Express 0.5673 ch. in yd., ft., in., to nearest inch.
5. (i) Find the ratio of 7 cwt. 2 qr. to 1 ton.  
(ii) What percentage is 4½d. of 5 shillings?
6. Find to the nearest penny $7\frac{3}{4}$% of £65 12s.

Test 13
1. Express 18s. 4½d. as the decimal of £1, to 4 places.
2. (i) $0.3728 \div \frac{1}{2}$; (ii) $0.0124 \div 0.775$.
3. $(\frac{1}{2} - \frac{1}{4}) = (\frac{1}{2} + 3\frac{1}{2} - \frac{9}{4} + \frac{1}{6})$.
4. Find the price for 365 days at 17s. 10d. per day.
5. (i) What percentage is £2 10s. of £3 6s. 8d.?  
(ii) Find to the nearest £100 the value of $2\frac{1}{2}$% of £3,685,240.
6. An article is sold for 24s. 6d. at a gain of 40%; find the cost price.

Test 14
1. Express 0.03729 mi. in yd., ft., in., to the nearest inch.
2. (i) $(0.15)^2 \div (0.12)^2$; (ii) $1 \div 0.987$, to 2 places.
3. Cost of 2 mi. 3 fur. 7 ch. at £3 11s. 8d. per mile.
4. Find the ratio of 2 tons 12 cwt. 28 lb. to 4 tons 10 cwt. 84 lb.
5. Add: $133\frac{3}{4}$% of 2 yd.; 14½% of 5 ft.; 62½% of 1 ch.; answer in yd., ft., in.
6. The population of a town decreases from 63,573 to 59,184. Find the decrease per cent., to 1 place of decimals.
TESTS IN COMPUTATION

Test 15

1. \( \frac{1}{3} - \frac{1}{4} (\frac{1}{2} - \frac{1}{4}) + (\frac{1}{2} - \frac{1}{4} - \frac{1}{3}) \).
2. (i) 0.90807 \times 0.07093; (ii) 0.02 \div 3108, correct to 8 places.
3. Cost of 12,000 articles at 11½d. each.
4. Find to the nearest penny the cost of 3 qr. 16 lb. 10 oz. at £1.25 per cwt.
5. 56\% of a sum of money is £47 16s. 8d.; find the value of 18\% of the same sum.
6. An article is sold for £37 16s. at a loss of 16\%; find the loss.

Test 16

1. Evaluate to 2 places of decimals, \( \frac{17}{20} \times \frac{3}{4} \).
2. (i) \( 0.8 \times 0.027 \times 0.35 \); (ii) \( 63\% \) of 7 tons 3 cwt.
3. Express 42 ch. 17 yd. 1 ft. as a decimal of 1 fur., to 4 places.
4. Find the cost of 472\( \frac{3}{4} \) tons at £84 15s. 8d. per ton.
5. What percentage, correct to 1 place of decimals, is £1 17s. 10d. of £3 5s. 6d.?
6. What sum of money, to the nearest £1000, exceeds £3,584,265 by 23\( \frac{3}{4} \)%?

TESTS 17-24 (Ch. IV–XV)

Test 17

1. (i) \( 4\frac{1}{4} \times 4\frac{3}{4} \div (5\frac{1}{2} + 3\frac{3}{4}) \); (ii) \( 0.1702 \div 0.46 \).
2. Find the cost of 724 articles at 16s. 8\textfrac{1}{3}d. each.
3. (i) What percentage is 4d. of 1s. 3d.?
(ii) Find the ratio of 17 lb. 8 oz. to 18 lb. 2 oz.?
4. Express 0.1355 days in hr., min., sec., to nearest sec.
5. A beam of timber 7 ft. 6 in. long, 1 ft. 4 in. wide, 8 in. deep, weighs 240 lb. Find the weight of the timber per cu. ft.
6. In what time will the simple interest on £62 10s. amount to £611s. 3d. at \( \frac{3}{4} \)% p.a.?

Test 18

1. (i) \( 1\frac{7}{10} \div (\frac{1}{2} \times 3\frac{1}{3}) \); (ii) 7-5 \times 0.0625.
2. \( \frac{3}{2} \) of £3 4s. 9d. plus \( \frac{1}{4} \) of 10s. \( \frac{3}{4} \) of 1 guinea.

Test 19

1. (i) \( 17\frac{1}{2} - 10\frac{1}{4} + 3\frac{1}{4} \) of 8-4; (ii) 13-9472 \div 0.184.
2. (i) Express 11s. 7\textfrac{1}{2}d. as a decimal of £5.
(ii) Find, correct to the nearest penny, 39\% of 1 guinea.
3. 4\frac{1}{2} lb. of tea cost 7s. 9\textfrac{1}{2}d.; find the cost per lb.
4. Find the cost of 186 articles at 3s. 10\textfrac{1}{4}d. each.
5. A poultry-run is 86 ft. long, 32 ft. wide, and is surrounded with netting 4\frac{1}{4} ft. high. Find the cost of the netting at 3\frac{3}{4}d. per sq. yd.
6. By selling an article for £3 12s. a man loses 4\%. For what must he sell it to gain 24\%?

Test 20

1. (i) 30.05 \times 0.0346; (ii) 0.65 \times 1.08 \div 3.9.
2. Arrange in ascending order of magnitude: \( \frac{9}{8}, \frac{3}{5}, \frac{9}{8}, \frac{9}{28} \).
3. A man earns 19s. 6d. for 3\frac{1}{4} hr. work; how much does he earn per hour?
4. (i) 15\% of a number is 84. What is the number?
(ii) The weekly wage bill in a factory is £248. What will it become if all wages are raised by \( \frac{1}{4} \)%?
5. Find the cost of matting for a floor 31 ft. 6 in. long, 22 ft. 6 in. wide, at 2s. 4d. per sq. yd.
6. At what rate \( \frac{3}{5} \)% p.a. will the interest on £870 amount to £13 1s. in 4 months?

Test 21

1. (i) \( 5\frac{1}{2} \times 0.8 \div 12\frac{1}{2} \); (ii) Express 0.1375 of 1 mi. in yards.
2. (i) Find the ratio of 1 yd. 4 in. to 2 yd. 2 ft.
(ii) Find the value of 102\% of 12s. 6d.
3. Taking 1 gall. = 277.27 cu. in., express 10 gall. in cu. ft., correct to 2 places of decimals.

4. A bankrupt pays at the rate of 14s. 6d. in the £. How much will a creditor lose to whom he owes £1845?

5. Divide £1 into 3 parts proportional to 2, 3, 7.

6. A solid cylindrical metal bolt is $1\frac{1}{2}$ in. in diameter and 5 in. long. Find its weight to the nearest oz., if the metal weighs $3\frac{1}{2}$ oz. per cu. in. \[\text{Take } \pi = \frac{22}{7}.\]

**Test 22**

1. (i) \(\frac{2}{3} + 1\frac{1}{2} - \frac{3}{4} = \frac{3}{8}\); (ii) \(0.64 \times 0.035 = 0.014\).

2. (i) Express 7s. 3d. as a percentage of 9s. 8d.

(ii) Express 0.0043 ton in lb., oz., to nearest oz.

3. Find, to the nearest penny, the cost of 3 tons 12 cwt. 3 qr. at £1 14s. 8d. per ton.

4. The rent of a house is £132 a year, and rates at 8s. 9d. in the £ are demanded on \(\frac{3}{4}\) of the annual rent. Find the amount of the rates.

5. Divide £5 10s. between A, B, C so that A’s share is \(\frac{1}{2}\) of C’s share and B’s share is \(\frac{3}{2}\) of C’s share.

6. A sheet of lead 9.6 m. long, 2.25 m. wide weighs 1935 kg. Find the thickness of the sheet, correct to \(\frac{1}{10}\) mm., given that 1 c.c. of lead weighs 11.4 gm.

**Test 23**

1. (i) \(1 \div (4\frac{3}{4} \div 7\frac{1}{4})\); (ii) \(0.002769 \div 0.156\).

2. (i) Find the ratio of £1 2s. to £1 16s. 8d.

(ii) Find, to the nearest penny, 26\% of £1 11s. 2d.

3. How many pieces, each 0.15 yd. long, can be cut from 4\(\frac{1}{2}\) yd. of tape, and how much remains over?

4. The population of a town was 42,732 in 1921 and increased by 83\% in the next 10 years. What was it in 1931?

5. A man obtained 60 st. of potatoes from a plot of ground 90 ft. long, 24 ft. wide. What was the average yield in tons per acre?

6. The diameter of a garden roller is 2 ft. 6 in., and the roller is \(3\frac{1}{2}\) ft. wide. What area, to the nearest sq. yd., does it roll over in 24 revolutions? \[\text{Take } \pi = \frac{22}{7}.\]

**Tests 25-32 (Ch. X-XVII)**

**Test 24**

1. (i) What must be added to \(\frac{2}{7}\) to make \(\frac{3}{4}\) of 1\(\frac{1}{2}\)?

(ii) What whole number is nearest to \(\frac{7}{4} + \frac{8}{27}\)?

2. (i) Express 3\(\frac{1}{4}\) as a decimal of 4s. 2d.

(ii) Find, correct to the nearest penny, the value of 14\% of £2 17s. 10d.

3. 36\% of the contents of a chest of ten is worth £4 10s.; what is the value of 64\% of the contents?

4. A steamer travels at 22 knots; find its speed in miles per hour. \[1 \text{ knot} = 6080 \text{ ft. per hour.}\]

5. An alloy is made of copper weighing 550 lb. per cu. ft. and tin weighing 440 lb. per cu. ft.; the alloy contains 2 parts of copper to 3 parts of tin, by weight. Find the weight per cu. ft. of the alloy.

6. A tank containing water is 4 ft. 6 in. long, 3 ft. 4 in. wide. How many inches, correct to \(\frac{1}{10}\) in., does the water-level fall, when 20 gall. are drawn out of the tank? \[1 \text{ gal.} = 277.3 \text{ cu. in.}\]

**Test 25**

1. (i) Find the value of 22\(\frac{1}{2}\)% of 11s. 8d.

(ii) Find the ratio of 17s. 9d. to £2 19s. 2d.

2. Find the rent for 52 weeks at 13s. 10\(\frac{1}{4}\)d. a week.

3. Find, correct to the nearest penny, the simple interest on £15 17s. for 8 mo. at 4\% per annum.

4. Goods are bought at £8 a ton and sold at 1\(\frac{1}{4}\)d. per lb.; find the gain per cent.

5. In 1930, £748,187 was paid under the Workman’s Compensation Act for 2667 fatal accidents. Find, to the nearest £, the average amount paid per accident.

6. A plot of ground 80 yd. long, 32 yd. wide has a fence round it, and a pathway 4 ft. 6 in. wide is made round the edge of it inside the fence. Find the cost of the path at 1\(\frac{1}{4}\)d. per sq. ft.
Test 26

1. (i) $2\frac{3}{4} - 7\frac{2}{9}$; (ii) Express $\frac{10}{23}$ as a percentage.

2. In a sale, a discount of 1s. 8d. in the £ is given on the marked price. If the reduced price of a piano is £71 10s., what is its marked price?

3. Find the simple interest on £138 for 14 months at 3\% p.a.

4. The number of men unemployed on 28th September 1931 was 2,070,639, and on 22nd August 1932 was 2,289,043. Find the increase per cent., to the nearest whole number.

5. A man paid 62\% of the cost of a dinner-party. What did the party cost if he paid £4 2s. 6d.?

6. The diameter of the cross-section of a tube railway tunnel is 11 ft. 6 in. Find the area of the cross-section, to the nearest sq. ft. [Take $\pi = 3.14.$]

Test 27

1. (i) Find the ratio of 3 qr. 16 lb. to 7\frac{1}{2} cwt.

   (ii) Divide £1620 into two parts such that one is 70\% greater than the other.

2. Evaluate correct to 4 places of decimals:

   \[
   1 \frac{1}{7} + 1 \frac{1}{8} + 3 \frac{1}{7} \cdot 4 \frac{1}{7} + \cdots
   \]

3. Find the pay for 365 days at 17s. 4\frac{3}{4}d. per day.

4. A man can do \(\frac{2}{3}\) of a piece of work in 35 hr.; how long will he take to do 60\% of it at the same rate?

5. A photograph 16\frac{1}{4} cm. wide, 12\frac{7}{8} cm. high, is mounted on a card 22\frac{1}{4} cm. wide, 17\frac{3}{4} cm. high. Find the area of the part of the card left uncovered.

6. Find, to the nearest penny, the true present worth of a bill for £274 due in 7 months time, interest being reckoned at 4\% p.a.

Test 28

1. \[
2 \frac{3}{4} + 2 \frac{2}{8} - 5 \frac{1}{3};
\]

2. \[
36 \times 0.003 \times 0.0035 = 0.00036;
\]

3. Find, to the nearest penny, the cost of 11 cwt. 1 qr. 16 lb. at £45 per ton.

4. The profits of a business for 1934 exceeded those for 1933 by 26\%. The profits were £4620 for 1934; find the profits for 1933.

Test 29

1. \[
\pi = 3.1415926\ldots \quad \text{and} \quad e = 2.7182818\ldots;
\]

   (i) Find, correct to 4 figures, the value of \(\pi + e\); (ii) \(\pi - e\).

2. On a map of scale 4 in. to the mile, an estate occupies an area of 9\frac{3}{4} sq. in. Find the actual area, to the nearest acre.

3. Subtract \(\frac{1}{2}\) of 2\frac{1}{2} guineas from 87\frac{1}{2} of £1 10s.

4. The weight of a lorry is 12\% of the load it carries. Find in lb. the weight of the lorry if the combined weight of the lorry and its load is 2 tons.

5. Find, to the nearest penny, the true discount on a bill for £608 due in 4 months time, reckoning interest at 4\frac{1}{2}\% p.a.

6. From a circular disc of diameter 5 in., a semicircular portion of diameter 3 in. is removed. Find the area of the remainder, correct to 3 figures. [\(\pi = 3.142\).]

Test 30

1. In the period June–August 1934, 9,570,000 telegrams were despatched in Great Britain, and for the corresponding period in 1935, after the reduction in cost, 12,240,000 telegrams were despatched. Find the increase per cent. to the nearest whole number.

2. Evaluate, correct to 5 places of decimals:

   \[
1 \frac{1}{1} + 1 \frac{1}{1} + 1 \frac{1}{1} + 1 \frac{1}{1} + \ldots
\]

3. The average breadth of a railway is 32 yd. How many acres are required per mile of railway?

4. Divide £12 into 4 parts proportional to 3, 5, 11, 13.

5. A bankrupt's liabilities are £29,570 and his assets are £3850 10s. How much in the £ can the bankrupt pay? Answer to a farthing.

6. A speculator lost 35\% of his capital in one operation, and lost 40\% of the remainder in a second operation. If he then had £1560, how much had he at first?
Test 31

1. Taking 1 m. = 1.0936 yd., find to the nearest foot, the difference between 161 km. and 10 mi.
2. A grocer sells 4 lb. 10 oz. of butter for 6s. 2d.; find the sale price per lb. If the grocer bought the butter at 5s. 16s. 8d. per cwt., find his gain per cent.
3. Street accidents in Great Britain increased from 156,793 in 1930 to 181,077 in 1931. Find the increase per cent., to the nearest whole number.
4. Gunpowder is composed (by weight) of 75% nitre, 10% sulphur, 15% charcoal. Find in cwt. the amount of each substance used in making 1 ton of gunpowder.
5. The catalogue price of an article is £36; this is subject to a discount of 22 1/2%, and the price so reduced is subject to a further discount of 12 1/2%. What is the final price?
6. A halfpenny may be regarded as a circular cylinder of diameter 1 in., thickness 7/8 in., and weight 1/4 oz.; find, correct to 7/8 oz., the weight per cu. in. of the alloy of which it is made. [Take \( \pi = 3.14 \).]

Test 32

1. An express is timed to leave Swindon at 3:45 p.m. and to arrive at Paddington (77 3/4 mi. away) at 4:50 p.m.; find its average speed in miles per hour, to the nearest mile.
2. The Chancellor of the Exchequer received for the 7 yr. 1925 to 1931 the following amounts of "Consignment Money": £1427 1s.; £931 18s. 4d.; £90,745 5s. 8d.; £14,985 16s. 7d.; £7594 13s. 6d.; £2116 7s. 6d.; £1195. Find the average annual receipt from this source, to the nearest penny.
3. A bankrupt pays at 7s. 8 1/8d. in the £. How much will a man receive, to whom he owes £1348?
4. A grocer buys tea at £10 per ton and retails it at 1s. 8d. per lb., but there is a wastage of 7% of the weight in the retail trading. Find the grocer’s gain per cent.
5. If a man’s capital increases each year by 20% of its value at the beginning of the year, find the percentage increase in 3 years.
6. A swimming-bath 100 ft. long, 40 ft. wide, is being filled at the rate of 120 gal. of water per minute. At what rate in inches per hour, correct to 7/8 in., does the water-level rise? [1 gal. = 277.3 cu. in.]

REVISION PAPERS

PAPERS 25-32 (Ch. I-XII)

Paper 25

1. (i) Simplify \( \frac{\pi}{4} - \frac{\pi}{8} \).
2. (i) Divide 0.4280719 by 0.043.
3. (i) Find the value of \( \frac{1}{3} + \frac{1}{6} \).
(iii) Find the cost of 173 articles at £2 16s. each.
3. (i) What percentage is 24 of 40?
3. (ii) What is 43% of £75?
4. Find the area of a path 8 ft. wide which surrounds a rectangular lawn, 18 yd. long, 13 yd. wide.
5. £116 is shared between A, B, C, so that A gets twice as much as B, and B gets twice as much as C. What does C get?
6. A tailor allows a customer to deduct 10% from the amount of a bill for cash. If the customer settles the bill by paying £12 3s. cash, find the original amount.

Paper 26

1. (i) Express 67320 in prime factors.
(ii) Divide 24 yd. 1 ft. 9 in. by 15.
2. (i) How many articles at 4s. 7d. each can be bought for £4?
(iii) How much money remains?
(ii) Find the cost of 126 tons at £2 6s. 3d. per ton.
3. (i) What percentage is 4 3/4d. of 1s.?
(iii) The duty on a watch is 35% of its value. What is the duty on a watch valued at £7 10s.?
4. Express in acres, correct to 2 places of decimals, the area of a field 140 yd. long, 86 yd. broad.
5. The railway fare for 36 people for 60 mi. is £11 5s.; find the fare for 80 people for 40 mi. at the same rate.
6. A man buys goods for £475 and sells them for £532. Find his gain per cent.

Paper 27

1. (i) The height of Everest is 29,142 ft. Express this in mi., yd.
(i) Find the square root of 27225.
2. (i) Taking 1 m. = 39-37 in., express 3 m. 65 cm. in ft., in., to the nearest inch.
   (ii) Find the cost of 3 tons 7 cwt. 2 qr. at £2 6s. 8d. per ton.
3. (i) Find the ratio of 7 cwt. 2 qr. to 1 ton.
   (ii) Express 62 1/3% as a fraction, and 62 1/3% as a percentage.
4. The population of a town increases from 11,200 to 13,100. Find the increase per cent., to the nearest whole number.
5. A metal sheet, 8-75 in. long, 3-2 in. wide, weighs 10 1/2 oz. If it is cut down so as to be 8 in. long, 3 in. wide, what will it weigh?
6. By selling a car for £132, I lose 28%. What is my loss?

Paper 28

1. (i) By what must 3 1/2 be multiplied to give 9 1/2?
   (ii) Express 17s. 10d. as a decimal of £1, correct to 4 places.
2. (i) Divide £422.4s. 8d. by 53.
   (ii) Find the cost of 3 yd. 1 ft. 8 in. at £1 5s. 6d. per yard.
3. (i) Find the value of 11 1/3 of £266 13s. 4d.
   (ii) 73 boys in a school get measles, and this is just less than 28% of the school. How many are there in the school?
4. A man owes £176, and only possesses £33 1s. 6d.; at what rate in the £, correct to the nearest penny, can he pay his creditors?
5. The diagram represents the top of a metal sheet 9 in. thick. The corners are right-angled and the dimensions are shown in inches. Find the volume of the sheet in cu. in., and its weight if the metal weighs 4-8 oz. per cu. in.

6. The speed of a tube train at various times after the start is as follows:

<table>
<thead>
<tr>
<th>Time in seconds</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed in m.p.h.</td>
<td>9</td>
<td>17</td>
<td>23</td>
<td>27.5</td>
<td>30</td>
<td>30</td>
</tr>
</tbody>
</table>

Represent these facts by a graph, and estimate (i) the time when the train first attains a speed of 20 m.p.h., (ii) its speed 3 sec. after the start.

Paper 29

1. (i) Simplify \( \left( \frac{1}{2} + \frac{3}{4} + \frac{5}{8} \right) \div \left( \frac{1}{8} + \frac{3}{4} + \frac{5}{8} \right) \).
   (ii) Express 0-4283 mi. in ch., yd., ft., to the nearest foot.
2. (i) Find the cost of 247 yd. at 3s. 11 1/2d. per yard.
   (ii) If 1 rupee = 1s. 6d., express 32 rupees 12 annas 2 pice in £. s. d., to the nearest penny, given that 1 rupee = 16 annas = 64 pice.
3. (i) 36% of a certain number is 117; find the number.
   (ii) A boy’s height increased from 5 ft. 2 in. to 5 ft. 3 1/2 in. in a year. Find the increase per cent., to one place of decimals.
4. How long will a column of soldiers 450 yd. long take to pass a given point if the rate of marching is 120 paces of 30 in. each per minute?
5. On a map of scale 4 in. to the mile an estate has an area of 12 35 sq. in. Find its area in acres.
6. A grocer buys 15 tons of sugar for £350 and sells it at 2 3/4d. per lb. Find his gain per cent. At what price would he have to sell it to gain 40 per cent.?

Paper 30

1. (i) Find the H.C.F. of 25245 and 64350.
   (ii) Divide 0-0028213 by 0-634.
2. (i) Find the dividend on £532 at 2s. 10d. in the £.
   (ii) The weekly output of 1,124,000 miners is 4,813,000 tons of coal. Find to the nearest cwt. the weekly output per miner.
3. (i) What percentage is £1 10s. 9d. of £6 16s. 8d.?
   (ii) After a man’s salary had been increased by 8%, it became £375 a year. What was the increase in his salary?
4. How long does a train 72 yd. long, travelling at 30 mi. an hour, take to pass completely through a station 126 yd. long?
5. A metal box without a lid is 15 cm. long, 11 cm. wide, 5 cm. high, measured externally, and the metal is 5 mm. thick. Find the weight of the box if 1 c.c. of the metal weighs 7-2 gm.
6. A obtains 35% of the total, 160 marks, assigned to a paper; B is ahead of A by 25% of A’s marks, C is ahead of B by 20% of B’s marks. What percentage of the total did C get?
Paper 31

1. (i) Express $\frac{1}{2} + \frac{1}{3}$ as a decimal, correct to 3 places.
   (ii) Find the value of $(0.05)^3 \times (0.16)^2 \div 0.2$.

2. (i) A locomotive uses $5\frac{1}{2}$ tons of coal at £1 18s. 6d. per ton for a journey of 240 mi. Find the cost per mile, to the nearest 1/2d.
   (ii) Find to the nearest penny the cost of 4 cwt. 3 qr. 21 lb. at £1 16s. 9d. per cwt.

3. (i) When railway fares were increased in the ratio 7 : 4, the fare for a journey became 24s. 6d.; what was the increase?
   (ii) A length is stated to be 5 cm. when it is really 6:25 cm.; find the error per cent.

4. A gas company charges 1s. 2½d. per therm. If 100 cu. ft. of gas are equivalent to 0:45 therm, find to the nearest penny the cost of 3800 cu. ft. of gas.

5. £7 11s. 8d. is paid for a job which three men working separately can do in 8, 10, and 15 days respectively. If all work together, how should the payment be shared between them?

6. In 1933, a house is rated at £80, and the rate payable is 8s. 4d. in the £. How much has to be paid? In 1934, the amount at which the house is rated is increased by 5%; and the rate is decreased by 10%. Find the decrease per cent. in the amount to be paid.

Paper 32

1. (i) Divide 9 cwt. 1 qr. 15 lb. 5 oz. by 89.
   (ii) Taking 1 m. as 40 in., find the nearest whole number the number of acres in 1 ac. [1 ac = 100 sq. m.]

2. Find correct to the nearest penny:
   (i) the value of $7\frac{1}{2}$% of £5 17s. 3d.;
   (ii) the cost of 2 tons 17 cwt. 1 qr. 10 lb. at £3 11s. 6d. per ton.

3. (i) What sum exceeds 16s. 8d. by 24 per cent.?
   (ii) The income of the Southern Railway fell from £6,960,000 in 1922 to £6,090,000 in 1923. Find the decrease per cent.

4. An elastic spring is held at one end; its natural length is 12 cm., and its stretched lengths when different weights are attached to the other end are as follows:

<table>
<thead>
<tr>
<th>Weight in gm.</th>
<th>20</th>
<th>50</th>
<th>70</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length in cm.</td>
<td>13</td>
<td>14.5</td>
<td>15.5</td>
<td>17</td>
</tr>
</tbody>
</table>

Illustrate these facts by a graph and find (i) the length of the spring when a weight of 85 gm. is attached, (ii) the weight which will stretch it to a length of 14.8 cm.

5. 10 men can construct a motor track 220 yd. long, 10 yd. wide, in 6 days of 8 hr. each. How many men will be required to construct a similar track 18 mi. long, 12 yd. wide in 72 days of 9 hr. each?

6. A property is divided between A, B, C so that the shares of A, B are in the ratio 5 : 3, and the shares of B, C are in the ratio 4 : 3. If A receives £825 more than C, how much does B receive?

PAPER S 33-40 (Ch. I-XIV)

Paper 33

1. Express $6552 \times 1150$ in prime factors. Find the least whole number by which it must be multiplied to give a perfect square.

2. A “bag” of white pepper contains 168 lb. (i) How many bags go to the ton? (ii) What is the cost per bag at 1s. 7d. per lb.?

3. Find the value of $4\frac{3}{4}$% of £75.

4. My household expenses in 1930 were: Food £300, clothing £140, rent £70, sundries £190. In 1934, the costs of food and clothes had increased by 50% and 40% respectively, and sundries by £24; the rent was unchanged. Find the new total expenditure and the increase per cent.

5. Find the amount of wood used to make an open box, 25 cm. long, 14 cm. wide, 8 cm. high, external measurements, the wood being 5 mm. thick.

6. Find the length of the circumference of a circle of diameter 5 in., correct to $\frac{1}{10}$ in., and the area of the circle, correct to $\frac{1}{10}$ sq. in. [Take $\pi = 3.142$.]
Paper 34
1. If oranges are reduced from 1s. 6d. a dozen to 1s. 3d. a dozen, how many more can be bought for 7s. 6d.?
2. Arrange in ascending order of magnitude:
   \( \frac{7}{8}, \frac{3}{4}, \frac{1}{2}, \frac{5}{6}, \frac{1}{3}, \frac{1}{4} \).
3. On a map whose scale is 1 in. to the mile the area of an estate is \(3\frac{1}{2}\) sq. in. Find its actual area in acres.
4. A tradesman accepts £50 8s. cash for a bill for £52 10s. What percentage of the bill did he deduct for cash?
5. A bankrupt owes £1250 to A and £2300 to B; his assets are £2710. Three-fifths of his debt to A is first paid in full, and the remainder of his assets are then used to pay the rest of what he owes A and what he owes B at the same rate in the £. How much does B receive?
6. Find the volume of a circular cylinder of diameter 3 in., and height 8 in., correct to \(\frac{1}{100}\) cu. in.; and find the area of its curved surface correct to \(\frac{1}{10}\) sq. in. [Take \(\pi = 3.142\).]

Paper 35
1. Edinburgh is 390 mi. from London. How much time is saved by going by air at 120 mi. an hour instead of by train at 54 mi. an hour?
2. (i) What percentage is 13s. 6d. of £1?
   (ii) A man buys 140 eggs at 3s. 3d. per score and 180 eggs at 2s. 1d. a dozen, and sells the whole at 5 for 1s. Find his gain per cent.
3. A motorist uses 389 gall. of petrol in driving 8253 mi. and his expenditure on petrol is at the rate of 7d. every 9 mi. Find the average cost of the petrol per gall., to the nearest 1d.
4. I can buy cigarettes at 100 for 4s. 8d. or at 30 for 1s., but each of the second kind contains only \(\frac{4}{7}\) of the amount of tobacco in the first kind. Which kind should I buy to obtain the greater amount of tobacco for a given sum of money, and how much more weight per cent. shall I get by doing so?
5. A coal cellar measures 12 ft. by 7 ft. by 8 ft. If 90 lb. of coal go to the cu. ft., find the cost of filling the cellar at £2 5s. a ton.
6. Find correct to the nearest inch the diameter of a wheel whose rim is 85 in. long. [Take \(\pi = 3.14\).]

Paper 36
1. The following tickets were sold for a pageant: 1189 at £1 1s.; 1622 at 10s. 6d.; 2411 at 5s.; 778 at 2s. 6d. What was the total amount paid and the average price per ticket?
2. 1 cwt. of artificial manure is sufficient for 1000 sq. yd. of land. Find in cwt. lb., to the nearest lb., how much is sufficient for a field of area 5 ac. 3\(\frac{1}{4}\) ro.
3. (i) What percentage is scored by a boy who obtains 81 marks out of 150?
   (ii) A man gains 20\% by selling an article for 19s.; what did it cost?
4. There are 20 members of a committee, and the ratio of the number of men to the number of women is 3 : 1. How many women must be added to the committee to make the ratio of men to women 3 : 2?
5. A room is 16 ft. long, 12 ft. wide, 9 ft. high. Find the cost of the wall-paper needed for it, allowing 84 sq. ft. for window space, etc., if the paper costs 4s. 9d. per piece, each piece being 12 yd. long, 28 in. wide.
6. A cylindrical telegraph pole is 10 in. in diameter and 24 ft. high. If the wood weighs 48 lb. per cu. ft., find the weight of the pole to the nearest lb. [Take \(\pi = 3.142\).]

Paper 37
1. (i) Divide £173 19s. 6d. by 197.
   (ii) The duty on cigarettes is 14s. 7d. per lb.; how much is it per cwt.?
2. A grocer buys tea at £7 per cwt. and sells it at 1s. 10\(\frac{1}{2}\)d. per lb. What is his gain per cent.? At what rate must he sell it to gain 60\%?
3. A gramophone record, price 1s. 6d., contains two dance tunes, each of which takes 2\(\frac{1}{2}\) min. to play. What does the dance music cost by the hour, if records are worn out after being played 100 times?
4. In firing at a target, A scores 2 hits in every 3 shots, B 3 hits in every 4 shots, C 4 hits in every 5 shots. Each fires the same number of shots and the target is hit altogether 931 times. How many shots were fired altogether? How many hits did A score?
5. Find to the nearest gallon how much water is required to fill a cistern 6 ft. by 4 ft. by 3 ft. [1 cu. ft. = 6.23 gall.] 

6. The bore (internal diameter) of an iron pipe is 7 cm. and the iron is 5 mm. thick. If the pipe is 40 cm. long and the iron weighs 7.2 gm. per c.c., find in kg., correct to 2 in 36 kg., the weight of the pipe. [Take $\pi = 3.142$.]

Paper 38

1. (i) The area of Denmark is 15,000 sq. mi. and its population is 3,600,000. Find the population per sq. mi.

(ii) A car runs 2640 yd. in 2 $\frac{3}{4}$ min. Find its average speed in miles per hour.

2. A path from the foot to the top of a hill ascends for 600 yd. at a gradient of 1 in 18, then for 300 yd. at 1 in 12, then for 150 yd. at 1 in 9, and lastly 200 yd. at 1 in 8. Find the height of the hill in feet.

3. When railway fares were increased in the ratio 7 : 4, the fare for a certain journey was increased by 16s. 6d.; what was the new fare?

4. A man X leaves a town P at 1 p.m. and travels to a town Q, 40 mi. away, going at 20 m.p.h. for 1 hr. and at 10 m.p.h. for the rest of the way. Another man Y leaves Q at 2 p.m. and motors to P at 30 m.p.h. Draw the travel graphs of X and Y, and find when and where the two men pass one another.

5. Water flows at 2 m.p.h. through a pipe whose cross-section is of area 7 sq. in. How long will it take to fill a cistern, 7 ft. by 4 ft. by 2 ft. 9 in., if the pipe is kept full?

6. The inner edge of a circular running-track is 400 m. long. Find the diameter of this circle to the nearest metre, and the area enclosed by it in hectares to the nearest are. [1 are = 1 sq. Dm.; take $\pi = 3.142$.]

Paper 39

1. (i) An engine uses 4 cwt. of fuel every 3 hr.; how long will 2 $\frac{1}{2}$ tons of fuel last?

(ii) A man who takes 7 steps in 3 sec. walks 6.5 km. per hour; find the length of his stride to the nearest cm.

2. How much is produced by a rate of 5s. 8d. in the £, on a rateable value of £348,600?

3. An 18-carat gold watch-chain weighs 940 grains (18 parts out of every 24 are pure gold). If the price of gold rises from £4 4s. per oz. to £7 per oz., find the increase in value of the gold in the chain. [1 oz. = 480 grains.]

4. The air-pressure on the screen of a car for various speeds is as follows:-

<table>
<thead>
<tr>
<th>Speed in m.p.h.</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>45</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure in lb. per sq. ft.</td>
<td>0.6</td>
<td>1.6</td>
<td>3.6</td>
<td>6.4</td>
<td>8.1</td>
<td>10</td>
</tr>
</tbody>
</table>

Represent these facts by a graph and find (i) the pressure when the speed is 35 m.p.h., (ii) the speed when the pressure is 3 lb. per sq. ft.

5. The outside measurements of an open wooden tray of rectangular shape to hold foolscape are 15 in. by 10 $\frac{1}{4}$ in. by 2 $\frac{3}{8}$ in.; the wood is $\frac{1}{8}$ in. thick. Find to the nearest cu. in. the volume of wood used in making the tray.

6. A cylinder closed at both ends is 12 cm. long, and 4 cm. in diameter. Find the area of its total external surface. [Take $\pi = \frac{22}{7}$.]
6. The floor of a hall is a rectangle 60 ft. long, 30 ft. wide. The side walls are 12 ft. high and the cross-section of the ceiling is a semicircle which springs from the side walls. Find the volume of the hall to the nearest 100 cu. ft., and the area of the ceiling to the nearest 10 sq. ft. [Take π=3.142.]

PAPERS 41-48 (Ch. I-XVII)

Paper 41

1. (i) Simplify \(\frac{\sqrt{3} + \sqrt{2}}{1 + \sqrt{2} \times \sqrt{3}}\).

(ii) Divide £1 into 3 parts proportional to 2, 5, 9.

2. A car travels 1056 yd. in 1\(\frac{1}{2}\) min. Express this speed in miles per hour.

3. In 1928, the rateable value of Oxford was £734,450 and rates were paid at 7s. 8d. in the £. How much altogether was paid? The population was 80,500; find the amount paid per head to the nearest penny.

4. Carpet 27 in. wide is sold at 5s. 6d. per yard length. Find the cost of the carpet for a piece 16 ft. 6 in. long, 11 ft. 3 in. wide.

5. Find, correct to the nearest penny, the simple interest on £284 for 16 mo. at 4\(\frac{1}{2}\)% p.a.

6. In what ratio must coal at 32s. per ton be mixed with coal at 44s. per ton to make the mixture worth 36s. 6d. per ton?

Paper 42

1. (i) Express \(2\frac{3}{7}\) as a percentage.

(ii) B exceeds A by 60\%; C exceeds B by 12\(\frac{1}{2}\)%; find the ratio of A to C.

2. An advertisement containing 43 words costs £1; find the charge per word correct to the nearest penny.

3. A bankrupt pays at the rate of 16s. 3d. in the £. How much will a man receive, if he is owed £45 13s. 4d. by the bankrupt? What percentage of the bankrupt’s debts remain unpaid?

4. If the diameter of the wheel of a car, travelling at 30 mi. an hour, is 2\(\frac{1}{4}\) ft., find the number of revolutions per minute made by the wheel. [Take π=\(\frac{22}{7}\).]

Paper 43

1. (i) Simplify \(\frac{\sqrt{2} - 3\sqrt{3}}{2\sqrt{2} + 3\sqrt{3}}\).

(ii) Find the value of 2\(\frac{1}{2}\)% of £41 16s., to the nearest penny.

2. Taking 35 yd. equal to 32 m., express 7 mi. in km.

3. The number of S.O.S. messages broadcast by the B.B.C. was 858 in 1933, and 889 in 1934. 44\(\frac{3}{4}\)% were successful in 1933, how many was this? 503 messages were successful in 1934, what percentage, correct to 3 figures, was this?

4. Find the weight of a rectangular metal plate 6\(\frac{1}{4}\) cm. long, 4\(\frac{1}{4}\) cm. wide, 6 mm. thick, if the metal weighs 7\(\frac{1}{2}\) gm. per c.c.

5. Find, to the nearest penny, the true discount on a bill for £2526 due in 3 months time, reckoning interest at 7\(\frac{1}{2}\)% p.a.

6. A well can be pumped dry in 1\(\frac{3}{4}\) hr. by two pumps which discharge respectively 80 gall., 120 gall. per min. If the smaller pump is used alone, the water sinks at 1 in. per min.; how deep is the well?

Paper 44

1. (i) Find the cost of 87 tons of coal at £1 17s. 8d. per ton.

(ii) The duty on an article is 17\(\frac{3}{4}\)% of its value. If the duty is 12s. 10d., find the value.

2. Find the cost to the nearest penny of 39,765 articles at 12s. 6d. per thousand.

3. 1 cwt. of condensed milk is equivalent to 35 gall., and 1 cwt. of powder-milk to 95 gall. In 1931, 483,600 cwt. of condensed milk and 352,700 cwt. of powder-milk were imported. To how many gallons, correct to 3 figures, was the total imported equivalent?
4. Find, to the nearest penny, the simple interest on £453 18s. for 24 yr. at 31% p.a.

5. Find, to the nearest gallon, the amount of water in a cylindrical tank 6 ft. in diameter, if the water is 3 ft. 4 in. deep. [Take 1 cu. ft. = $\frac{5}{4}$ gallon, and $\pi = 3.142$.]

6. A man makes a profit of 20% by selling an article for 15s.; he sells $\frac{3}{4}$ of his stock at this price and the rest at 10s. each. Find his gain per cent. on the whole transaction.

**Paper 45**

1. Find, to 3 places of decimals, the values of $\sqrt[3]{17.8297}$ and arrange them in ascending order of magnitude.

2. (i) The area of the British Dominions is 14,200,000 sq. mi.; find, correct to 2 figures, what percentage this is of the area of the Earth, 55,500,000 sq. mi.

   (ii) Find, correct to 1 figure, the error per cent. in the following rule: to convert metres to inches, subtract $\frac{3}{4}$ of the number and multiply the remainder by 40. [1 m. = 39.37 in.]

3. Planks 4 in. wide, 3 ft. 6 in. high are placed side by side to form a fence round a piece of ground 24 yd. long, 20 yd. wide. Find the length of the plank required, and the cost at 1s. 6d. per sq. ft.

4. In what ratio by volume must two metals, one weighing 485 lb. per cu. ft., the other 520 lb. per cu. ft., be combined to obtain an alloy weighing 506 lb. per cu. ft.?

5. Find the sum of money which will amount in 7 months time to £230 5s. at 4% p.a.

6. A starts a business with £400, and at the end of 4 mo., takes in B with £3000, and 3 mo. later C joins with £5000. The profits 1 yr. from the start are £242 10s. Find A's share if the profits are divided so that the shares are proportional to the "L-months" of capital each has in the business.

**Paper 46**

1. (i) Simplify $\frac{3}{4} - \frac{5}{18} \times (\frac{1}{2}) + (\frac{1}{2} - \frac{1}{2}) \div 1 \times \frac{1}{2}$.

   (ii) Express 0.007 as a percentage of 0.25.

2. Find the cost of 126 linen sheets at 28s. 6d. per pair.

3. Railway receipts for a certain year were as follows: from passenger traffic £2,487,700; from goods traffic £3,440,800; find to the nearest integer what percentage of the receipts came from goods traffic.

4. If £760 10s. amounts in 3 yr. at simple interest to £897 7s. 10d., find the rate per cent. p.a.

5. Find to the nearest gm. the weight of 12 m. of wire, diameter 3 mm., if the wire weighs 8 gm. per c.c. [Take $\pi = 3.14$.]

6. A boy, engaged to work for 30 days, is paid 5s. for each day he works and is fined 6d. for each day he stays away. At the end he receives £5 11s. 6d.; how many days did he work?

**Paper 47**

1. (i) Express 1s. 5½d. as a percentage of 3s. 6d.

   (ii) Express 1 ft. 5 in. in cm., correct to 3 figures, given that 1 yd. = 0.9144 m.

2. 8 tons 2 cwt. 2 qr. of coke cost £14 12s. 6d.; find the cost per ton.

3. A non-stop train leaves London at 10.30 a.m. and travels via Taunton (143 mi. from London) to Plymouth (226 mi. from London), arriving at 2.37 p.m. At what time to the nearest minute does it pass Taunton, assuming it travels at a uniform speed.

4. Which of the following approximations are obviously wrong? Give reasons.

   (i) $\sqrt{0.4816} \approx 0.2194$; (ii) $(4.86)^2 \approx 11.7$;

   (iii) $(0.3397)^2 \approx 1.154$; (iv) $27 \times 7 \times 23 \approx 2.41$.

5. A tank, 7 ft. 6 in. long, 5 ft. 4 in. wide, contains 700 gal. of water. Find the depth of the water. [Take 1 cu. ft. = $\frac{1}{6}$ gal.]

6. A man buys a house for £1450 and rents it so that, after allowing £9 a year for insurance and repairs, he obtains interest at 5% p.a. on his money. Find the rent.
Paper 48

1. (i) Express \( \frac{\pi}{2} - \sqrt{2} \) as a decimal, correct to 3 figures.
   (ii) Find the duty on an article valued at £6 4s. 4d., if duty is charged at the rate of 8s. 4d. in the £.

2. 120 cartridges cost 15s. 6d. Find the cost per million, if there is a reduction of 30\% in the price for this quantity.

3. Simplify \( \left( \frac{2\frac{1}{3} \times 4\frac{1}{3} - 3\frac{1}{2} + 5\frac{1}{2}}{1\frac{3}{8} - 5\frac{3}{4} - 4\frac{3}{8}} \right) + \left( \frac{4\frac{1}{2}}{3\frac{1}{2}} \times \left( \frac{7}{2} - \frac{1}{4} \right) \right) \).

4. An open rectangular tank is made of cement, the walls and base being 6 in. thick. Externally, the tank is 10 ft. long, 7 ft. wide, 3 ft. high. Find its weight in tons if the concrete weighs 168 lb. per cu. ft.

5. A shopkeeper allows a discount of 17\%\% on the marked price of a chair for cash in a sale. The sale price is £2 17s. 9d.; find the marked price.

6. A can do a piece of work in 21 days, B can do it in 31\frac{1}{3} days, and C in 63 days. How long will it take if all work together? If £30 is paid for the work, how much should each receive?

PART III

CHAPTER XVIII

SQUARE ROOT AND USE OF TABLES

The graph printed below represents the squares of numbers from 0 to 10. It may be used to read off approximate values of square roots of numbers between 0 and 100.

For oral work. Use the graph to write down approximate values of: (i) 3.4, 6.8, 8.6, 9.2; (ii) the numbers whose squares are 34, 52, 74, 88; (iii) the square roots of 46, 58, 92, 12.
Finding Square Roots. The problem of finding the square root of, say, 1369 is equivalent to finding the length of a square containing 1369 units of area.

Since $30^2 = 900$ and $40^2 = 1600$, the side of the required square is between 30 and 40 units long. We therefore start with a square whose side is 30 units long and find the breadth of the border which will convert it into a square of area 1369 units.

Area of required border $(1369 - 900) = 469$ units.

If the breadth of the border is $x$ units, the area of the border is $x(30 \times 2 + x)$ units; therefore $(60 + x)x = 469$.

The value of $x$ is found by trial: $x$ is small compared with 60, i.e. $60x = 469$; $x = 469/60 = 7$; but $67 \times 7 = 469$, exactly;

i.e., the required border is 7 units wide;

i.e., $\sqrt{1369} = 37$, exactly.

The reader should verify that $37 \times 37 = 1369$.

Example 1. Find the square root of 69169.

Mark off 69169 in periods of two digits starting from the decimal point, in this case from the right.

The greatest integer whose square is not more than 6 is 2; write 2 in the first period of the answer and 2 squared, that is 4, under the 6, and subtract. Bring down the digits 91 from the next period. Write down twice 2, that is 4,

and find the approximate value of $291 \div 40$, that is 7.

$47 \times 7 = 329$, but this is too large; $46 \times 6 = 276$.

Write 6 in the trial divisor and the second period of the answer, 276 under 291, and subtract. Bring down the digits 69 from the next period. Write down twice 26, that is 52, and find the approximate value of $1569 \div 520$, that is 3.

$523 \times 3 = 1569$. Write 3 in the trial divisor and in the third period of the answer.

$\sqrt{69169} = 263$, exactly.

The reader should verify that $263 \times 263 = 69169$.

Exercise 129

Find the square roots of:

1. 529.  
6. 3249.  
11. 6409.  
16. 169744.  
21. 4224900.
2. 1156.  
7. 3721.  
12. 137641.  
17. 157609.  
22. 9548100.
3. 1444.  
8. 7396.  
13. 117649.  
18. 277729.  
23. 166464.
4. 2209.  
9. 9801.  
14. 15129.  
19. 405769.  
24. 369664.
5. 53361.  
10. 97969.  
15. 556516.

Square Root of a Decimal. The same method can be used for finding the square root of a decimal.

Example 3. Find the square root of 0.00191844.

Mark off the number in periods of two digits, starting from the decimal point.

Put the decimal point in the answer above the decimal point in the given number.

The period 00 after the decimal point gives 0 in the answer after the decimal point; then proceed as with whole numbers.

i.e., $\sqrt{0.00191844} = 0.0438$, exactly.
Example 6. Find correct to 2 places of decimals the value of \( \frac{1}{\sqrt{3}} \)

\( \sqrt{3} \) means the number which when multiplied by itself makes 3, that is

\[ \sqrt{3} \times \sqrt{3} = 3 \]

\[ \therefore \frac{1}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{1}{3} \]

\[ \therefore \frac{1}{\sqrt{3}} \] is the square root of \( \frac{1}{3} \)

We therefore find the square root of \( \frac{1}{3} \):

\[ \frac{1}{3} = 0.333 \ldots \]

\[ \therefore \frac{1}{\sqrt{3}} = 0.58, \text{ correct to 2 places.} \]

Then \( \frac{1}{\sqrt{3}} = 0.577, \text{ as before.} \)

This process is called rationalising the denominator.

For example, to find the value of \( \frac{\sqrt{7}}{\sqrt{11}} \)

either say \( \frac{\sqrt{7}}{\sqrt{11}} = \frac{\sqrt{7}}{\sqrt{11}} = \sqrt{0.636} \ldots \), and work out the square root;

or say \( \frac{\sqrt{7}}{\sqrt{11}} = \frac{\sqrt{7} \times \sqrt{11}}{\sqrt{11} \times \sqrt{11}} = \frac{\sqrt{77}}{11} \), then work out the square root \( \frac{\sqrt{77}}{11} = \frac{\sqrt{77}}{11} = \sqrt{77} \) and divide it by 11.

Do not work out separately \( \sqrt{7} \) and \( \sqrt{11} \); this would involve, besides finding two square roots, an awkward long division.

Example 5. Find correct to 2 places of decimals the square root of \( \frac{2}{3} \).

To obtain the square root correct to 2 places of decimals, you will have to have

3 periods, i.e., 6 decimal places, after the decimal point:

\[ \frac{2}{3} = 0.666 \ldots \]

\[ \sqrt{\frac{2}{3}} = 1.66, \text{ correct to 2 places.} \]

To find the square root of \( \frac{1}{3} \),

\[ \frac{1}{\sqrt{3}} \]

we have:

\[ \frac{1}{\sqrt{3}} = 0.58 \]

\[ \therefore \frac{1}{\sqrt{3}} = 0.577, \text{ as before.} \]

This process is called rationalising the denominator.

For example, to find the value of \( \frac{\sqrt{7}}{\sqrt{11}} \)

either say \( \frac{\sqrt{7}}{\sqrt{11}} = \frac{\sqrt{7}}{\sqrt{11}} = \sqrt{0.636} \ldots \), and work out the square root;

or say \( \frac{\sqrt{7}}{\sqrt{11}} = \frac{\sqrt{7} \times \sqrt{11}}{\sqrt{11} \times \sqrt{11}} = \frac{\sqrt{77}}{11} \), then work out the square root \( \frac{\sqrt{77}}{11} = \frac{\sqrt{77}}{11} = \sqrt{77} \) and divide it by 11.

Do not work out separately \( \sqrt{7} \) and \( \sqrt{11} \); this would involve, besides finding two square roots, an awkward long division.

Exercise 132

Find correct to 3 significant figures, the square roots of:

1. 1.2
2. 2.20
3. 3.10
4. 4.03
5. 5.74
6. 6.074
7. 7.094
8. 8.03942
9. 9.5321
10. 10.9947
11. 11.0947
12. 12.0816
13. 13.8632
14. 14.0909
15. 15.084
16. 16.084

Find, correct to the number of decimal places indicated in brackets, the square roots of:

17. 17.094 (2)
18. 18.06085 (3)
19. 19.01 (3)
20. 20.0932 (3)
21. 21.001656 (4)
22. 22.000036 (3)
23. 23.4 (2)
24. 24.25 (2)
25. 25.1 (3)
26. 26.1 (2)
27. 27.3 (2)
28. 28.7 (3)
Express the following fractions so that their denominators do not contain √ signs:

29. \(\frac{1}{\sqrt{2}}\) \hspace{1cm} 30. \(\frac{6}{\sqrt{3}}\) \hspace{1cm} [31] \(\frac{\sqrt{5}}{\sqrt{13}}\) \hspace{1cm} 32. \(\frac{\sqrt{18}}{\sqrt{10}}\)

Evaluate, correct to 3 significant figures:

33. \(\frac{1}{\sqrt{3}}\) \hspace{1cm} [34] \(\frac{3}{\sqrt{2}}\) \hspace{1cm} [35] \(\frac{1}{\sqrt{12}}\) \hspace{1cm} 36. \(\frac{\sqrt{2}}{\sqrt{5}}\)

[37] \(\frac{\sqrt{5}}{\sqrt{8}}\) \hspace{1cm} *38. \(\frac{\sqrt{1.2}}{\sqrt{0.5}}\) \hspace{1cm} *39. \(\frac{10}{\sqrt{5}} + \frac{6}{\sqrt{3}}\) \hspace{1cm} *40. \(\frac{2}{\sqrt{6}} + \frac{3}{\sqrt{24}}\)

Use of Tables. Approximate values of the squares and square roots of numbers may be found by using printed tables, the degree of accuracy depending on the number of significant figures given by the tables. Decimal points are usually omitted from the tables and must be inserted by making a rough estimate of the answer.

Table of Squares. The arrangement of the tables is best explained by taking an example.

By direct multiplication, \(237^2 = 237 \times 237 = 56169\).

This result can be obtained, correct to 4 figures, from 4-figure tables. In the table of squares, look for the number 23 in the left-hand column; the figures in the “23” row, together with the headings at the top of the page, are:

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>23</td>
<td>5290</td>
<td>5336</td>
<td>5382</td>
<td>5429</td>
<td>5476</td>
<td>5523</td>
<td>5570</td>
<td>5617</td>
<td>5664</td>
<td>5712</td>
</tr>
</tbody>
</table>

There is also at the end of this row a column of “mean differences” whose use will be explained later.

In the “23” row, under the 7 in the main columns, we find the figures 5617; this means that the first 4 significant figures in \(237^2\) are 5617.

Since \(200^2 = 40000\) and \(300^2 = 90000\), the square of 237 lies between 40000 and 90000.

\[\therefore 237^2 = 56170\] correct to 4 figures.

This statement can be checked by comparing it with the exact value 56169 obtained by direct multiplication.

The figures obtained in working out the square of 237 are exactly the same as those obtained in working out the square of 237, the only difference being the position of the decimal point.

Since \(200^2 = 40000\) and \(300^2 = 90000\), the square of 237 lies between 400 and 900; therefore from the tables

\[237^2 = 56170, \text{correct to 4 figures.}\]

Similarly \(2.37^2 = 5.617, 0.237^2 = 0.05617, \text{etc., to 4 figures.}\)

EXERCISE 133 (Oral)

Find from 4-figure tables the squares, to four figures, of:

1. 47. 2. 47. 3. 72. 4. 72. 5. 83. 6. 290.

7. 3100. 8. 23. 9. 0.24. 10. 660. 11. 4.13. 12. 72.5.

13. 619. 14. 1.76. 15. 2870. 16. 31.6. 17. 3.17. 18. 0.346.

19. 0.895. 20. 695. 21. 0.327. 22. 5040. 23. 0.0101.

24. 0.703. 25. 0.094. 26. 0.0633. 27. 70.8. 28. 3180.

29. 897. 30. 0.709.

Mean Differences Column. We now proceed to explain how 4-figure tables are used to find approximately the squares of numbers of four significant figures.

Example 7. Use tables to find approximately the value of \(2.374^2\).

An extract from a table of squares showing the “23” row was given on p. 278; at the end of this row in the tables, the mean differences for that row are added:

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>23</td>
<td></td>
<td>59</td>
<td>14</td>
<td>19</td>
<td>23</td>
<td>28</td>
<td>33</td>
<td>38</td>
<td>42</td>
<td></td>
</tr>
</tbody>
</table>

From the main columns, we have

\[2.37^2 = 5.4617\] and \[2.38^2 = 5.6644.\]

The value of \(2.374^2\) lies between 5.4617 and 5.6644.

Under the heading 4 in the mean differences, \(2.37^2 = 5.4617\).

We find 19, which means that 19 must be added to 5617 to obtain the first 4 significant figures in \(2.374^2\).

\[\therefore 2.374^2 = 5.6363\]

But direct multiplication shows that \(2.374^2 = 5.635876\), so that the result given by the tables is actually correct to 4 figures. In general, when mean difference columns are used, the 4th figure is not absolutely reliable, but the error will not usually be more than a unit in the fourth figure; this fact may be indicated by writing the result in the form, \(2.374^2 = 5.63 \pm 0.06\) (6).
EXERCISE 134 (Oral)

Find to 4 significant figures, as given by 4-figure tables, the squares of:

1. 3826.  
2. 3812.  
3. 3874.  
4. 3868.  
5. 6453.  
6. 6467.  
7. 06472.  
8. 6484.  
9. 0931.  
10. 9076.  
11. 9058.  
12. 09011.  
13. 8094.  
14. 5308.  
15. 4007.  
16. 07047.  
17. 1798.  
18. 01656.  
19. 1488.  
20. 02067.  
21. 3162.  
22. 3163.  
23. 3108.  
24. 3187.  
25. 0209.  
26. 3008.  
27. 1007.  
28. 1009.  
29. 8487.  
30. 9208.  
31. 09007.  
32. 9001.  
33. 45753.  
34. 16367.  
35. 22784.  
36. 030523.

Table of Square Roots. If a number is multiplied by 10 the digits in its square root are altered, but if it is multiplied by 100 the digits remain the same. For example, it can be shown that, correct to 4 figures,

\[ \sqrt{692} = 26.31; \quad \sqrt{692} = 26.31; \quad \sqrt{0.0692} = 0.2631; \]

but

\[ \sqrt{6920} = 83.19; \quad \sqrt{692} = 83.19; \quad \sqrt{0.00692} = 0.08319. \]

For this reason, tables of square roots are arranged on pairs of pages: on one page the reading corresponding to 692 is 2631 and on the other page it is 8319; decimal points are usually omitted. It is therefore necessary to find by inspection, or by the square root rule, the first significant figure of the square root and the position of the decimal point.

Example 8. Use square root tables to find to 4 figures, (i) \(\sqrt{537};\) (ii) \(\sqrt{537}.\)

(i) The rule shows that \(\sqrt{537} = 24.4.\)

Look on the page where the square root of 537 starts with 2. \(\sqrt{537} = 23.1(7),\) to 4 figures.

(ii) The rule shows that \(\sqrt{537} = 74.\)

Look on the page where the square root of 537 starts with 7. \(\sqrt{537} = 73.1(8),\) to 4 figures.

The mean difference column is used in the same way for square roots as for squares.

PYTHAGORAS' THEOREM

Example 9. Use tables to find \(\sqrt{0.1138},\) to 4 figures. \(\sqrt{0.1138} = 0.3374\)

Look on the page where the square root starts with 3. \(\sqrt{0.1138} = 0.3374\)

When mean differences are used, the 4th figure is not reliable. Here, the result is more likely to be correct to 4 figures, if 1138 is treated as 1140 - 2, instead of 1130 + 8.

The rule for square root gives \(\sqrt{0.1138} = 0.3374\)

to 5 figures, so that the second result is actually correct to 4 figures.

EXERCISE 135 (Oral)

Find to 4 significant figures, as given by 4-figure tables, the square roots of:

1. 26.  
2. 26.  
3. 268.  
4. 0268.  
5. 73.  
6. 730.  
7. 7340.  
8. 00734.  
9. 616.  
10. 597.  
11. 0923.  
12. 00864.  
13. 507.  
14. 3040.  
15. 000529.  
16. 0000609.  
17. 1743.  
18. 1743.  
19. 3564.  
20. 03564.  
21. 7074.  
22. 6086.  
23. 92050.  
24. 09407.  
25. 3863.  
26. 4007.  
27. 5718.  
28. 005074.  
29. 77060.  
30. 8046.  
31. 004297.  
32. 01093.  
33. 001976.  
34. 57387.  
35. 20738.  
36. 31567.

Pythagoras' Theorem. Examine the following diagrams:

In (i), the shaded portion of the square is a square whose side is the hypotenuse of the triangle; and in (ii) the shaded portion is made up of two squares whose sides are the other two sides of the triangle. Hence it follows that

\[ \text{The area of the square on the hypotenuse of a right-angled triangle is equal to the sum of the areas of the squares on the other two sides.} \]

But the areas of these squares are \(a^2\) sq. in., \(b^2\) sq. in., \(c^2\) sq. in.,

\[ c^2 = a^2 + b^2. \]
Example 10. If the lengths of the sides of a rectangle are 4-7 in., 3-2 in., find to $\sqrt[4]{5}$ in. the length of a diagonal.

If the length of a diagonal is $c$ in.,

$$c^2 = 4.7^2 + 3.2^2 = 22.09 + 10.24 \text{ (table of squares)} = 32.33$$

$$c = \sqrt{32.33} = 5.686 \text{ (table of square roots)}$$

Hence the length of the diagonal is 5-69 in., to $\sqrt[4]{5}$ in.

Example 11. A room is 17 ft. long, 14 ft. wide, 9 ft. high. Find the distance from a corner A of the floor to the opposite corner N of the ceiling.

Suppose the diagonal AC of the floor is $x$ ft. long and the diagonal AN of the room is $y$ ft. long.

From the right-angled triangle ABC,

$$x^2 = 17^2 + 14^2;$$

from the right-angled triangle ACN,

$$y^2 = x^2 + 9^2 = 298 + 81 = 379;$$

$$y = \sqrt{379} = 21.88 \text{ (table of square roots)};$$

AN is 23-8 ft., to 3 figures.

Notice that it is unnecessary to find the value of $x$.

EXERCISE 136

[Give answers correct to 3 significant figures, unless otherwise stated]

Find the length of a side of a square field of area:

1. 8000 sq. yd.  9. 13750 sq. yd.  3. 50 sq. km.  4. 1 ac. 5. 160 sq. ch.  6. 4 ac. 3 ro.  7. Half a sq. mile.

In Nos. 8–11, the length of the hypotenuse of a right-angled triangle is $c$ in., and the lengths of the other sides are $a$ in. and $b$ in.

8. If $a = 5-2$, $b = 7-8$, find $c$.  9. If $a = 4-73$, $b = 6-14$, find $c$.

10. If $b = 5-67$, $c = 8-06$, find $a$.  11. If $a = 23-26$, $c = 37-08$, find $b$.

12. The sides of a rectangle are 7-85 cm., 6-42 cm. long, find the length of a diagonal.

13. The length of a diagonal of a rectangular courtyard is 23-7 yd., and the length of one side is 18-8 yd.; find the perimeter.
Example 12. Find the reciprocal of 0.365 to 4 figures.

The reciprocal of 0.365 is \( \frac{1}{0.365} = \frac{10}{3} = 3.3333 \).

By long division, \( 1 \div 0.365 = 2.740 \), to 4 figures.

It is, however, quicker to look up the result in a table of reciprocals which is arranged on the same plan as tables of squares, etc.

In a table of reciprocals in the "36" row under the heading 5 in the main columns, we find the figures 2740; the position of the decimal point must be found by making a rough approximation as above.

\[ \therefore \text{from the tables, } 0.365 = 2.740 \text{ to 4 figures.} \]

Example 13. Find from tables the reciprocal of 0.3654, to 4 figures.

The reciprocal of 0.3654 is less than the reciprocal of 0.365, and therefore the difference given in the mean difference columns must be subtracted.

When mean differences are used,

\[ \begin{align*}
\text{Reciprocal of 0.365} & = 2.740 \\
\text{Subtract diff. for 4} & = 0.004 \\
\text{Reciprocal of 0.3654} & = 2.737
\end{align*} \]

\[ \therefore \text{from tables, } 0.3654 = 2.737 \text{ (7).} \]

Example 14. Find the reciprocal of 18460 to 4 figures.

\[ \begin{align*}
\text{Number} & \\
\text{Reciprocal} & \\
\frac{1}{18460} & = 0.0001 \\
& = 0.000053 \\
& = 0.000053 \\
& = 0.000053
\end{align*} \]

\[ \therefore \text{from tables, } 0.000053, 1 \text{ (7).} \]

Exercise 137 (Oral)

Find to 4 significant figures, as given by 4-figure tables, the values of:

1. \( \frac{1}{2.8} \) 
2. \( \frac{1}{28} \) 
3. \( \frac{1}{0.28} \) 
4. \( \frac{1}{280} \) 
5. \( \frac{1}{2.9} \) 
6. \( \frac{1}{390} \) 
7. \( \frac{1}{77} \) 
8. \( \frac{1}{707} \) 
9. \( \frac{1}{43.6} \) 
10. \( \frac{1}{632} \)

Miscellaneous Examples

Example 15. Find the value of \( \frac{3}{27-68} \) to 3 figures.

From tables, \( \frac{1}{27-68} \approx 0.03612 \),

\[ \therefore \frac{3}{27-68} \approx 0.03612 \times 3 \approx 0.2528 \approx 0.253. \]

Example 16. Evaluate to 3 figures, \( \frac{3}{\sqrt{47.15} + \sqrt{(6.15)^2 + (13.24)^2}} \).

From square root tables and table of reciprocals,

\[ \begin{align*}
\sqrt{47.15} & \\
& = 6.867 \\
& = 2.289 \approx 0.4369
\end{align*} \]

\[ \begin{align*}
\sqrt{(6.15)^2 + (13.24)^2} & \\
& = 1753 \\
& = 0.02654 - 0.00574 \approx 0.02075
\end{align*} \]

\[ \therefore \text{expression } \approx 0.4369 + \sqrt{0.02075} \approx 0.4369 + 0.1441 \approx 0.581, \text{ to 3 figures.} \]

Exercise 138

Find, to 3 significant figures, the values of:

1. \( \frac{1}{17.87} \) 
2. \( \frac{1}{400.8} \) 
3. \( \frac{1}{\sqrt{16.74}} \) 
4. \( \frac{3}{\sqrt{40.96}} \) 
5. \( \frac{1}{(0.4077)^2} \) 
6. \( \frac{5}{\sqrt{0.1107}} \) 
7. \( \frac{1}{0.03} \) 
8. \( \frac{1}{0.02} \) 
9. \( \frac{1}{27.38 + 19.03} \) 
10. \( \frac{1}{2072 - 9163} \)

11. If \( \frac{1}{u} + \frac{1}{v} = f \), find \( f \) if \( u = 8.63 \), \( v = 6.07 \).
12. If \( \frac{1}{u} - \frac{1}{v} = \frac{1}{y} \), find \( f \) if \( u = 27.4, v = 40.8 \).

13. If \( \frac{1}{a^n} + \frac{1}{b^2} = \frac{1}{c^2} \), find \( c \) if \( a = 3.84, b = 4.07 \).

Find, to 3 significant figures, the values of:

\[ \frac{1}{\sqrt{4.93}} + \frac{1}{\sqrt{6.48}} \]
\[ \sqrt{1 - \frac{1}{9}} + \sqrt{1 + \frac{1}{9}} \]
\[ (6.307)^4 + (5.213)^4 \]

18. Given 1 ft. = 0.3048 m, express 1 m. in feet.

19. Given 1 kg. = 2.205 lb., express \( \frac{1}{2} \) lb. in kg.

20. Given 1 cm. = 0.3937 in., express 1 sq. cm. in sq. in., and express 1 sq. in. in sq. cm.

*21. Find the smallest integer greater than 146290 which is a perfect square.

22. Find, to one significant figure, the error per cent. in taking \( \sqrt{3} \) as \( 1.7 \).

*23. Express 100 yd. as a percentage of 1 mi., to 4 figures.

*24. The lengths of the two shorter sides of a right-angled triangle are \( \sqrt{2} \) in., \( \sqrt{3} \) in.; find the perimeter of the triangle, correct to 1 fig. in.
Example 4. What is the meaning of $a^2$?
\[a^2 \times a^3 = a^2 \times a^4 = a^{2+3} = a^6;\]
\[\therefore \ a^6 \ is \ the \ fourth \ root \ of \ a^2; \quad \therefore \ a^6 = \sqrt[4]{a^2}.\]

But since $a^4 = a^4 \times 1 \times a^4$ and since $a^4 = \sqrt[4]{a^2}$, it follows that
\[a^6 = (\sqrt[4]{a^2})^3; \quad \therefore \ a^6 = (\sqrt[4]{a^2})^3.\]

For example, $16^2 = \sqrt[4]{16^8} = \sqrt[4]{16 \times 16 \times 16}$
\[= \sqrt[4]{(8 \times 2) \times (8 \times 2) \times (8 \times 2)} = \sqrt[4]{8 \times 8 \times 8 \times 8} = 8;\]
but it is simpler to say, $16^2 = \sqrt[4]{16} \times \sqrt[4]{16} = 2 \times 2 = 4$.
\[\therefore \ 16^3 = (\sqrt[4]{16})^3 = 2^3 = 8,\]
that is, take the root first, if the result is exact.

Example 5. What meaning must be given to $10^{-2}$?
\[10^3 \times 10^{-2} = 10^{3-2} = 10^1 = 10;\]
but $10^3 \times 10^{-2} = 1$, \[\therefore \ 10^{-2} = \frac{1}{10^2}\]

Example 6. What is the meaning of $a^{-5}$, where $a$ is not zero?
\[a^6 \times a^{-5} = a^{6-5} = a^1 = a;\]
but $a^5 \times \frac{1}{a^5} = a$, \[\therefore \ a^{-5} = \frac{1}{a^5}.\]

Example 7. What meaning must be given to $6^0$?
\[6^3 \times 6^0 = 6^3 + 0 = 6^3; \quad \therefore \ 6^0 = \frac{6^3}{6^3} = 1.\]
Similarly $10^0 = 1$, and more generally $a^0 = 1$, if $a$ is not zero.

Example 8. Find the value of (i) $(\sqrt[3]{2})^{-2}$; (ii) $9^{-1.6}$.

(i) The method of Example 5 shows that
\[\sqrt[3]{2}^{-2} = 1 \div 2^2 = 1 \div \sqrt[3]{2^2}.\]
(ii) Similarly $9^{-1.5} = 9^{-2} \times 9^{1.5} = 9^{-2} \times 9^{2.3} = 9 \times \sqrt{9} \times 9^{2.3} = 9 \times 3 = 27$.
\[\therefore \ 9^{-1.5} = \frac{3}{\sqrt{9}}.\]
Write down the numerical values of:

44. $100^{0.5}$. 45. $100^{0.5}$. 46. $32^{0.2}$. 47. $32^{0.8}$.
48. $(0.2)^{-1}$. 49. $(0.1)^{-2}$. 50. $(0.027)^{4}$. 51. $(0.027)^{-3}$.

The methods used in Examples 2–7, p. 287, may be used to establish the following results:

(i) If $p$, $q$ are any positive integers,

$$\frac{1}{p} = \frac{1}{q}; \quad \frac{p}{q} = \left(\frac{a}{b}\right)^{p} = \left(\frac{a}{b}\right)^{q}.$$

(ii) If $n$ is any number, and $a$ is not zero, $a^{-n} = \frac{1}{a^n}$.

(iii) If $a$ is not zero, $a^0 = 1$.

**Powers of 10.** Since $10^2 = \sqrt{10}$, the value of $10^2$ can be calculated to as many decimal places as desired by using the square-root process.

Thus $10^2 = \sqrt{10} = 3.162 \ldots$;

similarly $10^4 = \sqrt{10^2} = 3.162 \ldots = 1.779 \ldots$.

and $10^8 = \sqrt{10^4} = 1.779 \ldots = 1.333 \ldots$.

Also $10^3 = \frac{1}{10^2} = \frac{1}{1000} = \sqrt{31.62} \ldots = 5.623 \ldots$.

and in a similar way we can find approximate values $10^4$, of $10^8$, $10^3$; also $10^0 = 1$.

We therefore have the following table of values which can be used to draw the graph of $10^x$:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$0.125$</th>
<th>$0.25$</th>
<th>$0.375$</th>
<th>$0.5$</th>
<th>$0.625$</th>
<th>$0.75$</th>
<th>$0.875$</th>
<th>$1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^x$</td>
<td>$1$</td>
<td>$1.33$</td>
<td>$1.78$</td>
<td>$2.37$</td>
<td>$3.16$</td>
<td>$4.22$</td>
<td>$5.62$</td>
<td>$7.30$</td>
</tr>
</tbody>
</table>

**EXERCISE 141 (Oral)**

[Use the printed graph of $10^x$ for the following examples]

Write down the values of:

1. $10^{0.3}$. 2. $10^{0.7}$. 3. $10^{0.42}$. 4. $10^{0.84}$.

Express as powers of 10:

5. 2. 6. 3. 7. 6. 8. 7.8. 9. 1.5.

10. Find the values of $a$, $b$, $c$, $N$ in the following argument:

$$-2.8 = 10^2; \quad 3.2 = 10^3;$$

$$\therefore 2.8 \times 3.2 = 10^2 \times 10^3 = 10^5 = 10^5 = N.$$ Check the result by ordinary multiplication.

11. Use the method of No. 10 to find approximately:

(i) $4.6 \times 1.6$; (ii) $2.7^2$.

12. Find the values of $a$, $b$, $c$, $N$ in the following argument:

$$8.5 = 10^2; \quad 3.8 = 10^3; \quad \therefore 8.5 \times 3.8 = 10^2 \times 10^3 = 10^5 = N.$$

Check the result by ordinary multiplication.

13. Use the method of No. 12 to find approximately:

(i) $7.4 \div 2.9$; (ii) $8.2 \div 6.6$.

Find the values of $a$, $b$, $N$ in the following arguments:

14. $8 = 10^2$; $\therefore \sqrt{8} = 10^0 = N$. 15. $6 = 10^2$; $\therefore \sqrt[3]{6} = 10^0 = N$.

16. $9 = 10^2$; $\therefore \sqrt[3]{9} = 10^0 = N$. 

17. $10^2$; $\therefore \sqrt[3]{10^2} = 10^0 = N$. 

Logarithms. If a number is expressed as a power of 10, the index is called the common logarithm of the number.

For example, from the graph on p. 291, $2 = 10^{0.30}$.
\[\therefore \text{the logarithm of } 2 \text{ is approximately } 0.30.\]

Numbers between 1 and 10. The following portion of a four-figure table of logarithms will be used to explain how the logarithm of a number is obtained from a table:

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>. . .</th>
<th>9</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>. . .</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>3010</td>
<td>3022</td>
<td>3032</td>
<td>. . .</td>
<td>3054</td>
<td>3201</td>
<td>2.46</td>
<td>. . .</td>
<td>3404</td>
<td>15</td>
<td>17</td>
<td>19</td>
</tr>
<tr>
<td>2.1</td>
<td>3200</td>
<td>3222</td>
<td>3243</td>
<td>. . .</td>
<td>3263</td>
<td>3404</td>
<td>2.46</td>
<td>. . .</td>
<td>14</td>
<td>16</td>
<td>18</td>
<td></td>
</tr>
</tbody>
</table>

From this extract we see that:
\[2.00 = 10^{0.3010},\quad 2.01 = 10^{0.3032},\quad 2.09 = 10^{0.3201},\]
\[2.10 = 10^{0.3222},\quad 2.12 = 10^{0.3263},\quad 2.19 = 10^{0.3404}.\]

The logarithms of numbers given to 4 figures are obtained by using the columns of mean differences in the ordinary way: for example, to find the logarithm of 2.013,

The working set out here in full should be done mentally.

\[\text{Number} \quad \text{Logarithm} \quad \text{Diff. for } 3 \quad \text{6} \quad 2.013 \quad 0.3032\]

\[2.013 = 10^{0.3032}.\]

To save space, the decimal points in the left-hand column and in the main columns are usually omitted in printed tables and must be supplied by the reader. The extract given above is usually printed in the form:

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>. . .</th>
<th>9</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>. . .</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>3010</td>
<td>3032</td>
<td>3054</td>
<td>. . .</td>
<td>3201</td>
<td>2.46</td>
<td>. . .</td>
<td>15</td>
<td>17</td>
<td>19</td>
<td></td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>3200</td>
<td>3243</td>
<td>3263</td>
<td>. . .</td>
<td>3404</td>
<td>2.46</td>
<td>. . .</td>
<td>14</td>
<td>16</td>
<td>18</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Since $1 = 10^0$ and $10 = 10^1$, the logarithm of any number between 1 and 10 lies between 0 and 1, and is therefore of the form $0.\text{***}$, and the figures printed in the four-figure columns are the figures of the logarithm which follow the decimal point, though the decimal point is usually not printed.

EXERCISE 142 (Oral)

Use tables to express as powers of 10:

1. 3.2. 2. 6.4. 3. 9.6. 4. 3. 5. 9.

6. 2.5. 7. 5. 8. 4.1. 9. 8.2. 10. 7.7.

11. 4.26. 12. 6.95. 13. 3.04. 14. 5.60. 15. 1.11.

16. 1.07. 17. 1.20. 18. 9.06. 19. 4.44. 20. 6.03.

21. 2.563. 22. 2.568. 23. 7.451. 24. 7.468. 25. 4.645.

26. 4.657. 27. 4.678. 28. 4.606. 29. 6.347. 30. 6.358.

31. 8.876. 32. 9.708. 33. 7.067. 34. 7.004. 35. 5.081.

36. 5.005. 37. 3.050. 38. 3.005. 39. 1.086. 40. 1.074.

41. 1.003. 42. 1.048. 43. 1.006. 44. 1.995.

The same tables may be used by reversing the process to find the value of any given power of 10.

Example 9. Find the value of

(i) $10^{7076}$; (ii) $10^{7143}$; (iii) $10^{7148}$.

(i) Look for the figures 7076 in the four-figure columns; these occur in the "51" row in the column headed 0; therefore the significant figures in the value of $10^{7076}$ are 510; but the value of $10^{7078}$ lies between $10^3$ and $10^4$, that is between 1 and 10; \[\therefore \text{the value of } 10^{7078} = \text{5}10, \text{to } 4 \text{ figures}.\]

(ii) The figures 7143 occur in the "51" row in the column headed 8, \[\therefore 10^{7143} = \text{5}180, \text{to } 4 \text{ figures}.\]

(iii) The actual figures 7148 do not occur in the four-figure columns; \[10^{7148} = \text{5}180 \quad \text{and} \quad 10^{7150} = \text{5}190,\]
\[\therefore \text{the value of } 10^{7148} \text{ lies between } 5180 \text{ and } 5190.\]

The difference between 7143 and 7148 is 5, and in the column of mean differences for this row 5 occurs under the column headed 6, \[\therefore 10^{7148} = \text{5}186, \text{to } 4 \text{ figures}.\]

Note. Whenever mean differences are used in 4-figure tables, the 4th figure is not reliable; this may be indicated by enclosing the 4th figure in brackets.
INDICES AND LOGARITHMS

Example 10. Find the value of

(i) $10^{0.123}$; (ii) $10^{0.548}$.

(i) The figures less than 0135 which are nearest to it and occur in the four-figure columns are 0128.

$10^{0.128} = 1.030$ to 4 figures.

The difference between 0128 and 0135 is 7: but 7 does not occur in the column of mean differences for this row, we therefore take the nearest difference, namely 8, which is in the column headed 2.

.. $10^{0.128} = 1.03(2)$ to 4 figures.

(ii) From the tables, $10^{0.548} = 9.000$, to 4 figures.

For this row, the difference 1 occurs in two columns, namely, those headed 2 and 3; we cannot therefore tell whether it is better to take the value as 9.002 or as 9.003; the result may be written $10^{0.548} = 9.00(2)$.

EXERCISE 143 (Oral)

Use logarithm tables to find the values of:

1. $10^{0.332}$, 2. $10^{0.7709}$, 3. $10^{0.7782}$, 4. $10^{0.5428}$
   5. $10^{0.8542}$, 6. $10^{0.7085}$, 7. $10^{0.8106}$, 8. $10^{0.7128}$
   9. $10^{0.7016}$, 10. $10^{0.574}$, 11. $10^{0.781}$, 12. $10^{0.8042}$
   13. $10^{0.4019}$, 14. $10^{0.5258}$, 15. $10^{0.7581}$, 16. $10^{0.8642}$

Use logarithm tables to find the numbers whose logarithms are:

   32. -0.298, 33. -0.314, 34. -0.025, 35. -0.003, 36. -9.546

Multiplication and Division

Example 11. Find the value of $3 \times 18 \times 2.17$.

Rough Estimate: $3 \times 2 = 6$.

$3 \times 18 \times 2.17 = 10^{0.24} \times 10^{0.3665} = 10^{0.6064} = 10^{0.5024 + 0.1038}$

$= 10^{0.8062} = 6.90(1)$ or $6.90(2)$

NUMBERS GREATER THAN 10

Example 12. Find the value of $8.37 \div 5.09$.

Rough Estimate: $8 \div 5 = 1.6$.

$8.37 \div 5.09 = 8 \times 0.6617 \times 10^{-1} = 0.6617 \times 10^{-1}$

$= 0.662 \approx 1.64(1)$ or $1.64(2)$

Note. To obtain the 4th figure accurately, it is necessary to use 5-figure tables; the error in the 4th figure obtained from 4-figure tables will usually be small. But in the exercises that follow, the reader is asked to give the answer to four figures, 'as given by 4-figure tables,' in order to make it easier to check the accuracy of his use of the tables.

The fourth figure should be enclosed in brackets to show its approximate character.

EXERCISE 144

Find the value to 4 figures, as given by 4-figure tables, of:

1. $2.36 \times 2.24$, [2] $2.73 \times 3.18$, 3. $4.19 \times 1.84$
   4. $3.624 \times 2.315$, 5. $5.278 \times 1.406$, [6] $2.073 \times 4.108$
   7. $2.086 \times 4.104$, [8] $7.263 \times 1.173$, 9. $3.708 \times 2.046$
   10. $8.67 \div 3.29$, [11] $7.49 \div 4.08$, 12. $9.07 \div 2.61$
   13. $7.408 \div 2.165$, [14] $9.032 \div 4.705$, 15. $8.007 \div 5.023$
   16. $9.804 \div 5.073$, 17. $7.006 \div 1.088$, 18. $6.704 \div 5.908$
   *19. $1.78 \times 2.04 \times 2.56$, *20. $7.03 \div 6.74 \div 8.26$

Numbers greater than 10

<table>
<thead>
<tr>
<th>Number</th>
<th>expressed as power of 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>7243</td>
<td>$10^{3.8599}$</td>
</tr>
<tr>
<td>7243 $\times 10^3$</td>
<td>$10^{3.8599} \times 10^3$</td>
</tr>
<tr>
<td>7243 $\times 10^2$</td>
<td>$10^{3.8599} \times 10^2$</td>
</tr>
<tr>
<td>7243 $\times 10$</td>
<td>$10^{3.8599} \times 10$</td>
</tr>
</tbody>
</table>

Thus, so long as the order of the digits of a number is unaltered, the decimal portion of the logarithm, called its mantissa, remains the same.

The integral portion of the logarithm, called its CHARACTERISTIC, is obtained by counting the number of digits between the decimal point and where it would be in the standard form.

For example, for 7243-3, two digits, the characteristic is 2;
for 7243, four digits, the characteristic is 4.
Example 15. Find the value of $816.3 \times 37.42$.

Rough Estimate: $800 \times 40 = 32000$.

$816.3 \times 37.42 = 10^2 \times 1.819 \times 10^1 = 10^4 = 10000$.

$= 10^4 \times 3500 = 35000$.

Example 16. Find the value of $816.3 \div 37.42$.

Rough Estimate: $800 \div 40 = 20$.

$816.3 \div 37.42 = 10^2 \div 1.819 \div 10^1 = 10^1 = 10$.

$= 10^1 \times 99 = 990$.

Example 17. Find the value of $(37.06)^4$.

Rough Estimate: $40^4 = 2560000$.

$(37.06)^4 = (10^{1.5689})^4 = 10^{6.2756}$.

Example 18. Find the value of $\sqrt[4]{5614}$.

$\sqrt[4]{5614} = 10^{0.9164} = 9164$.

Rough Check: $89^4 = 512$; $98^4 = 729$.

We shall now repeat Example 15, to show how the work may be arranged, after the principles have been grasped; for other examples of this method of arrangement, see pp. 298, 304.

Example 19. Find the value of $816.3 \times 37.42$.

Rough Estimate: $800 \times 40 = 32000$.

$816.3 \times 37.42 = 3055 \times 10^4 = 305500$.

Logarithmic Notation. The fact that the logarithm of 2 is 3.010 is often written in the form, log 2 = 3.010; and in general if $x = 10^p$, that is, if $p$ is the logarithm of $x$, then log $x = p$.

Examples 15-19 illustrate the following general properties:

- $\log (xy) = \log x + \log y$;
- $\log \left(\frac{x}{y}\right) = \log x - \log y$;
- $\log (x^n) = n \log x$. 

---

**EXERCISE 145 (Oral)**

Read off the characteristics of the logarithms of the numbers:

1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. 14. 15. 16. 17. 18. 19. 20.

$\begin{array}{cccccccccccc}
3623 & 3623 & 3623 & 3623 & 3623 & 3623 & 3623 & 3623 & 3623 & 3623 & 3623 & 3623 & 3623 & 3623 & 3623 & 3623 & 3623 \end{array}$

State the number of digits to the left of the decimal point in the numbers whose logarithms are:


$\begin{array}{ccccccc}
10 & 11 & 12 & 13 & 14 & 15 & 16 \\
1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array}$

**EXERCISE 146 (Oral)**

Use tables to write down to 4 figures the logarithms of:

1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. 14. 15. 16. 17. 18. 19. 20.

$\begin{array}{ccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 \\
624 & 7480 & 41 & 72 & 62 & 4 & 72 \end{array}$

Use tables to write down to 4 figures the numbers whose logarithms are:

21. 22. 23. 24. 25. 26. 27. 28. 29. 30. 31. 32. 33. 34. 35. 36. 37. 38. 39. 40.

$\begin{array}{ccccccc}
21 & 22 & 23 & 24 & 25 & 26 & 27 \\

$\begin{array}{ccccccc}
41 & 42 & 43 & 44 & 45 & 46 & 47 \\
0.34 & 0.35 & 0.36 & 0.37 & 0.38 & 0.39 & 0.40 \end{array}$
EXERCISE 147

Find the value to 4 figures, as given by 4-figure tables, of:

1. 53.2 x 67.4. \[ 2. 724 \times 15.3. \]
2. 6070 x 1.08. \[ 3. 843 \times 9.15. \]
4. 387 \times 71.6. \[ 5. 413 \times 8.62. \]
7. 2683 x 37.4. \[ 8. 409 \times 64.9. \]
[9. 2072 \times 18.37. \]
[10. 6724 x 8.36. \]
11. 5070 \times 21.68. \[ 12. 61320 \times 483.4. \]
13. 36828 x 4.073. \[ 14. 50 \div 3.0643. \]
15. 4527 \times 3406. \[ 16. 1000 \div 28.447. \]
17. 10.73 \times 2004. \[ 18. 73600 \div 909. \]
19. 21.76a. \[ 20. (7.294)^a. \]
22. \( \sqrt[3]{734} \); \( \sqrt[3]{734} \). \[ 23. (i) \sqrt[3]{618}; \text{ (ii) } \sqrt[3]{618}; \text{ (iii) } \sqrt[3]{618}. \]
24. \( \sqrt[3]{453} \). \[ 25. \sqrt[3]{507.3}. \]
26. \( \sqrt[3]{8672}. \)
27. \( \sqrt[3]{10.43. \}
28. \( \sqrt[3]{2007}. \)
29. \( \sqrt[3]{40.78}. \)
30. \( \sqrt[3]{100. \}
31. \( \sqrt[3]{1000. \}

If the expression is complicated, the working may be arranged in the more concise form illustrated by the following examples:

Example 20. Find the value of \( \frac{65130 \times 37.14}{7932 \times 4.186}. \)

\[ \text{Rough Estimate: } \frac{60000 \times 40}{800 \times 4} = \frac{6000000}{800} = 700. \]

<table>
<thead>
<tr>
<th>Number</th>
<th>Logarithm</th>
<th>Number</th>
<th>Logarithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>65130</td>
<td>4.8113</td>
<td>7932</td>
<td>2.8994</td>
</tr>
<tr>
<td>37.14</td>
<td>1.5699</td>
<td>4.186</td>
<td>0.6218</td>
</tr>
<tr>
<td>Numerator</td>
<td>6.3837</td>
<td>Denominator</td>
<td>3.5212</td>
</tr>
<tr>
<td>Expression</td>
<td>2.8625</td>
<td>\text{. expression } = 7.286 \times 10^2 = 7286 (6).</td>
<td></td>
</tr>
</tbody>
</table>

Example 21. Find the value of \( \frac{(3541)^a \times 4.783}{365 \times \sqrt[4]{41.8}}. \)

\[ \text{Rough Estimate: } \frac{30^2 \times 30^2}{30^2} = 30^2 = 900. \]

<table>
<thead>
<tr>
<th>Number</th>
<th>Logarithm</th>
<th>Logarithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3541)^a</td>
<td>1.5491 \times 3</td>
<td>4.6473</td>
</tr>
<tr>
<td>4.783</td>
<td>-6797</td>
<td>418</td>
</tr>
<tr>
<td>Numerator</td>
<td>5.3270</td>
<td>Denominator</td>
</tr>
<tr>
<td>34.65</td>
<td>1.5397</td>
<td>2.9767</td>
</tr>
<tr>
<td>\sqrt[4]{41.8}</td>
<td>1.6212 \times \frac{1}{3}</td>
<td>-8106</td>
</tr>
<tr>
<td>Denominator</td>
<td>2.3503</td>
<td>Expression</td>
</tr>
</tbody>
</table>

\[ \text{. expression } = 9.478 \times 10^9 = 9478 (9). \]
Given 1 kg. = 2-05 lb., express 50 lb. in kg.

The world production of tea in 1930 was 813,500 tons, of which 287,500 tons came from India and Ceylon. What percentage was this?

If 1 c.c. of lead weighs 11-4 gm., find the weight of a cube of lead, 7-45 cm. by 5-08 cm. by 3-16 cm.

If a cuboid of iron 6-13 cm. by 4-95 cm. by 2-05 cm. weighs 462 gm., find the weight of 1 c.c. of iron.

In Egyptian measures, 1 Fedden = 24 qirats, and 1 qirat = 209-3 sq. yd.; express 1 Fedden in acres.

Entertainment tax yielded £6,952,088 in 1931 and £7,868,908 in 1932. Find the increase per cent.

The Trades Union membership was 3,388,286 in 1926 and 2,060,043 in 1931. Find the decrease per cent.

The area of a circle, radius $r$ in., is $A$ sq. in. where $A = \pi r^2$, and $r = \sqrt{\frac{A}{\pi}}$.

(i) Find $A$ if $r = 4-163$; (ii) find $r$ if $A = 10$.

The time of a complete oscillation of a pendulum $l$ ft. long is $t$ sec. where $t = 2\pi\sqrt{\frac{l}{g}}$ and $l = \frac{g^2}{4\pi^2}$ and $g = 32.2$. (i) Find $l$ if $t = 2-35$; (ii) find $t$ if $l = 40$.

The weight of a brass cylinder, height $h$ cm., base-diameter $d$ cm. is $w$ gm. where $w = 2-09d^2h$. (i) Find $w$ if $d = 2-516$, $h = 127-4$; (ii) find $h$ if $w = 856-4$, $d = 3-85$.

Find $k$ from the formula, $w = \pi k(D + d)(D - d)$ if $w = 4528$, $l = 10-7$, $D = 6-31$, $d = 4-92$.

Find in yards the diameter of a circular enclosure of area $2\frac{1}{2}$ ac.

The volume of a cube is 7 cu. in., find the area of its surface.

The skating record for 500 m. is 42-6 sec. If 1 m. = 39-37 in., express the average speed in miles per hour.

Find the number of acres in 1 sq. km., given that 1 m. = 39-37 in.

If $\frac{2}{3}\pi r^2 = 100$, find the value of $4\pi r^2$.

If 1 litre contains 61-04 cu. in., express 1 ft. in cm.

**23.** Evaluate $\frac{a^2 + b^2 - c^2}{ab}$ if $a = 7-65$, $b = 6-48$, $c = 5-19$.

**24.** Evaluate $\sqrt{(s-a)(s-b)(s-c)}$ if $a = 3-15$, $b = 4-27$, $c = 5-14$, and $s = \frac{1}{2}(a + b + c)$.

**25.** Evaluate $\sqrt{((ab + cd)(ad + bc))}$ if $a = 17-03$, $b = 21-42$, $c = 18-16$, $d = 22-07$.

**Positive Numbers less than 1**

**Example 22.** Find the logarithm of 0-0648,

\[ 0-0648 \div 10^2 = 0-00648 \div 10^2 \]

\[ = 0-00648 \div 100 = 0-00648 \]

Similarly, \[ 0-648 \div 100 = 0-00648 \div 100 \]

and \[ 0-00648 \div 100 = 0-00648 \div 100 \]

Logarithms of numbers between 0 and 1 are negative, but are always written so that the decimal portion is positive; for example, the logarithm of 0-0648 is taken as $-2 + \cdot 8116$, instead of $-1-1884$, and for brevity is written $2-8116$, the "minus" being placed above the 2 to show that it refers only to the 2 and not to 8116. But at first it is best to write out such logarithms in full when making any calculations and to speak of "minus 2 plus point 8116," although later the shorter phrase "bar 2 point 8116" will be used.

*Thus for any number between 0 and 1, the characteristic is negative and is obtained by counting the number of digits between the decimal point and where it would be in the standard form.*

For example, for 0-648, one digit, the characteristic is $-1$;

for 0-00648, four digits, the characteristic is $-4$.

Notice that this is the same rule as that given on p. 295 for numbers greater than 10.

There is no logarithm of a negative number.

**Example 23.** Find the number whose logarithm is

(i) \[ 3-6749 \]; (ii) \[ 1-9027 \].

Move the figures 3 places to the right, \[ 10^{3-6749} = 0-00473 \].

(ii) \[ 10^{1-9027} = 2-008 \].

Move the figures 1 place to the right, \[ 10^{1-9027} = 0-200(8) \].
EXERCISE 150 (Oral)

[It is suggested that the characteristics of the logarithms of the numbers in Nos. 1-16 should be read off, before Exercise 150 is taken.]

Find the logarithms of:

1. 0.342. 2. 0.0483. 3. 0.0076. 4. 0.902.
5. 2.04. 6. 0.0075. 7. 10.01. 8. 0.0003.
9. 0.0025. 10. 0.101. 11. 0.001. 12. 0.0704.
13. 0.04503. 14. 0.4013. 15. 0.007138. 16. 0.01101.
17. \( \sqrt[3]{5} \). 18. \( \sqrt[3]{6} \). 19. \( \sqrt[3]{8} \). 20. \( \sqrt[3]{9} \).
21. \( 4 \times 10^{-6} \). 22. \( 6 \times 10^{-1} \). 23. \( 423 \times 10^{-16} \).

Find the numbers whose logarithms are:

24. \( -1 + 0.5922 \). 25. \( -2 + 0.8645 \). 26. \( -4 + 0.9085 \).
27. \( -1 + 0.4771 \). 28. \( -2 + 0.6085 \). 29. \( -3 + 0.7057 \).
30. \( -1 + 0.8476 \). 31. \( -5 + 0.9703 \). 32. \( -1 + 0.5229 \).
33. \( -4 + 0.2939 \). 34. \( -2 + 0.1650 \). 35. \( -3 + 0.1315 \).
36. 3.016. 37. 1.0425. 38. 3.0055. 39. 3.028.
40. 1.0000. 41. 10077. 42. 10594. 43. 1.9990.
44. 5.06. 45. 2.0000. 46. 1.44. 47. 5.044.

Some preliminary practice in working with negative characteristics is desirable. The working given in the illustrative examples may be abbreviated as soon as the processes are understood.

Example 24. Express with the decimal portion positive:

(i) \( 2.89 + 5.47 \); (ii) \( 3.76 + 1.58 \).

(i) \( 2 + \frac{89}{50} \); (ii) \( -3 + \frac{76}{50} \).

In subtraction, the reader should use for the decimal portion his ordinary method, it may be either of those given below; for the integral portion, if there is any difficulty, it is best to use the rule "change the sign of the lower line and add."
Example 27. Evaluate $0.000645 \times 82.3$.

Rough Estimate: $0.0006 \times 80 = 0.048$.

<table>
<thead>
<tr>
<th>Number</th>
<th>Logarithm</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000645</td>
<td>3.8096</td>
<td>-4 + .8096</td>
</tr>
<tr>
<td>82.3</td>
<td>1.9154</td>
<td>1 - .9154</td>
</tr>
<tr>
<td>Expression</td>
<td>5.7250</td>
<td>-2 + .7250</td>
</tr>
</tbody>
</table>

.: expression $= 0.0530(9)$.

Example 28. Evaluate $429.3 \div 0.00736$.

Rough Estimate: $400 \div 0.0007 = 50,000$.

<table>
<thead>
<tr>
<th>Number</th>
<th>Logarithm</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>429.3</td>
<td>2.6328</td>
<td>2 - .6328</td>
</tr>
<tr>
<td>0.00736</td>
<td>3.8669</td>
<td>-3 + .8669</td>
</tr>
<tr>
<td>Expression</td>
<td>4.7659</td>
<td>4 + .7659</td>
</tr>
</tbody>
</table>

.: expression $= 58,330$.

Example 29. Evaluate $(0.08644)^2$.

Rough Estimate: $(0.099)^2 = 0.00092$.

<table>
<thead>
<tr>
<th>Number</th>
<th>Logarithm</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.08644</td>
<td>2.9367</td>
<td>-2 + .9367</td>
</tr>
<tr>
<td>$0.08644^2$</td>
<td>4.8101</td>
<td>-6 + 2.8101</td>
</tr>
</tbody>
</table>

.: expression $= 0.000645(8)$.

Example 30. Evaluate $\sqrt[3]{0.5173}$.

<table>
<thead>
<tr>
<th>Number</th>
<th>Logarithm</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt[3]{0.5173}$</td>
<td>1.1032</td>
<td>4 - 1.1032</td>
</tr>
<tr>
<td>$0.5173^3$</td>
<td>1.9284</td>
<td>-1 + .9284</td>
</tr>
</tbody>
</table>

.: expression $= 0.848(0)$.

Rough Check: $(0.6)^3 = (0.64)^2 = (0.6)^2 = 0.36$

$(0.9)^3 = (0.81)^2 = (0.8)^2 = 0.64$

Exercise 152

Find the value to 4 figures, as given by 4-figure tables, of:

1. $0.243 \times 3.12$.
2. $0.816 \times 4.37$.
3. $0.729 \times 5.62$.
4. $0.215 \times 0.186$.
5. $0.743 \times 0.814$.
6. $0.315 \times 6.17$.
7. $0.0863 \times 0.924$.
8. $0.0072 \times 0.091$.
9. $0.000389 \times 47.4$.

Example 31. Find to 3 figures the value of $\sqrt[3]{(0.05871)^4 \div \sqrt[3]{0.7128}}$.

<table>
<thead>
<tr>
<th>Number</th>
<th>Logarithm</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$(0.05871)^4$</td>
<td>2.7678 x 3</td>
<td>4 - 3.7678</td>
</tr>
<tr>
<td>$\sqrt[3]{0.7128}$</td>
<td>1.8580 + 2</td>
<td>-3 + 1.8580</td>
</tr>
</tbody>
</table>

Numerator: $4.2326$
Denominator: $1.8911$

.: expression $= 0.000778$, to 3 figures.

Example 32. Find to 3 figures the value of $\frac{1}{\sqrt[3]{0.9-0.326}}$.

<table>
<thead>
<tr>
<th>Number</th>
<th>Logarithm</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt[3]{0.9-0.326}$</td>
<td>1.5132 x 4</td>
<td>-3 + 1.5132</td>
</tr>
<tr>
<td>Expression</td>
<td>0.1217</td>
<td></td>
</tr>
</tbody>
</table>

.: expression $= 1.323 = 1.32$, to 3 figures.

If the calculation of the logarithm is itself difficult, as in the next example, the working should be set out in full.
Example 33. Find, to 3 figures, the value of \((0.0728)^{1.4}\).

<table>
<thead>
<tr>
<th>Number</th>
<th>Logarithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0728</td>
<td>0.8621</td>
</tr>
<tr>
<td>1.4</td>
<td>-1.6</td>
</tr>
<tr>
<td>((-2+0.8621)\times(-1.6)) &amp; ((-1.1379)(-1.6))</td>
<td></td>
</tr>
<tr>
<td>1.1379</td>
<td>1.6</td>
</tr>
<tr>
<td>1.1379</td>
<td>68374</td>
</tr>
<tr>
<td>(=-1.2064)</td>
<td></td>
</tr>
</tbody>
</table>

\[ \therefore \text{expression } = 10^{1.1379} = 66.16 = 66.2, \text{ to } 3 \text{ figures.} \]

EXERCISE 153

Find the value to 4 figures, as given by 4-figure tables, of:

1. \(0.387 \times 0.473 \times 7.08\)
2. \((0.01031 \times 0.2074 \times 1.702)\)
3. \((0.314 \times 0.7285) \times 0.3762\)
4. \(0.5178 \times 4.917 \times 1.4971\)
5. \(0.4271 \times 0.00726\)
6. \((0.9193 \times 1.63)\)
7. \((0.7356)^a\)
8. \((3.142 \times 0.0067)\)
9. \((0.00728)^a \times 15.07\)
10. \((0.0728)^a \times 37\)
11. \(\sqrt{(6.372 \times 15.08)}\)
12. \(16.39 \div \sqrt{(0.8372)}\)
13. \(247.3 \div (1.634)^a\)
14. \(\sqrt{(0.7164) \div (0.4285)}\)
15. \((0.7054)^a\)
16. \((0.0107)^a\)
17. \((0.6362)^{-1.8}\)
18. \((0.835)^{-1.4}\)
19. \((-0.5)\)
20. \((-0.1)^{-0.2}\)
21. \((-0.09)^{-a}\)
22. \((-0.076)^{-a}\)
23. \((-0.624)^{-a}\)
24. \((-12.8)^{0.5}\)
25. \((-0.075)^{-a}\)
26. \((-0.62)^{-0.3}\)

Find the value to 3 figures, of:

27. \((0.438)^a\)
28. \((0.057)^a\)
29. \((0.342)^a\)
30. \((0.0361)^a\)
31. \((0.524)^a\)
32. \((0.172)^a\)
33. \((0.08143)^a\)
34. \((0.05361)^a\)
35. \((0.0434)^a\)
36. \((0.0434)^a\)
37. \((0.0434)^a\)
38. \((0.0434)^a\)
39. \((0.0434)^a\)
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49. \((0.0434)^a\)
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83. \((0.0434)^a\)
84. \((0.0434)^a\)
85. \((0.0434)^a\)
86. \((0.0434)^a\)
87. \((0.0434)^a\)
88. \((0.0434)^a\)
89. \((0.0434)^a\)
90. \((0.0434)^a\)
91. \((0.0434)^a\)
92. \((0.0434)^a\)
93. \((0.0434)^a\)
94. \((0.0434)^a\)
95. \((0.0434)^a\)
96. \((0.0434)^a\)
97. \((0.0434)^a\)
98. \((0.0434)^a\)
99. \((0.0434)^a\)
100. \((0.0434)^a\)

Index Problems. The general properties of logarithms were stated on p. 297; the relation, \(\log (x^n) = n \log x\), is useful in problems where the value of an index is required.
CHAPTER XX

HARDER MENSURATION

Areas

If ABCD is a parallelogram on the same base and between the same parallels as the rectangle ABHK, the areas of the parallelogram and rectangle are equal. 

Area of parallelogram ABCD = area of ABHK = AB × BH.

This fact is often expressed in the form:

area of parallelogram = base × height.

If ABCD is a parallelogram and if AH is its height, area of ∆ ABC = \( \frac{1}{2} \) area of ABCD = \( \frac{1}{2} \) BC × AH.

This fact is often expressed in the form:

area of triangle = \( \frac{1}{2} \) base × height.
The area of a triangle can also be expressed in terms of the lengths of its sides:

If the lengths of the sides are \(a\), \(b\), \(c\) in., and if the semi-perimeter is \(s\) in., that is if \(s = \frac{1}{2}(a + b + c)\), it can be proved that the area of the triangle is

\[
\sqrt{s(s-a)(s-b)(s-c)} \text{ sq. in.}
\]

If the parallel sides \(AB\), \(DC\) of the trapezium \(ABCD\) are of lengths \(a\), \(b\) in. and at a distance \(h\) in. apart,

area of \(\triangle ABC = \frac{1}{2} ah \text{ sq. in.}\),
area of \(\triangle ACD = \frac{1}{2} bh \text{ sq. in.}\),

\(\therefore\) area of trapezium \(ABCD = \frac{(a + b)h}{2} \text{ sq. in.} = \frac{1}{2}(a + b)h \text{ sq. in.}\).

This fact is often expressed in the form:

area of trapezium = average width \(\times\) height.

**Example 1.** Find the area of the isosceles triangle, whose sides are of lengths 7.4 cm, 7.4 cm, 6.2 cm.

The altitude \(AD\) bisects the base \(BC\), \(\therefore BD = 3.1\) cm;
but \(AD^2 + DB^2 = AB^2\),

\[
AD = \sqrt{(7.4^2 - 3.1^2)} \text{ cm.} = \sqrt{(54.76 - 9.61)} \text{ cm.} = \sqrt{45.15} \text{ cm.} = 6.720 \text{ cm.}
\]

\(\therefore\) area of \(\triangle ABC = \frac{1}{2} \times 6.2 \times 6.2 = 20.8 \text{ sq. cm.}, \text{ to 3 figures.}\)

**Example 2.** Find in acres the area of a triangular field, whose sides are of lengths 366 yd., 407 yd., 485 yd.

Denoting the lengths by \(a\), \(b\), \(c\), we have

\[
\begin{align*}
a &= 366 & s &= 629 \\
b &= 407 & s &= 263 \\
c &= 485 & s &= 222 \\
2s &= a + b + c &= 1258 & s &= 144
\end{align*}
\]

\(\therefore\) area = \(\sqrt{(629 \times 263 \times 222 \times 144)} \text{ sq. yd.} \approx 97235 \text{ sq. yd.}\)

but 4840 sq. yd. = 1 ac.,

\(\therefore\) area = 15.9 ac., to 3 figures.

Note. \((s-a)+(s-b)+(s-c) = 3s - a - b - c = 3s - 2s = s\). The values obtained for \(s-a\), \(s-b\), \(s-c\), should be checked by verifying that their sum is \(s\). Here, 263 + 222 + 144 = 629.

---

**EXERCISE 156**

[Do not give answers to more than 3 figures. Logarithm tables should be used, where suitable.]

In Nos. 1-14, \(AH\), \(CK\) are altitudes of the parallelogram \(ABCD\).

Find the area of \(ABCD\), if:

1. \(BC = 8\) cm., \(AH = 5\) cm.
2. \(AB = 3\) in., \(CK = 4\) in.
3. \(AD = 2.34\) in., \(AH = 1.56\) in.
4. \(CD = 7.28\) cm., \(CK = 9.46\) cm.
5. If the area of \(ABCD\) is 28.46 sq. in., and if \(BC = 7.35\) in., find the length of \(AH\).
6. If the area of \(ABCD\) is 37.24 sq. cm., and if \(CK = 8.14\) cm, find the length of \(AB\).
7. If \(BC = 4.76\) in., \(CD = 3.28\) in., \(AH = 2.84\) in., find the area of \(ABCD\) and the length of \(CK\).
8. If \(CD = 5.74\) cm., \(AH = 2.85\) cm., \(CK = 3.92\) cm, find the area of \(ABCD\) and the length of \(AD\).

Find the area of the triangle \(ABC\), if:

9. \(BC = 7\) in., \(AH = 5\) in. [10] \(AB = 6.14\) cm., \(CK = 8.72\) cm.
11. \(BC = 3.45\) in., \(AH = 4.85\) in.
12. \(AB = 10.17\) cm., \(CK = 12.08\) cm.
13. If the area of \(\triangle ABC\) is 8.32 sq. in., and if \(BC = 6.35\) in., find the length of \(AH\).
14. If the area of \(\triangle ABC\) is 59.7 sq. cm., and if \(CK = 13.4\) cm, find the length of \(AB\).

Find the area of a triangle, whose sides are:

15. 5 in., 5 in., 6 in. [16] 7 cm., 7 cm., 6 cm.
17. 3.64 in., 3.64 in., 4.96 in. *18. Each 7.5 cm.
19. Find the area of a rhombus whose diagonals are of lengths 3.9 in., 4.7 in. [The diagonals cut at right angles.]
20. The parallel sides of a trapezium are 1.47 ft., 1.15 ft., and are 0.85 ft. apart. Find the area of the trapezium.
21. The parallel sides of a trapezium are 4.36 cm, 3.18 cm., and its area is 18.72 sq. cm.; find the distance between the parallel sides.
22. The end wall of a barn consists of a rectangle 25 ft. wide, 14 ft. high, surmounted by an isosceles triangle. The ridge of the roof is 17 ft. above the ground. Find the area of the wall.

23. AH, CK are perpendiculars to the diagonal BD of the quadrilateral ABCD. If AH = 4-63 in., CK = 2-84 in., BD = 5-37 in., find the area of ABCD.

24. DP, CQ are the perpendiculars from D, C to AB. If DP = 238 yd., CQ = 316 yd., AP = 95 yd., PQ = 174 yd., QB = 82 yd., find in acres the area of ABCD.

25. ABCD is a quadrilateral, right-angled at A and at B; AD = 3 in., BC = 2-2 in., CD = 1-7 in. Find (i) the area of ABCD, (ii) the area of △ BCD, (iii) the length of the perpendicular from B to DC produced.

Find the area of a triangle and the length of its greatest altitude, if the sides are:

26. 5 cm., 6 cm., 7 cm. [27] 4-6 in., 5-3 in., 6-9 in.

28. 207 yd., 246 yd., 291 yd. (area in acres).

29. 7-36 ch., 8-49 ch., 9-15 ch. (area in acres).

280. The parallel sides of a trapezium are 7-5 in., 3-9 in.; and the other sides are each 2-6 in. Find its area.

Example 8. The surface of the water in a swimming-bath is a rectangle 155 ft. long, 40 ft. wide, and the depth of the water increases uniformly from $3\frac{1}{4}$ ft. at one end to $10\frac{1}{4}$ ft. at the other end. Find the volume of water in the bath.

The side-face of the bath in contact with the water is a trapezium, parallel sides $3\frac{1}{4}$ ft., $10\frac{1}{4}$ ft., at a distance 155 ft. apart.

Therefore the area of side-face of bath in contact with the water is $\frac{155}{2} (3\frac{1}{4} + 10\frac{1}{4}) = 155$ sq. ft.

But the water is in the form of a prism, of cross-section equal to area of side-face, and of width 40 ft.;

\[ \text{Volume of water} = \frac{1}{2} (3\frac{1}{4} + 10\frac{1}{4}) \times 155 \times 40 \text{ cu. ft.} \]
\[ = \frac{1}{2} \times 14 \times 155 \times 40 \text{ cu. ft.} \]
\[ = 43,400 \text{ cu. ft.} \]

EXERCISE 157

[Give answers to 3 figures. Logarithm tables should be used where suitable]

1. A triangular set square is 5 mm. thick and its two shorter sides measure 22-5 cm., 38-7 cm. Find its volume.

2. The cross-section of a trough 7 ft. long is a triangle whose sides are 3 in., 4 in., 5 in. Find how much water it can hold.

3. The heights of the front and back walls of a lean-to shed are 8 ft. and 10 ft., and are 7 ft. apart. The shed is 16 ft. long. Find its volume.

4. The cross-section of a trench, 24 ft. long, is a trapezium 5 ft. 6 in. wide at the bottom and 5 ft. 4 in. wide at the top. The trench is 6 ft. 3 in. deep. Find in cu. ft. the volume of soil removed in making it.

5. A rectangular brass plate 4-6 in. wide, 0-15 in. thick weighs 3 lb. Find its length if the brass weighs 528 lb. per cu. ft.

6. The depth of water in a swimming-bath increases uniformly from $2\frac{1}{4}$ ft. at one end to $8\frac{1}{4}$ ft. at the other end. The bath is 120 ft. long and 35 ft. wide. Find the volume of the water.

7. Find the air-space in a hall, 60 ft. long, $2\frac{1}{2}$ whose cross-section has the dimensions shown in the diagram.
8. The base of a prism, 15 cm. long, is an isosceles triangle, sides 5 cm., 5 cm., 6 cm. Find (i) its volume, (ii) the total area of its surface.

9. The cross-section of a writing-desk, 3 ft. wide, is shown in the diagram; three of the corners are right-angled. Find its volume in cu. ft.

10. The depth of water in a swimming-bath increases uniformly from 3 ft. at one end to 10 ft. at the other end. The bath is 80 ft. long and 30 ft. wide. Find the number of gallons of water in the bath. [1 cu. ft. = 6 1/2 gal.]

11. Find the number of gallons of water left in the bath described in No. 10, when the water is allowed to run out until only half the floor remains covered.

[12] The base of a metal prism is an equilateral triangle, side 12 cm., and the height of the prism is 35 cm. Find (i) the total area of its surface, (ii) its weight in Kg., if 1 c.c. of the metal weighs 8.45 gm.

13. The base of a reservoir is a horizontal rectangle 100 yd. long, 50 yd. wide. The two shorter end-faces are vertical and the two longer side-faces are inclined outwards at 45° with the vertical. Find the volume of water in gallons when the depth is 6 ft. [1 gall. = 277.3 cu. in.]

14. The diagram represents the cross-section of a barn, 45 ft. long; the three marked corners are right-angled. Find the volume of the barn.

15. The cross-section of a prism, 8 in. high, is a triangle whose sides are 6 in., 7 in., 9 in. Find its volume.

16. A vessel of uniform horizontal cross-section contains water to a depth of 16 cm. When 50 marbles, each of volume 0.876 c.c., have been dropped into it, the depth is observed to be 18.65 cm. Find the area of the cross-section.

17. A trough is 15 ft. long; its cross-section is an isosceles trapezium, 27 in. wide at the top, 9 in. wide at the bottom, and its slant edges are each 15 in. Find the amount of water in cu. ft. the trough will hold.

18. Water is discharged at 3500 gall., per min., from a reservoir of surface area 7200 sq. ft. into one with surface area 5600 sq. ft. The sides of both reservoirs are vertical planes. By how much does the difference of the levels of the two surfaces alter in 15 min.? [1 cu. ft. = 6 1/2 gal.]

VOLUME OF A PYRAMID

If the base of a solid is a polygon ABCDE..., and if the other faces are triangles VAB, VBC, VCD, ..., with a common vertex V, the solid is called a pyramid, with ABCDE... as base and V as vertex; and the distance VK of the vertex V from the plane of the base is called the height of the pyramid. The form of a pyramid is best explained by the use of models. If the lines joining the vertex to the corners of the base are all equal, the solid is called a right pyramid, and each of these lines is called a slant edge of the pyramid. It can be proved that

Volume of pyramid = 1/3 x area of base x height.

Example 4. The base ABCD of a right pyramid, vertex V, is a rectangle, 4 1/2 in. by 3 1/2 in., and the length of a slant edge is 3 1/2 in. Find (i) the volume of the pyramid, (ii) the total area of its surface.

(i) If the diagonals AC, BD of the base cut at K, VK is the height of the pyramid. Let VK = h in., KC = x in., then AC = 2x in.

Since $\angle ABC = 90^\circ$, $(2x)^2 = 4^2 + 3^2 = 16 + 9 = 25$;
\[4x^2 = 32/32; \quad x^2 = 8/8 = 8 \text{ in.}
\]

Since $\angle VKC = 90^\circ$, $h^2 + x^2 = 3^2 + 8^2 = 9 + 64 = 73$;
\[h = \sqrt{73} = 8.53 \text{ in.}
\]

\[\text{Volume} = \frac{1}{3} \times \text{area of base} \times \text{height} = \frac{1}{3} \times (4.5 \times 3.5 \times 8.53) = 164.5 \text{ cu. in.}
\]

(ii) As in Example 1, p. 310, the height VN of the isosceles triangle VAB is given by

\[\text{VN} = \sqrt{(3.99 - 2.29)^2} = \sqrt{(15.21 - 4.84)^2} = \sqrt{10.37} = 3.22 \text{ in.}
\]

\[\text{Area of } \triangle VAB = \frac{1}{2} \times 3.99 \times 3.22 = 7.084 \text{ sq. in.}
\]

Similarly, the area of $\triangle VBC = \frac{1}{2} \times 3.6 \times \sqrt{(3.99 - 1.83)^2}$ sq. in.

\[= 1.8 \times \sqrt{(15.21 - 3.24)^2} = 1.8 \times 12.49 = 22.498 \text{ sq. in.}
\]

\[\text{Area of } \triangle VCD = \frac{1}{2} \times 3 \times \sqrt{(3.99 - 2.8^2)} = 26.67 \text{ sq. in.}
\]

\[\text{But area of base } ABCD = (4.5 \times 3.5) = 15.84 \text{ sq. in.}
\]

\[\text{Total area of surface} = (26.67 + 15.84) = 42.51 \text{ sq. in., to 3 figures.}
\]
EXERCISE 158

[Do not give answers to more than 3 figures. Use logarithm tables where suitable]

Find the volume of a pyramid, given:

1. Height, 5 in.; square base, side 3 in.
2. Height, 6 cm.; rectangular base, 4 cm. by 5 cm.
3. Height, 8 cm.; triangular base, sides 3 cm., 4 cm., 5 cm.
4. Find the height of a pyramid of volume 100 cu. in., if its base is a square, side 5 in.
5. Find the area of the base of a pyramid of volume 48 cu. cm., if its height is 8 cm.

Find the volume of a right pyramid, given:

6. Slant edge, 13 cm.; rectangular base, 6 cm. by 8 cm.
7. Slant edge, 2 in.; square base, side 2 in.
8. Height, 9 cm.; base, equilateral triangle, side 8 cm.
9. Height, 1 ft.; triangular base, sides 5 in., 6 in., 7 in.
10. The volume of a right pyramid whose base is 4 in. square is 32 cu. in.; find its height and the length of a slant edge.
11. The volume of a right pyramid, 8 in. high, standing on a square base, is 96 cu. in.; find the length of a side of the base and of a slant edge.

Find the total area of the surface of a right pyramid, given:

12. Height, 4 cm.; square base, side 6 cm.
13. Height, 5 in.; rectangular base, 4 in. by 6 in.
14. Find the volume of a right pyramid standing on a square base, if its height is 6 in. and the length of its slant edge is 8 in.

15. In a cubical tank, each edge 4 ft., there is a solid metal pyramid, 4 ft. high, whose base is a square of side 3 ft. The tank is filled with water. If the pyramid is removed, find the distance the water-level sinks.

CIRCLES AND CYLINDERS

Circles and Cylinders. The mensuration of the circle has been discussed in Chapter XIV, see p. 195. The principal formulæ are repeated here for the convenience of the reader:

For a circle, radius $r$ in.,

- Circumference $= 2\pi r$ in.; area $= \pi r^2$ sq. in.

For a circular cylinder, radius $r$ in., height $h$ in.,

- Area of curved surface $= 2\pi rh$ sq. in.; volume $= \pi r^2 h$ cu. in.

Calculations are generally shortened by using logarithms:

$$\log{\pi} = 0.4971.$$

Example 5. The diameter of the section of a steel wire, weighing 492 lb. per cu. ft., is 0-104 in. Find, to 3 figures, the weight of a mile length of the wire.

The radius of the cross-section is 0.052 in., therefore the area of the cross-section is $\pi \times (0.052)^2$ sq. in.

- The volume of a length of 1 mi. of wire is

$$\frac{(\pi \times (0.052)^2 \times 144) \times 1760 \times 3}{27160} = \frac{34320}{4971}.$$

- The weight of a length of 1 mi. of wire

$$= \frac{\pi \times (0.052)^2 \times 5280 \times 492}{144} = \frac{4537}{26920} = 153.2$$

N.B. Obtain a numerical expression for the result before using logarithms.

Example 6. A cylindrical tankard holds 1 qt. If its height equals the radius of its base, find the height in inches to 1/10 in., given that 1 gal. = 277.27 cu. in.

If the radius of the base is $r$ in., the height is $r$ in.;

- The volume $= \pi r^2 r$ cu. in.
- $\pi r^3 = 277.27 \div 4 = 69.32$;
- $r^2 = 69.32 + \pi$;
- $r = \sqrt{(69.32 + \pi)} = 2.804$;
- $r = 2.80$ in., to 1/10 in.
Example 7. A length of 2400 ft. of paper is wrapped on a wooden cylinder of radius 3 in.; the thickness of the paper is \(\frac{1}{16}\) in. Find the radius of the whole roll to \(\frac{1}{16}\) in.

If the radius of the whole roll is \(r\) in., the area of the cross-section of the roll is \(\pi r^2\) sq. in.; but the area of the cross-section of the wooden cylinder is \(\pi \times 3^2\) sq. in.; \(\therefore\) area of cross-section of paper = \((\pi r^2 - 9\pi)\) sq. in.

But the cross-section of the paper is a rectangle 2400 ft. long. \(\frac{1}{16}\) in. deep.

\[
\begin{align*}
\pi r^2 - 9\pi &= 2400 \times \frac{1}{16} \times \frac{1}{16} = 240; \\
\therefore \quad \pi r^2 &= 240 + 9; \\
\therefore \quad r^2 &= \frac{249}{\pi} + 9; \\
\therefore \quad r &= \sqrt{(85.40) - 9.241}. 
\end{align*}
\]

\(\therefore\) the radius of the whole roll is 9\frac{1}{2} in., to \(\frac{1}{16}\) in.

EXERCISE 159

[Give answers to 3 figures; \(\log \pi = 0.4971\); 1 gallon = 277.3 cu. in.]

1. Find (i) the circumference, (ii) the area of a circle of radius 13\frac{1}{2} in.

2. Find (i) the radius, (ii) the area of a circle whose circumference is 100 yd.

3. Find in yards (i) the radius, (ii) the circumference of a circle whose area is 1 ac.

4. Find the number of revolutions of a wheel of a car, 33 in. in diameter, when the car travels 1 mi.

5. A circular hole of diameter 7.5 cm. is punched in a tin sheet 9 cm. long, 8 cm. wide. Find the area of the upper surface of the sheet.

6. Taking the mean distance of the Earth from the Sun to be \(1.5 \times 10^6\) km. and the length of the year to be 365\frac{1}{2} days, find the speed of the Earth in its orbit (assumed circular) in miles per hour.

[1 mi. = 1.609 km.]

7. A cylindrical ruler, 1\frac{1}{2} in. in diameter, is 18 in. long. Find its volume.

8. Find the weight of a cylindrical iron bar, 5\frac{1}{2} dm. long, 6.4 cm. in diameter, if the iron weighs 7.72 gm. per c.c.

9. Find the inside area in sq. ft. of a wall 4 ft. 9 in. high, enclosing a circular courtyard, diameter 65 ft.

10. An oil drum, diameter 2 ft. 3 in., holds 50 gal. Find its height.

11. Find the area of the ring bounded by two concentric circles, radii 15.7 cm., 21.4 cm. respectively.

12. An open jam-jar is 4\frac{1}{2} in. in diameter and 5\frac{1}{2} in. high, external measurements. Find the total area of the external surface.

13. A circular lead disc of radius 10\frac{1}{2} in. weighs 824.6 oz. Find its thickness if 1 cu. in. of lead weighs 6.52 oz.

14. Find the perimeter of a semicircular plate of radius 3\frac{1}{2} in.

15. The section of a tunnel 1\frac{1}{2} mi. long is a semicircle 23 ft. in diameter. Find in sq. yd. the area of the internal curved surface.

16. The volume of a cylinder 6 in. long is 1 cu. in. Find the diameter of the cross-section.

17. A cylindrical boiler holds 176 gal. and is 3 ft. long. Find its internal radius in inches.

18. Find the area of sheet tin required to make a closed cylindrical tin 4 in. in diameter, 9 in. high, if the lid overlaps \(\frac{1}{2}\) in. all round. [No allowance is to be made for the join up the side of the tin.]

19. A cylindrical tin holds 1 pt. A second tin is half the height but \(\frac{1}{3}\) times the diameter of the first; how much does it hold?

20. Water flows at 6 ft. per second through a pipe of diameter \(2\frac{1}{2}\) in. How many hours will it take to fill a tank 40 ft. long, 30 ft. broad, 8 ft. deep if the pipe remains full?

21. A tank 12 ft. by 8 ft. by 6 ft. is filled in 2 hr. by a pipe through which water flows at 3 ft. per second. Find the diameter of the pipe.

22. Find the weight in cwt. of 100 ft. of lead pipe, internal diameter 1 in., and 0.2 in. thick, if 1 cu. ft. of lead weighs 708 lb.

23. Find in sq. yd. the area of a path 3 ft. wide which runs round a circular pond of area \(\frac{1}{4}\) ac.

24. Paper 3000 ft. long, \(\frac{1}{16}\) in. thick, is wrapped round a cylinder, radius 3 in. Find the radius of the whole roll in inches.

25. 5 mi. of paper is wound on to a roller, diameter 6 in., the complete roll being 3 ft. in diameter. Find the thickness of the paper in inches.
Circular Cone. Cut out a sector of a circle and fold it into the form of a funnel. The surface so obtained is called a circular cone; we can regard it as a right pyramid with a circular base.

If \( V \) is the vertex of the cone, and if \( O \) is the centre of the base and \( AB \) a diameter of the base, the line \( VO \) is called the axis of the cone and its length is called the height of the cone; \( VA \) is called a slant side of the cone and its length is called the slant length or the slant height of the cone.

If a cut is made along \( VA \) and if the curved surface is then folded flat, we obtain a sector of a circle, centre \( V \), radius \( VA \), as shown.

If the slant length of the cone is \( l \) in., and if the base-radius is \( r \) in., then \( VA = l \) in. and arc \( ABA \) of sector = \( 2\pi r \) in.

Since the circumference of the complete circle, of which the sector is a part, is \( 2\pi r \) in., the area of the sector is \( \frac{2\pi r}{2\pi} \) of this complete circle.

\[ \therefore \text{area of sector} = \frac{2\pi r}{2\pi} \text{ of } \pi r^2 \text{ sq. in.} = \pi rl \text{ sq. in.} \]

\[ \therefore \text{area of curved surface of cone} = \pi rl \text{ sq. in.} \]

Also since a cone may be regarded as a pyramid, its volume is given by the rule, \( \frac{1}{3} \) area of base \( \times \) height;

\[ \therefore \text{volume of cone} = \frac{1}{3} \pi r^2h \text{ cu. in.} \]

By Pythagoras, the values of \( l, r, h \) are connected by the relation, \( r^2 + h^2 = l^2 \).

EXERCISE 160

[Give answers to 3 figures; \( \log \pi = 0.4971 \)]

Find the volume of a circular cone, given:

1. Height, 4 in.; area of base, 15 sq. in.
2. Height, 8 cm.; radius of base, 3 cm.
3. Height, 3 in.; slant length, 5 in.

4. Height, 5 in.; perimeter of base, 8 in.
5. Height, 6\footnote{1/4} in.; slant length, 8\footnote{3/8} in.

Find the area of the curved surface of a circular cone, given:

6. Slant length, 8 cm.; base-radius, 6 cm.
7. Slant length, 13 cm.; height, 12 cm.
8. Slant length, 7 in.; perimeter of base, 15 in.
9. Height, 8 in.; base-diameter, 1 ft.
10. Slant length, 9\footnote{3/4} in.; height, 7\footnote{1/2} in.
11. Find the height of a cone whose base-radius is 4\footnote{3/2} in., and whose volume is 56\footnote{2/8} cu. in.
12. The length of the arc of a sector of a circle is 6 cm. and the radius of the circle is 4 cm.; find the area of the sector.
13. The area of a sector of a circle of radius 5 in. is 20 sq. in.; find the length of the arc of the sector.
14. The base of a conical tent is 15 ft. in diameter and the height is 9 ft. Find (i) the volume of the tent, (ii) the area of the canvas used for making it.
15. Find the height of a cone if the area of the curved surface is 100 sq. in. and the base-radius is 4\footnote{1/2} in.
16. A conical block of silver has a slant edge of 20 in. and base-radius 12 in. How many coins \( \frac{1}{2} \) in. thick and \( 1\frac{1}{2} \) in. in diameter can be made from it?
17. A conical tent of capacity 600 cu. ft. stands on a circular base of area 160 sq. ft. Find in sq. ft. the area of the canvas.
18. The inside of a glass is an inverted cone, height 5 in.; diameter of top 3 in.; it contains \( \frac{3}{4} \) pt. of wine. What is the depth at the centre? Use the fact that the depth is proportional to the diameter of the surface of the wine; if the depth is 5\footnote{1/2} in., the diameter of the surface is 3\footnote{1/2} in. [1 gal. = 277\footnote{1/2} cu. in.]

The Sphere. An object shaped like a tennis ball is called a sphere.

If the radius of a sphere is \( r \) inches, it can be proved that

\[ \text{area of surface of sphere} = 4\pi r^2 \text{ sq. in.} ; \]

\[ \text{volume of sphere} = \frac{4}{3} \pi r^3 \text{ cu. in.} \]
Example 8. The volume of a sphere is 8 cu. in.; find, to 3 figures, the area of its surface.

If the radius of the sphere is \( r \) in.,

\[
\frac{4}{3}\pi r^3 = 8; \quad \therefore \quad r^3 = \frac{6}{\pi}; \quad \therefore \quad r = \left(\frac{6}{\pi}\right)^{\frac{1}{3}}.
\]

\[
\text{Area of surface} = 4\pi r^2 \text{ sq. in.}
\]

\[
\frac{4}{\pi} \left(\frac{6}{\pi}\right)^{\frac{2}{3}} = \frac{4}{\pi} \left(\frac{36}{\pi^2}\right)^{\frac{1}{3}} = \frac{4}{\pi} \left(\frac{6}{\pi}\right)^{\frac{2}{3}} = \frac{4}{\pi} \times 19.3 \text{ sq. in.}
\]

\[
= 4(6 + \pi) = 19.3 \text{ sq. in.}
\]

\[
2 \times \frac{4}{6 + \pi} \approx 2 \times \frac{4}{9.42} = 0.876 \text{ sq. in.}
\]

\[
= \frac{4}{6 + \pi} \times 19.3 \approx 4 \times 19.3 \times 0.876 = 60.2 \text{ sq. in.}
\]

\[
4 \pi r^2 = 4 \pi \times 19.3 \approx 4 \times 60.2 = 240.8 \text{ sq. in.}
\]

\[
\text{Area} = 1.2866 \text{ sq. in.}
\]

Example 9. A hollow copper sphere weighs 10 lb. and its external diameter is 6 in. If the cavity is spherical, find its radius, to 3 figures, given that 1 cu. ft. of copper weighs 548-2 lb.

If the radius of the cavity is \( r \) in., the volume of the cavity is \( \frac{4}{3}\pi r^3 \) cu. in.; also the volume of the sphere is \( \frac{4}{3}\pi \times 3^3 \) cu. in.;

\[
\therefore \quad \text{the volume of the copper is} \quad \frac{4}{3}\pi (27 - r^3) \text{ cu. in.;}
\]

but 1 cu. in. of copper weighs 548-2 lb.,

\[
\therefore \quad \text{the weight of the copper is} \quad \frac{4}{3}\pi (27 - r^3) \times 548.2 \text{ lb.}
\]

\[
\begin{align*}
\text{Logarithms} & \quad 1.4771 & \quad -6021 \\
\text{Log.} \quad \frac{4}{3}\pi (27 - r^3) & \quad 1.4771 & \quad -6021 \\
\text{Log.} \quad 548.2 & \quad 2.7390 & \quad 0.8764
\end{align*}
\]

\[
\therefore \quad \text{the radius of the cavity is} \quad 2.69 \text{ in., to 3 figures.}
\]

EXERCISE 161

[Give answers to 3 figures; log \( \pi = -4.971 \)]

Find the volume and area of the surface of a sphere, given:

1. Radius, 2.63 in.  
2. Diameter, 7.28 cm.

Find the radius of a sphere, given:

3. Volume, 4256 cu. cm.  
4. Area of surface, 984 sq. in.

5. Find the volume of a sphere whose surface is 74.5 sq. cm.

EXERCISE 161

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5. Find the volume of a sphere whose surface is 74.5 sq. cm.
Similar Figures and Similar Solids. Two plane figures or two solids are called similar if the larger can be regarded as a magnification of the smaller; that is, if all corresponding angles are equal and if the ratio of any two corresponding lengths is constant, this constant being equal to the (linear) magnification.

Thus if the sides of a rectangle, 6 in. by 9 in., are magnified in the ratio $\frac{3}{2}$, we obtain a rectangle, 10 in. by 15 in., which is said to be similar to the original rectangle.

The ratio of their areas $= \frac{10 \times 15}{6 \times 9} = \frac{5 \times 5}{3 \times 3} = \left(\frac{5}{3}\right)^2$;

thus the ratio of their areas equals the square of the magnification.

It can be proved geometrically that if any two plane figures are similar, the ratio of the area of the larger to the area of the smaller is equal to the square of the magnification; that is, the square of the ratio of the length of any line in the larger figure to the length of the corresponding line in the smaller figure.

For example, if two triangles are similar, the ratio of their areas equals the square of the ratio of corresponding sides, or the square of the ratio of corresponding altitudes.

Any two circles are similar; the ratio of their areas equals the square of the ratio of their radii. If the radii are $a$ in., $b$ in., ratio of areas $= \frac{\pi a^2}{\pi b^2} = \left(\frac{a}{b}\right)^2$.

If the edges of a rectangular box, 6 in. by 9 in. by 12 in., are magnified in the ratio $\frac{3}{2}$, we obtain a rectangular box, 10 in. by 15 in. by 18 in., which is said to be similar to the original rectangular box.

The ratio of their volumes $= \frac{10 \times 15 \times 18}{6 \times 9 \times 12} = \frac{5 \times 5 \times 5}{3 \times 3 \times 3} = \left(\frac{5}{3}\right)^3$; thus the ratio of their volumes equals the cube of the magnification.

Also since their surfaces are composed of similar figures, the ratio of the areas of their surfaces equals the square of the magnification.

It can be proved geometrically that if any two solids are similar, the ratio of the volume of the larger to the volume of the smaller is equal to the cube of the magnification, and that the ratio of the area of the surface of the larger to that of the smaller is equal to the square of the magnification.
9. A water-can, height 15 in., diameter 10 in., holds $4\frac{1}{2}$ gall. How much does a can of the same shape, 10 in. high, hold?

10. Two similar tanks are respectively 5 in. and 7 in. high; the smaller holds a pint; how much does the larger hold?

11. It costs 36s. to gild a sphere of diameter 4$\frac{1}{2}$ in.; how much will it cost to gild a sphere of diameter 7$\frac{3}{4}$ in.?

12. A plaster model of a statue is 2 ft. high and weighs 24 lb. If the statue is 9 ft. high and is made in stone which is $2\frac{1}{2}$ times as heavy as the plaster, find the weight of the statue in tons.

13. The areas of two circles are 96 sq. in. and 15 sq. in.; find the ratio of their diameters.

14. The volumes of two spheres are 500 c.c. and 108 c.c.; find the ratio of (i) their diameters, (ii) the areas of their surfaces.

15. The tobacco in a tin 6 in. high lasts me 8 days. How long will the tobacco in a tin of the same shape 9 in. high last me?

16. The ratio of the areas of the surfaces of two hemispheres is 5 : 4. Find in the form $n : 1$, giving $n$ to 3 figures, the ratio of (i) their diameters, (ii) their volumes.

17. The ratio of the volumes of two similar cones is 3 : 2. Find in the form $n : 1$, giving $n$ to 3 figures, the ratio of (i) their heights, (ii) the areas of their surfaces.

18. A golf course occupies 185 acres. By what area is it represented on a map of scale (i) 2 inches to the mile, (ii) 1 inch to the mile?

19. An estate is represented by an area of 375 sq. in. on a map whose scale is 100 : 1. What is the area of the estate represented on a map whose scale is 250 : 1?

20. If the surface of a soap bubble increases by 21%, find the percentage increase in (i) its diameter, (ii) its volume.

21. If 1 pt. of water is poured into a conical vessel, the depth of the water is 4 in. How much more water must be added to make the depth 6 in.?

22. If 1 pt. of water is poured into a conical vessel, the area of the wet surface is 40 sq. in.; what is the increase in the area of the wet surface if 7 more pints are added?

23. A tank can be filled by a 3-in. pipe in 40 min. How long will it take if a 4-in. pipe is used, the rate of flow being the same?

**MISCELLANEOUS EXAMPLES**

**EXERCISE 163**

[Give answers to 3 figures. Logarithm tables should be used where suitable]

1. A bushel of grass seed, costing 2s. per lb., weighs 25 lb. Find the cost of seed for a lawn 78 ft. by 36 ft. if 8 bushels are required per acre.

2. ABCD is a field. B is 284 yd. east of A; C is 215 yd. north of B; D is 194 yd. west of C. Find in acres the area of the field.

3. The unshaded part of the square in the diagram consists of a square and four equal trapeziums. Find its area.

4. Find the area of the figure represented in the diagram if all the corners, except two, are right-angled.

5. A corridor is 20 yd. long, 7 ft. 6 in. wide. It is covered with carpet at 6s. 3d. per sq. ft. down the middle, with a margin 18 in. wide along each side which is laid with wooden blocks $4\frac{1}{2}$ in. by 9 in. at £2 10s. per 100. Find the cost of (i) the carpet, (ii) the blocks.

6. The internal length and breadth of a petrol can are $9\frac{3}{4}$ in., $5\frac{1}{4}$ in. Find the depth of a can which holds $2\frac{1}{2}$ gall. [1 gall. = 277-3 cu. in.]

7. A railway embankment is 20 ft. wide at the top, 40 ft. wide at the bottom, and 11 ft. high. Find the number of tons required for 50 yd. of embankment, assuming that 1 cu. ft. of earth weighs 120 lb.

8. The base of a wooden wedge 9 in. long is a triangle whose sides are 2 in., 4 in., 4 in. long. Find the weight of the wedge if 1 cu. in. of the wood weighs 0-365 oz.

9. Find the radius of a circle whose circumference is 8-34 cm.

10. Find the circumference of a circle whose area is 20 sq. yd.

11. A cylindrical tank, without a cover, holds 100 litres. The internal diameter is 40 cm.; find the total area of the internal surface.
12. Find the area of the quadrilateral ABCD, given that AB = 6 in., BC = 8 in., CD = 9 in., DA = 7 in., \( \angle ABC = 90^\circ \).

13. A rectangular area on a map measures 2.87 in. by 4.18 in., approximately and represents an area of 58,000 sq. yd. Find the scale of the map in the form, 1 : n.

14. The diagram represents the vertical section of a bridge, in the form of a rectangle from which a semicircle has been removed. The bridge is 14 ft. wide and is made of material weighing 112 lb. per cu. ft. Find the weight of the bridge in tons, and the cost of painting the curved surface under the bridge at 1s. 3d. per sq. yd.

15. A circular cylinder, height 20 cm., diameter 12 cm., is half full of water. If 5 circular discs, each 4 cm. in diameter and 3 mm. thick, are dropped into it and totally submerged, find the rise of the water-level.

16. Find the volume of a circular cone if its height is 6-54 in., and the diameter of its base is 8-28 in.

17. The length of the arc of a sector of a circle, of radius 5 in., is 18 in. If the sector is folded to form the curved surface of a cone (without overlap), find (i) the base-radius of the cone, (ii) the area of the curved surface of the cone, (iii) the height of the cone.

18. A circular cylinder, internal diameter 8 in., height 10 in., contains water to a depth of 6 in.; two solid lead spheres each of diameter 4 in. are placed in it. Find the height the water-level rises.

19. The base of a tank is a square ABCD, side 5 ft., and the tank is 3 ft. high. It contains water to a depth of 2 ft., the base being horizontal. The tank is now tilted slowly about the edge BC until the edge AD is in the surface of the water. How much water has run out?

20. A piece of ground in the shape of a rectangle 100 yd. by 80 yd., with a semicircle on each short side as diameter, is surrounded by a running-track 15 ft. wide. The track is to be covered to a depth of 3 in. with cinders weighing 70 lb. per cu. ft. Find in tons the weight of the cinders required.

**CHAPTER XXI**

**SPECIFIC GRAVITY**

The mass of a unit of volume of a substance is called its density. Any units may be selected, but must be specified. Masses are estimated by using the fact that the ratio of the masses of two bodies is equal to the ratio of their weights at the same place. The statement that a body weighs 2 lb. is an abbreviation for the statement that its weight equals the weight of a body of mass 2 lb. at the same place.

Thus for copper, 1 cu. in. weighs 5-1 oz., 1 cu. cm. weighs 8-8 gm.; therefore the density of copper can be given as

\[
5-1 \text{ oz. per cu. in. or } 8-8 \text{ gm. per cu. cm.}
\]

For many purposes it is useful to compare the weight of a substance with the weight of the same volume of water. In the metric system the weight of 1 gm. was chosen as the weight of 1 c.c. of water (at 4°C).

\[\therefore \text{the density of water is } 1 \text{ gm. per c.c.}\]

In British units, it is found that 1 cu. ft. of water weighs about 1000 oz., or, more accurately, 62-3 lb. (weighed in air at ordinary temperature).

\[\therefore \text{the density of water is } 62-3 \text{ lb. per cu. ft.}\]

The ratio of the weight of any volume of a substance to the weight of an equal volume of water (at 4°C) is called the specific gravity of the substance. Since the specific gravity is a ratio, it does not depend on the units chosen. The abbreviation for specific gravity is sp. gr.

For example, the statement that “the sp. gr. of silver is 10-5” means that any volume of silver is 10-5 times as heavy as the same volume of water. Since 1 c.c. of water weighs 1 gm., 1 c.c. of silver weighs 10-5 gm.; since 1 cu. ft. of water weighs 62-3 lb., 1 cu. ft. of silver weighs (62-3 \times 10-5) lb.

**Example 1.** A sheet of plate glass is 15 in. long, 8 in. wide, and \( \frac{1}{8} \) in. thick. Find its weight, to the nearest oz., if the density of the glass is 1-63 oz. per cu. in.

The volume of the glass = \((15 \times 8 \times \frac{1}{8}) \text{ cu. in.} = 30 \text{ cu. in.} ;
\]

\[\therefore \text{its weight} = (1-63 \times 30) \text{ oz.} = 49 \text{ oz., to nearest oz.}\]
Example 2. The density of steel is 486 lb. per cu. ft.; find, to 2 figures, its specific gravity.

1 cu. ft. of steel weighs 486 lb. Logarithms
1 cu. ft. of water weighs 62-3 lb.
\[ 2-6866 \]
sp. gr. of steel \[= \frac{486}{62-3} = 7-8 \] to 2 figures.
\[ 1-7945 \]
\[ \therefore \text{sp. gr. of steel} = 7-8 \] to 2 figures.
\[ -8921 \]

Example 3. A rectangular tin, 16 cm. by 12 cm., contains petrol (sp. gr. 0-68) to a depth of 25 cm. Find, to 2 figures, the weight of the petrol.

1 c.c. of water weighs 1 gm., \[ \therefore 1 \text{ c.c. of petrol weighs 0-68 gm.} \]

But volume of petrol \( = (16 \times 12 \times 25) \text{ c.c.} = 4800 \text{ c.c.} \)

\[ \therefore \text{weight of petrol} = (0-68 \times 4800) \text{ gm.} = 3264 \text{ gm.} \]
\[ = 3-3 \text{ kg.}, \] to 2 figures.

Example 4. Find, to the nearest cu. in., the volume of a lump of lead weighing 50 lb., given that the sp. gr. of lead is 11-4.

1 cu. ft. of water weighs 62-3 lb.,

\[ 2-3237 \]
1 cu. ft. of lead weighs \( = (62-3 \times 11-4) \text{ lb.} \)
\[ 3-2735 \]
\[ \therefore (62-3 \times 11-4) \text{ lb. is the weight of 1728 cu. in. of lead.} \]
\[ 1-6690 \]
\[ \therefore 50 \text{ lb. is the weight of } \frac{1728 \times 50}{62-3 \times 11-4} \text{ cu. in.,} \]
\[ 4-9365 \]
\[ \text{that is, 121-6 cu. in.;} \]
\[ 2-8514 \]
\[ \therefore \text{volume} = 120 \text{ cu. in.}, \] to 2 figures.

Example 5. A flask when empty weighs 16-3 gm., when full of water weighs 61-7 gm., and when full of spirit weighs 52-6. Find, to 2 figures, the sp. gr. of the spirit.

The weight of the empty flask = 16-3 gm.,

\[ \therefore \text{the weight of the water in the flask} = (61-7 - 16-3) \text{ gm.} = 45-4 \text{ gm.;} \]
and the weight of the same volume of spirit = (52-6 - 16-3) gm. = 36-3 gm.

\[ \therefore \text{the ratio of the weight of a certain volume of spirit to} \]
the weight of the same volume of water
\[ = \frac{36-3}{45-4} = 0-7995. \]
\[ = 1-5599 \]
\[ \therefore \text{the sp. gr. of the spirit} = 0-80, \] to 2 figures.

Note. It was unnecessary to state the volume of the alcohol in the flask, because the specific gravity is the ratio of the weight of any volume to the weight of the same volume of water. A flask used in this way to find the specific gravity of a substance is called a specific gravity bottle.

DENSITY AND SPECIFIC GRAVITY

The approximate data contained in the following table should be used, where necessary, in Exercise 164:

<table>
<thead>
<tr>
<th>Specific Gravity Table</th>
</tr>
</thead>
<tbody>
<tr>
<td>Platinum, 21-2</td>
</tr>
<tr>
<td>Gold, 19-3</td>
</tr>
<tr>
<td>Mercury, 13-6</td>
</tr>
<tr>
<td>Lead, 11-4</td>
</tr>
<tr>
<td>Silver, 10-5</td>
</tr>
<tr>
<td>Copper, 8-85</td>
</tr>
</tbody>
</table>

1 cu. ft. of water weighs 62-3 lb.
1 gal. of water weighs 10 lb.

EXERCISE 164

[Give answers to 2 figures]

1. Find the weight of a rectangular block of wood, 8 in. by 6 in. by 5 in., if the density is 0-45 oz. per cu. in.
2. Find the weight of a rectangular block of stone, 40 cm. by 35 cm. by 3-6 cm.; sp. gr. 2-6.
3. A rectangular metal block, 4 cm. by 3 cm. by 2 cm., weighs 195-4 gm.; find the sp. gr. of the metal.
4. A cubic foot of a metal weighs 516-7 lb.; find its specific gravity.
5. A block of ebony weighs 62-4 gm.; find its volume in c.c.
6. A lump of gold weighs 38 oz.; find its volume in cu. in.
7. A metal bar, sp. gr. 7-18, has a circular cross-section of diameter 4-8 cm., and is 1 m. long. Find its weight.
8. A cylindrical wooden stick 91 cm. long is 1-8 cm. in diameter and weighs 170 gm. Find the sp. gr. of the wood.
9. A rectangular block of limestone, 23 ft. by 14 ft. by 10 ft., weighs 210 tons. Find its specific gravity.
10. A solid silver cone is 6 in. high and the diameter of its base is 3 in.; find its weight in oz.
11. The diameter of a lead sphere is 2-8 cm.; find its weight.
12. The outer diameter of a pipe is 5-6 cm. and its bore (i.e. internal diameter) is 4-9 cm. If the material is of sp. gr. 7-4, find the weight of a length of 10 m. of pipe.
A cylindrical tank, 10 ft. high, 6 ft. in diameter, inside measurements, is full of petrol. Find in lb. the weight of the petrol.

14. A cylindrical tube is 10 cm. long and holds 66 gm. of mercury when full. Find the internal diameter of the tube.

15. The external dimensions of a closed wooden box (sp. gr. 0.65) are as follows: length 3 ft., breadth 2 ft., depth 1 ft. 8 in. If the wood is $\frac{1}{4}$ in. thick, find in lb. the weight of the box.

16. A barometer tube 125 cm. long has internal diameter 1.2 cm.; find the cost of filling it with mercury at 10s. 6d. per kg.

17. Some metal shot weighing 48 gm. are dropped into a tall cylindrical glass, diameter 2 cm., half-full of water. If the water-level rises 1.4 cm., find the sp. gr. of the metal.

18. A flask weighs 23 gm. when empty, 141.7 gm. when full of water, and 102.6 gm. when full of another liquid. Find the sp. gr. of the liquid.

19. A bottle weighs 7 oz. when empty, 18 oz. when full of water, and 17.1 oz. when full of oil. Find the sp. gr. of the oil.

20. A flask weighs 15 gm. when empty and weighs 84.3 gm. when full of water. Find its weight when full of alcohol.

21. 20 equal ball-bearings and a small stoppered bottle full of water together weigh 53.62 gm. If the ball-bearings are placed inside the bottle which is then filled up with water, the weight is 49.87 gm. Find the volume of 1 ball-bearing.

22. If 8 c.c. of a liquid, sp. gr. 0.65, are mixed with 12 c.c. of a liquid, sp. gr. 0.85, find the sp. gr. of the mixture, assuming there is no chemical action.

23. The composition by weight of a mixture is 30% water, 70% alcohol. Find the sp. gr. of the mixture.

24. A bottle when empty weighs 32 gm. and when full of sulphuric acid weighs 115.6 gm. What will it weigh when full of alcohol?

25. An alloy is made of two metals, sp. gr. 18 and sp. gr. 10. Find its sp. gr. if (i) equal volumes, (ii) equal weights, of the two metals are used.

---

**PRINCIPLE OF ARCHIMEDES**

The Principle of Archimedes. It is assumed that the meaning of Archimedes' Principle will be demonstrated experimentally; the illustrative examples given below indicate its applications. The Principle may be stated as follows:

If a body is immersed, wholly or partially, in a fluid, the upward (vertical) thrust which the fluid exerts on the body is equal to the weight of the fluid displaced.

**Example 6.** A block of wood, sp. gr. 0.7, of volume 120 c.c. is floating in water. Find the volume of the submerged portion of the block.

The weight of water displaced = weight of the block

$=(120 \times 0.7 \text{ gm.}) = 84 \text{ gm.}$ ;

$\therefore$ the volume of the water displaced is 84 c.c.

$\therefore$ the volume of the submerged portion of the block is 84 c.c.

**Example 7.** A lump of metal weighs 73 gm. in air and appears to weigh 63.5 gm. in water and 64.7 gm. in another liquid. Find the sp. gr. of (i) the metal, (ii) the other liquid.

The upward thrust of the water on the lump

$=(73 - 63.5 \text{ gm.}) = 9.5 \text{ gm.}$ ;

$\therefore$ the weight of a volume of water equal to the volume of the lump is 9.5 gm.

$\therefore$ sp. gr. of metal = weight of any volume of metal

$\therefore$ sp. gr. of metal = weight of same volume of water

$73 \text{ gm.} \times \frac{1}{9.5} = 7.7$.

The upward thrust of the other fluid on the lump

$=(73 - 64.7 \text{ gm.}) = 8.3 \text{ gm.}$ ;

$\therefore$ the weight of a volume of the fluid equal to the volume of the lump is 8.3 gm; but the weight of the same volume of water is 9.5 gm.

$8.3 \text{ gm.} \times \frac{1}{9.5} = 0.87$.

**EXERCISE 165**

[Give answers to 2 figures; 1 cu. ft. of water weighs 62.3 lb.]

1. A block of wood floats in water so that the volume of the submerged portion is 36 c.c.; find the weight of the block.

2. A block of wood floats in a liquid of sp. gr. 0.8 so that the volume of the submerged portion is 55 c.c.; find the weight of the block.
3. A wooden cuboid, 5 cm. by 4 cm. by 3 cm., sp. gr. 0.65, is floating in water. Find the volume of water displaced.

4. A block of wood floats in water so that the volume of the submerged portion is 10 cu. in.; find the weight of the block.

5. A body is floating in a liquid of sp. gr. 1.35 and the volume of the portion submerged is 8.5 cu. in.; find the weight of the body.

6. A body of weight 100 lb. floats in water with 2/3 of its volume below the surface. Find the volume of the body in cu. ft.

7. A lead cuboid, sp. gr. 11.4, measures 2 cm. by 3 cm. by 4 cm. Find its weight (i) in air, (ii) in water, (iii) in glycerine, sp. gr. 1.25.

8. A lump of brass weighs 10 lb. Find its apparent weight in glycerine, sp. gr. 1.25, given that 1 cu. ft. of brass weighs 520 lb.

9. A lump of stone weighs 65 gm., and its apparent weight in water is 35 gm. Find (i) its volume, (ii) its sp. gr., (iii) its apparent weight in a liquid of sp. gr. 1.4.

10. A body weighs 157 gm. in air and 99 gm. in water. Find (i) its sp. gr., (ii) its apparent weight in a liquid of sp. gr. 0.8.

11. A body, sp. gr. 7.40, weighs 135 gm. in air. Find its weight (i) in water, (ii) in a liquid of sp. gr. 0.65.

12. Find the apparent weight in spirit, sp. gr. 0.78, of a copper cylinder 4.7 cm. long, 0.68 cm. in diameter, if the sp. gr. of the copper is 8.95.

CHAPTER XXII

COMPOUND INTEREST

The connection between simple and compound interest was explained in Chapter XV, see p. 208. The following examples indicate the method for computing the amount of a sum of money lent at compound interest. To obtain results correct to the nearest penny, it is necessary to keep 3 places of decimals, working in £, so that the final result can be written down correct to 3 places.
EXAMPLE 3. Find the compound interest on £287 15s. for 1½ yr. at 4½% per annum, interest being payable half-yearly.

\[
\begin{align*}
\text{£287}.75 & \quad \text{Princ. 1st period.} \\
5.7590 & \quad \text{Int. 2½%.} \\
\cdot71937 & \quad \text{Int. 1½%.} \\
\hline
294.22437 & \quad \text{Princ. 2nd period.} \\
5.89449 & \quad \text{Int. 2½%.} \\
\cdot73556 & \quad \text{Int. 1½%.} \\
300.84442 & \quad \text{Princ. 3rd period.} \\
6.01689 & \quad \text{Int. 2½%.} \\
\cdot75211 & \quad \text{Int. 1½%.} \\
307.61542 & \quad \text{Amount, 3 periods.} \\
287.75 & \quad \text{Deduct 1st Princ.} \\
19.86342 & \quad \text{Comp. Int.}
\end{align*}
\]

:. compound interest = £19.863, to 3 places = £19 17s. 3d., to nearest penny.

Note. As a rough check, it is advisable to note that the corresponding items of the interest increase steadily from period to period; a rough estimate is obtained by calculating the simple interest on £300 for 1½ years at 4½% p.a.

EXAMPLE 4. A man invests £6300 for 3 years at 4½% p.a. compound interest, compounded yearly. Income tax at 4s. in the £ is deducted at the end of each year. Find the amount at the end of the third year.

Allow for the income tax deduction by altering the rate %.

Nominal rate = 4½%:
but deduction = ½ of yearly interest;
:. net rate = ½ of 4½% = 3¾%.

We therefore calculate the compound interest at the rate of 3¾% p.a.; the reader should now use the ordinary method to show that the net amount after 3 years is £727 11s. 6d., to the nearest penny.

EXERCISE 166

[Give answers correct to the nearest penny]

Find the compound interest (payable yearly) on the following sums for the stated periods and rates of interest:

\begin{align*}
1. & \quad £240; 2 yr.; 4½%. & \quad [2] & \quad £302; 2 yr.; 3½%. \\
2. & \quad £362; 2 yr.; 5%. & \quad [4] & \quad £537; 2 yr.; 6%. \\
3. & \quad £328 10s.; 3 yr.; 6½%. & \quad [6] & \quad £271 4s. 6d.; 3 yr.; 4½%. \\
4. & \quad £108 8s. 7d.; 3 yr.; 5%. & \quad [8] & \quad £348 14s.; 2 yr.; 3½%. \\
5. & \quad £92 11s. 6d.; 2 yr.; 5½%. & \quad [10] & \quad £273 14s. 4d.; 2 yr.; 4½%. \\
6. & \quad £473; 2½ yr.; 6½%. & \quad [12] & \quad £547; 2½ yr.; 5½%. \\
7. & \quad £819; 2½ yr.; 7%. & \quad [14] & \quad £372; 2½ yr.; 6%. \\
\end{align*}

Find the amount at compound interest of:

\begin{align*}
15. & \quad £2004.25 for 2½ yr. at 3½%, payable yearly. \\
16. & \quad £927 7s. 6d. for 2½ yr. at 2½%, payable yearly. \\
17. & \quad £285 for 1 yr. at 5½% p.a., payable half-yearly. \\
18. & \quad £473 for 1½ yr. at 4½% p.a., payable half-yearly. \\
19. & \quad £639 for 1 yr. at 6½% p.a., payable quarterly. \\
20. & \quad £5720 for 1 yr. at 5½% p.a., payable quarterly.
\end{align*}
Find the difference between the simple and compound interest (payable yearly) on the following sums for the stated periods and rates of interest:

21. £350; 3 yr.; 3\%.

22. £220; 4 yr.; 5\%.

23. £195 6s.; 3 yr.; 4\%.

24. £407 14s.; 3 yr.; 4\%.

26. A district contains 64,000 inhabitants. If the population increases at the rate of 2\% per annum, find the number of inhabitants at the end of 3 yr.

27. A man invests £2500 at \(\frac{3}{4}\) p.a., compounded yearly. Income tax at 5\% in the £ is deducted at the end of each year. Find the amount at the end of the third year.

28. Find, as the decimal of £1, the compound interest on £1 for 3 yr. at 3\% p.a., payable yearly. Hence find what sum will amount to £5556 12s. in 3 yr. at 5\% p.a. compound interest.

29. Which is the better price for a property, £16,000 paid at once or £19,500 paid in 4 years' time, allowing a Compound interest?

30. One moneylender charges interest at 2\% per annum, payable monthly, and a second moneylender charges interest at 30\% per annum, payable every 2 months. If a man borrows £100 from each of them, how much will he owe at the end of 4 months? [Each month is reckoned as \(\frac{1}{12}\) of a year.]

31. A man borrowed £500 at 5\% p.a., the interest for any year being calculated on the total sum owing at the beginning of that year. He paid back £150 at the end of each of the first two years. What did he owe at the end of the third year?

32. Find, as the decimal of £1, the difference between the simple and compound interest on £1 for 2 yr. at 3\% payable yearly. Hence find the sum on which the difference between the simple and compound interest for 2 yr. at 3\% is £30 12s.

33. A man borrowed £650 on January 1, 1930, and repaid equal instalments of £243 on December 31 of each of the years 1930, 1931, 1932. Reckoning compound interest at 6\% p.a., find how much had still to be paid at the end of 1933.

---

**Example 6.** Find an expression for the amount of £250 at 3\% p.a. compound interest for 12 yr.

- £100 becomes £103 in 1 year's time,
- £1 becomes £1.03 in 1 year's time, that is, £1 has been made 1.03 times larger.
- £250 becomes £250 \(\times 1.03^n\) in 1 year's time.

If this sum of money remains at interest for another year, it is again made 1.03 times larger.

- In 2 years £250 becomes $250 \(\times 1.03 \times 1.03\), that is, £250(1.03)^2;
- and in 3 years £250 becomes $250(1.03^2) \times 1.03\), that is, £250(1.03)^3;
- and so on.

Thus in 12 years, £250 becomes £250(1.03)^12.

If this amount is evaluated by 4-figure tables, the value of 12 times log 1.03 will only be correct to 2 or 3 places of decimals. We therefore use 7-figure tables, which give (p. 342), log 1.03 = 0.0128372;

\[
\log 1.03 = 0.0128372
\]

\[
250 \times 12 \times 0.0128372 = 356.40
\]

But in this result, the number of shillings is not reliable.

### Compound Interest Table

<table>
<thead>
<tr>
<th>Number</th>
<th>Logarithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>250</td>
<td>2.979</td>
</tr>
<tr>
<td>(1.03)^12</td>
<td>0.1540</td>
</tr>
<tr>
<td>Amount</td>
<td>2.5519</td>
</tr>
</tbody>
</table>

If 7-figure tables are used throughout, we have

\[
\text{Amount} = 35640
\]

= £356 9s. to nearest shilling.

From the compound interest tables on p. 342, we have

\[
\text{Amount} = (250 \times 1.03^{12}) = 356.44 = £356 9s., \text{ to nearest shilling.}
\]

Similarly in 9 years, reckoning compound interest,

- at 3\% p.a., EP becomes £P(1.03)^9;
- at 4\% p.a., EP becomes £P(1.04)^9;
- at 4\% p.a., EP becomes £P(1.025)^9.
COMPOUND INTEREST

Oral Work. Read off expressions for what £400 becomes
(i) in 7 years at 2% p.a. compound interest;
(ii) in 8 years at 5% p.a. compound interest;
(iii) in 10 years at 3% p.a. compound interest;
(iv) in 15 years at 4% p.a. compound interest.

Hence we have the general formula:

The amount of £P for n years at r% p.a., compound interest, compounded yearly, is

\[ P(1 + \frac{r}{100})^n \]

This is called the compound interest law and applies to any quantity which increases or decreases so that the amount at the end of each period of constant length bears a constant ratio to the amount at the beginning of that period; this ratio is called the growth factor if it is greater than 1, and the decay factor if less than 1.

For example, if the population of a town increases steadily by 2% p.a. of the amount at the beginning of each year, the yearly growth factor is \((1 + \frac{0.02}{100}) = 1.02\), and the population after \(n\) yr. is \((1.02)^n\) times the population at the beginning of that period. Similarly, if the value of the machinery in a factory depreciates steadily by 8% p.a. of its value at the beginning of each year, the yearly decay factor is \((1 - \frac{0.08}{100}) = 0.92\), and the value after \(n\) yr. is \((0.92)^n\) times its value when new.

EXERCISE 167 (Oral)

1. Write down as a decimal the growth factor or multiplying factor for 1 year for the following annual percentage increases:
   - 4%; 3%; 2\(\frac{1}{2}\)%; 5\%; 10%; 3\(\frac{1}{2}\)%
2. Write down as a decimal the decay factor for 1 year for the following annual percentage decreases:
   - 5%; 4%; 3\(\frac{1}{2}\)%; 6\%; 10%; 12\(\frac{1}{2}\)%
3. Write down the rate per cent. increases which are equivalent to the following growth factors:
   - 1.02; 1.045; 1.0575; 1.2; 1.0325
4. Write down the rate per cent. decreases which are equivalent to the following decay factors:
   - 0.96; 0.88; 0.975; 0.75; 0.9825.

COMPOUND INTEREST FORMULA

If compound interest is reckoned at \(r\%\) p.a., compounded yearly, the annual multiplying factor is \(R\), where \(R = 1 + \frac{r}{100}\), and the amount of £P after \(n\) years may then be written \(£PR^n\).

Example 7. Find the sum of money which will amount to £3500 in 20 yr. at 4% p.a., compound interest.

If the sum of money is £P, the amount in 20 yr. at 4% p.a. is £P(1.04)^{20}\%

| \(P(1.04)^{20}\) = 3500 | \(P\) = 3500 / (1.04)^{20} ≈ 1597.
|-----------------|------------------|
| 3500 / 1597 | 320920
| the sum is approximately £1597, where the 4th figure is not reliable. | P = 320920 + 320920 = 15974.

Alternatively, from the compound interest table on p. 342, £1 amounts to £2.19112 in 20 yr. at 4% p.a.;

- the sum which amounts to £3500 = £3500 / £2.19112 = £15974.

Thus in this case the 4th figure obtained by using logarithms happens to be correct.

Example 8. At what rate per cent. p.a. will £480 amount to £677 in 10 yr. at compound interest?

If the rate \(\%\) p.a. is \(R\), the annual multiplying factor is \(1 + \frac{r}{100}\);

- if \(R = 1 + \frac{r}{100}\), the amount of £480 in 10 yr. is £480R^{10}, and this is £677.

\[ R^{10} = \frac{677}{480}; \: R = \left(\frac{677}{480}\right)^{\frac{1}{10}} = 1.035; \: \text{quotient} = 1.0149 \]

- the rate \(\%\) p.a. is 3\(\frac{1}{2}\)% approximately.

Example 9. In what time will £725 amount to £975 at 3\(\frac{1}{2}\)% p.a. compound interest?

The annual multiplying factor is 1.035, \(\therefore\) if £725 amounts to £975 in \(n\) yr.,

\[ 725(1.035)^n = 975; \]

- log 725 + log 1.035 = log 975;

\[ 2.8603 + n \times 0.01494 = 2.9890; \: n \times 0.01494 = 0.1287; \: n = \frac{01287}{0.01494} = 8.61; \]

- the time is approximately 8-6 yr.
Example 10. In what time will a sum of money double itself at 4% p.a. compound interest?

The annual multiplying factor is 1.04. \(\therefore\) if £1 amounts to £2 in \(n\) yr.,
\[
(1.04)^n = 2; \quad n \log 1.04 = \log 2; \quad 1.04 \times 0.01030 = 0.01010; \quad n = 0.01010 = 17.7; \quad \frac{17.7}{4} = 4.425.
\]
\(\therefore\) the time is approximately 17.7 yr.

Note. It can be proved by methods of advanced algebra that a sum of money at \(r\) per cent. per annum compound interest doubles itself in approximately \((\frac{70}{r})\) years.

Table for the amount of £1 at compound interest

<table>
<thead>
<tr>
<th>Year</th>
<th>2%</th>
<th>3%</th>
<th>4%</th>
<th>5%</th>
<th>6%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0200</td>
<td>1.0300</td>
<td>1.0400</td>
<td>1.0500</td>
<td>1.0600</td>
</tr>
<tr>
<td>2</td>
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<td>1.0609</td>
<td>1.0816</td>
<td>1.1025</td>
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<td>3</td>
<td>1.0616</td>
<td>1.0833</td>
<td>1.1052</td>
<td>1.1279</td>
<td>1.1511</td>
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<tr>
<td>4</td>
<td>1.0832</td>
<td>1.1055</td>
<td>1.1282</td>
<td>1.1518</td>
<td>1.1757</td>
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<td>5</td>
<td>1.1053</td>
<td>1.1282</td>
<td>1.1516</td>
<td>1.1759</td>
<td>1.2018</td>
</tr>
<tr>
<td>6</td>
<td>1.1283</td>
<td>1.1518</td>
<td>1.1764</td>
<td>1.2020</td>
<td>1.2285</td>
</tr>
<tr>
<td>7</td>
<td>1.1518</td>
<td>1.1764</td>
<td>1.2020</td>
<td>1.2285</td>
<td>1.2553</td>
</tr>
<tr>
<td>8</td>
<td>1.1760</td>
<td>1.2020</td>
<td>1.2285</td>
<td>1.2553</td>
<td>1.2825</td>
</tr>
<tr>
<td>9</td>
<td>1.2018</td>
<td>1.2285</td>
<td>1.2553</td>
<td>1.2825</td>
<td>1.3100</td>
</tr>
<tr>
<td>10</td>
<td>1.2285</td>
<td>1.2553</td>
<td>1.2825</td>
<td>1.3100</td>
<td>1.3379</td>
</tr>
</tbody>
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The following 7-figure logarithms are supplied here for use in Exercise 168:

<table>
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<th>No.</th>
<th>Log.</th>
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</thead>
<tbody>
<tr>
<td>1.025</td>
<td>-0107239</td>
</tr>
<tr>
<td>1.03</td>
<td>-0128372</td>
</tr>
</tbody>
</table>

Find (i) by logarithms, (ii) by using the compound interest table, the approximate amounts of the following sums, at compound interest for the stated periods and rates:

1. £100; 8 yr.; 3%.
2. £500; 12 yr.; 5%.
3. £650; 20 yr.; 4%.
4. £374; 11 yr.; 3\%\%.
5. £1250; 10 yr.; 5%.
6. £9275; 7 yr.; 2\%\%.

Find by logarithms the approximate amounts of the following sums, at compound interest for the stated periods and rates:

7. £724; 15 yr.; 4%.
8. £3050; 24 yr.; 3\%\%.

Find by logarithms the approximate value of the compound interest on the following sums for the stated periods and rates:

9. £638; 11 yr.; 3\%\%.
10. £590; 14 yr.; 2\%\%.

Find by logarithms the approximate sum of money which amounts at compound interest to:

11. £850 in 9 yr. at 4%.
12. £640 in 14 yr. at 5%.
13. £3000 in 16 yr. at 2\%\%.
14. £4075 in 25 yr. at 6%.

Find by logarithms the approximate rate per cent. if:

15. £600 amounts to £821 in 8 yr., compound interest.
16. £140 amounts to £271 in 15 yr., compound interest.
17. £360 amounts to £8000 in 16 yr., compound interest.

In what time, at compound interest, will:

18. £320 amount to £405 at 4\% p.a.?
19. £437 amount to £845 at 3\%\% p.a.?
20. Any sum of money double itself at 5\% p.a.?

*21. A Savings Certificate costs 16s. and is worth 24s. at the end of 10 yr. What rate \(r\) p.a. compound interest is allowed? How long would it take 16s. to increase to £2 at the same rate?

*22. The value of port wine increases by 5% every year at a compounded rate. If its value is £2 10s. a dozen when laid down, find its value 15 yr. later.
28. The value of a machine depreciates each year by 10% of its value at the beginning of that year. Its value when new is £750; find its value when it is 8 yr. old.

*24. Use the compound interest table on p. 342 to draw graphs showing the amount of £100 at any time in the first 12 yr. at (i) \(2\frac{1}{2}\) p.a., (ii) 5% p.a. Draw on the same diagram the corresponding simple interest graphs.

25. The value of a machine depreciates from £1800 to £680 in 12 yr. Find the yearly decay factor, assuming it to be constant.

*26. The population of a town increased from 37,526 in 1911 to 61,473 in 1931. Find the yearly growth factor assuming it to be constant. If the growth factor has been constant, what was the population in 1901?

CHAPTER XXIII

HARDER PERCENTAGE

Practice in the use of percentage factors is desirable.

Example 1. The sales of a book in 1931 exceeded those in 1930 by 10%, and the corresponding successive yearly percentage increases for 1932 and 1933 were 15% and 20%. By what percentage did the sales in 1933 exceed those in 1930?

If \(C\) copies were sold in 1930, and if \(C_1, C_2, C_3\) copies were sold in 1931, 1932, 1933,

\[ C_1 = C \times 1\frac{1}{10}; \]
\[ C_2 = C_1 \times 1\frac{1}{10} = C \times 1\frac{1}{10} \times 1\frac{1}{10}; \]
\[ C_3 = C_2 \times 1\frac{1}{2} = C \times 1\frac{1}{10} \times 1\frac{1}{2}; \]
\[ C_3 = C \times 1\frac{1}{8}; \]
\[ \therefore C_3 \text{ exceeds } C \text{ by } 51\frac{1}{8}%. \]

\(C_3\) exceeds those in 1930 by 51\%.

In actual practice, when there are several multiplying factors, the intermediate steps may be omitted. But it must be noted that the successive percentages cannot be added together; they are not percentages of the same thing.

Example 2. If a dealer charges £2 15s. for a chair, he gains 37\% at what price must he sell it to gain 45\%?

\[
\begin{array}{c|c|c}
\text{C.P.} & \text{1st S.P.} & \text{2nd S.P.} \\
100 & 137\frac{1}{2} & 145 \\
\hline
\end{array}
\]

\[
\therefore 2\text{nd S.P.} = \frac{145}{137\frac{1}{2}} \text{ of 1st S.P.} = \frac{290}{275} \text{ of 55s. = 58s.}
\]

\(\therefore\) the chair must be sold for £2 18s. to gain 45\%.

Example 3. A tradesman marks an article at 40\% above cost price, but gives a discount of 1s. in the £ on the marked price. What is his gain per cent.? He also offers 21 of the articles for the price of 20, allowing the same discount. Find his gain per cent. in this case.

\[
\begin{array}{c|c|c|c|c}
\text{C.P.} & \text{Marked Price} & \text{Discount} & \text{Net S.P.} \\
£100 & £140 & 140s. or £7 & £133 \\
\hline
\end{array}
\]

\(\therefore\) the tradesman gains 33\%.

(ii) When he sells 21 articles for the price of 20 articles, if the cost price is £(100 \times 21), the net sale price is £(133 \times 20),

\(\therefore\) if the cost price is £100, the net sale price is £133\frac{1}{3}, or £126\frac{2}{3};

\(\therefore\) the tradesman gains 26\%.

Example 4. When the cost of coal increases by 40\%, a man reduces his annual consumption by 20\%. Find the percentage change in his annual expenditure on coal.

First expenditure:
100 tons at 100 units of money per ton cost 10,000 units of money.

Second expenditure:
80 tons at 140 units of money per ton cost 11,200 units of money.

\(\therefore\) expenditure increases by \(\frac{1,200}{10,000} \times 100\) per cent., i.e. by 12\%. 

Example 5. A sold a car to B at a profit of 40%, B sold it to C at a profit of 60%, C sold it to D at a loss of 25%. If D paid £187 more than it cost A, find the profit A made.

Start with a car which cost A £100, then

<table>
<thead>
<tr>
<th>B's C.P.</th>
<th>C's C.P.</th>
<th>D's C.P.</th>
</tr>
</thead>
<tbody>
<tr>
<td>£140</td>
<td>£18 &amp; 80</td>
<td>£18 &amp; 30</td>
</tr>
</tbody>
</table>

\[ \text{D paid } \frac{18 & 30 - 140}{100} \times 140, \text{ i.e. } £168. \]

\[ \therefore \text{D paid } £68 \text{ more than A's C.P. if this was } £100, \]

\[ \therefore \text{A's profit was } \frac{68}{100} \times 140, \text{ that is } £110. \]

EXERCISE 169

1. A makes a car for £240 and sells it to B at a profit of 25%; B sells it to C at a profit of 30%. How much does C pay for it?

2. The value of a machine is £5000 when new, and depreciates each year by 10% of its value at the beginning of that year. Find its value when 3 years old.

3. If a man sells his house for £4500 he loses £300. What must he sell it for to gain 20%?

4. If a dealer sold a table for £14, he would gain 12%. Find his gain per cent. if he sells it for £15.

5. If a man sells a chair for 27s., he gains 8%. Find his loss per cent. if he sells it for 22s.

6. If a man sells a horse for 100 guineas, he gains 12 & & 1/2%. What profit or loss per cent. would he have made if he had sold it for 90 guineas?

7. By selling an article for 38s., a man loses 5%; for what must he sell it to gain 10%?

8. By selling an article for 29 guineas a man loses 13%; for what must he sell it to make a profit of 15%?

9. The length of a rod when cut down by 10% is 3 ft. What would its length have become if it had been cut down by 20%?

10. After the price of an article is reduced from 1s. to 6d., the seller still makes a profit of 30%. What profit per cent. did he make at first?

11. A merchant gains 15% by selling an article for £8 12s. 6d.; what must he sell it for to double his profit?
25. The export of cotton piece goods in 1920 exceeded the export in 1919 by 12% in quantity and by 43% in value. Find the average percentage increase in price of a given quantity of the goods.

26. If the price of petrol rises 20%, by how much per cent. must a motorist reduce his consumption so as not to increase his expenditure?

27. A man saves 20% of his income. If his expenditure increases by 35%, how much per cent. must his income be increased so that he may save 10% of it?

28. A manufacturer catalogues his goods at 80% above cost of production. He sells to a distributor allowing him 40% discount off catalogue prices, and the distributor allows the retailer 25% off catalogue prices. What is the percentage profit made by (i) the manufacturer, (ii) the distributor?

29. When the price of coal rose 20%, a householder reduced his annual consumption by 20%; his yearly expenditure then became £36, what was it before?

30. The cost of producing an article is made up as follows: cost of labour 65%, cost of material 35%. It is sold at a profit of 50%. If the cost of labour increases by 20%, and if the selling price is increased by 30%, the profit is still 50%; find the percentage change in the cost of material.

Mixtures. Inverse problems on mixtures were discussed in Chapter XVII, see p. 240; some rather harder examples (direct and inverse) are given in Exercise 170.

Example 6. In what ratio must tea at 1s. 9d. per lb. be mixed with tea at 2s. 7d. per lb. so that a profit of 36% is made by selling the tea at 2s. 10d. per lb.?

\[
2s. 10d. = \frac{180}{100} \text{ of cost price of 1 lb.}
\]

\[
\therefore \text{cost price of 1 lb.} = \frac{100}{180} \times 34d., \text{that is 25d.}
\]

\[
\therefore \text{tea at 21d. per lb. is mixed with tea at 31d. per lb. so that the mixture is worth 25d. per lb.}
\]

On every 1 lb. of tea at 21d., the gain is (25 - 21)d. = 4d.

On every 1 lb. of tea at 31d., the loss is (31 - 25)d. = 6d.

\[
\therefore \text{gains and losses balance if 3 lb. of tea at 21d. per lb. are mixed with 2 lb. of tea at 31d. per lb.}
\]

\[
\therefore \text{the tea at 1s. 9d. per lb. must be mixed with the tea at 2s. 7d. per lb. in the ratio 3:2.}
\]
12. The composition by weight of two alloys for railway-carriage
bearings is as follows:—

  English: copper 22%, tin 67%, antimony 11%.
  French: copper 82%, tin 18%.

If the English and French alloys are mixed in the ratio 3:1 by
weight, what percentage of copper is there in the mixture?

13. The values of 3 articles are as 5:6:7; the third remains
constant, but the first two increase so that the values become as
7:6:5. Find the increase per cent. in value of the first two.

14. In a battle the casualties were 16% of the total strength.
35% of the officers and 15% of other ranks were casualties.
Find the ratio of the number of officers to the number of other
ranks.

15. In what ratio must coffee costing £9 6s. 8d. per cwt. be mixed
with chicory costing £3 5s. 4d. per cwt. so as to make $12\frac{1}{2}$% profit
by selling the mixture at 1s. 6d. per lb.?

16. At a modern school 90% of the pupils learn French, 85% of
them learn German, and 65% of them learn Spanish. What is the
least percentage that learn all three languages?

17. A, B, C invest £8000, £4000, £2000 respectively in a business.
A and B receive respectively 20% and 10% of the annual profits as
salaries, and the remainder is shared between A, B, C in proportion
to the capitals they invested. If the profits are £3200, what does
each receive?

18. A tradesman buys 200 lb. of fruit; 10% of it is unsaleable,
and the rest is sold at 1s. per lb., but in order to "turn the scales"
he has to put 16\% oz. instead of 1 lb. into the scale-pan. If he
makes a profit of 15%, what did he pay altogether for the fruit to
the nearest shilling?

19. Five men agree to complete a piece of work in 48 days work-
ing 8 hr. a day. One ceased work at the end of 12 days and a
second at the end of 15 days. The rest then agreed to work 9 hr.
a day. By what percentage (to the nearest unit) must they increase
their rate of work to finish in the specified time?

20. A box when empty weighs 10 lb. It will hold 50 lb. of
clothes or 180 lb. of books. What will it weigh when packed full of
clothes and books if (i) 20% of the volume is occupied by books and
the rest by clothes, (ii) 20% of the weight of the contents is made up
by books and the rest by clothes?
SHARES AND STOCKS

A man can buy or sell shares just as he can buy or sell eggs or motor-cars, but he can usually only buy them through a stockbroker, and the price may alter from day to day according to the demand for the shares. When a man has bought some shares of a company, his name is registered as the owner of those shares, which are numbered just like Treasury notes or cloak-room tickets, and he receives a piece of paper from the company recording the fact that he owns certain shares of specified numbers; this piece of paper is called a share certificate, and is the only thing which is given to him to show that the shares belong to him; and when he wishes to sell the shares, he merely sends this share certificate to a stockbroker, who will then sell the shares for whatever value they may have at the time. Shares are bought either for income or for capital appreciation (i.e., increase in value due to a rise in their price). And shares are sold either because the money is required for some purpose or because it is believed that the money can be invested to greater advantage elsewhere.

A share is said to stand at a premium if its cash value is greater than its nominal value. Thus Woolworth's 6% (£1) preference shares at 10s. stand at "10s. premium," because the cash value exceeds the nominal value by 10s.

A share is said to stand at par, if its cash value is the same as its nominal value; that is, for example, if the cash value of a (10s.) share is 10s.

A share is said to stand at a discount, if its cash value is less than its nominal value; for example, when Moss' Empires 5% (£1) preference shares were standing at 13s. 3d., they were at 6s. 9d. discount.

If the price of a share is quoted in the form, "at 1g." this means that the price is £1 1/4: if the price is quoted in shillings or pence; this is stated explicitly. A quotation such as 12s., to 12s. 3d. means that a seller receives 12s. per share and that a buyer pays 12s. 3d. per share.

Questions about shares are answered merely by using unitary method or proportion, and present no difficulty if the meaning of the data is understood.

The phrase, a (5s.) share at 4s. paying a dividend of 4% p.a., means that
(i) the cash value of 1 (5s.) share is 4s.
(ii) the yearly interest on 1 (5s.) share is 41/2 of 5s.
(iii) an investment of 4s. cash yields 41/2 of 5s. yearly interest.
SHARES AND STOCKS

Write down the sum of money obtained by selling:
5. 40 (£1) shares at 12s.
6. 1000 (10s.) shares at 8s.

Write down the price of the following shares:
7. 100 (10s.) shares cost £20.
8. 60 (5s.) shares cost £30.

Write down the yield obtained by investing in:
9. (£1) shares at 2, paying 8% p.a.
10. (10s.) shares at 5s., paying 3% p.a.

EXERCISE 172

Find the cost of and the income from the following preference shares:
1. 60 Boots 7% (£1) shares at 27s.
2. 120 Lyons 5% (£1) shares at 25s. 3d.
3. 150 Hodsmans 8% (10s.) shares at 13s. 6d.
4. 50 Guest Keen 5% (£5) shares at 6s.
5. 180 London Tin 7½% (10s.) shares at 14s. 2d.

Find the number of shares that can be bought and the income obtained by investing:
6. £50 in (£1) shares at 25s., paying 8%.
7. £72 in (5s.) shares at 9s., paying 12%.
8. £210 in (2s.) shares at 1s. 6d., paying 3⅕%.
9. £1350 in (£1) shares at 2½, paying 12%.
10. £240 in (£5) shares at 8, paying 9%.

Find the sum of money obtained by selling:
11. 100 Dunlop 7½% (£1) shares at 31s.
12. 250 Trust Houses 6½% (£1) shares at 1½.
13. 420 Polikoff 8% (10s.) shares at 8s. 9d.
14. 160 United Tea 5% (£5) shares at 3⅚.
15. 120 Lawes 7½% (10s.) shares at 7s. 9d.

SHARE TRANSACTIONS

Find the yield obtained by investing money in:
16. (£1) shares at 1½, paying 9%.
17. (10s.) shares at 8s., paying 4%.
18. (2s.) shares at 3s. 9d., paying 12½%.
19. (£5) shares at 8½, paying 10½%.
20. (5s.) shares at 84s., paying 120%.

21. A man bought 160 (5s.) shares for £34. At what price did the shares stand? At what premium or discount were they quoted?
22. A man sold 600 (10s.) shares for £515. At what price did the shares stand? At what premium or discount were they quoted?
23. A man who owns 250 (£1) shares receives from them a dividend of £20. What is the rate % of the dividend?
24. A man who owns 1400 (10s.) shares receives from them a dividend of £42. What is the rate % of the dividend?
25. By investing £360 in (£1) shares paying 3%, a man obtained a dividend of £12; at what price did the shares stand?
26. By investing £700 in (10s.) shares paying 8%, a man obtained a dividend of £40; at what price did the shares stand?
27. By investing £600 in (5s.) shares paying 9%, a man obtained a dividend of £33 15s.; at what price did the shares stand?
28. London Brick 8½% (£1) shares stand at 36s., and Rio Tinto 5% (£5) shares stand at 4½. Which investment gives the larger yield?

Share Transactions. A stockbroker is merely a commission-agent who arranges to buy and sell shares for his clients.

For arranging the transaction, the stockbroker charges his client a commission, called brokerage, which varies according to the price of the share, the number of shares involved, and other considerations.

If the price of a (£1) share is, say, 22s. 8d., and if the broker's commission is 2d. a share, a man who is buying the shares pays (22s. 8d. + 2d.), that is 22s. 10d., for each share; but a man who is selling the shares receives only (22s. 8d. - 2d.), that is 22s. 6d., for each share.
SHARES AND STOCKS

There are also other charges which affect the transaction, viz. a transfer duty which is a Government tax, the stamp on the contract, and the registration fee charged by the company; but these charges will be ignored in the Exercises in this book unless specially mentioned. Further, it will be assumed that the brokerage has been included in the named prices of the shares unless the contrary is stated.

Example 4. A man sells 720 El Oro mining (5s.) shares at 6s. 3d. and invests the proceeds in Odmans Press (4s.) shares at 11s. 3d. How many Odmans Press shares does he buy?

1 El Oro share is sold for 6½s. cash;
∴ 720 El Oro shares are sold for (6½ × 720)s. cash.
∴ he invests (6½×720)s. cash in O.P. shares at 11½s.

4½s. cash buy 1 O.P. share,

∴ (4½×720)s. cash buy 4½ × 720 O.P. shares, =400 shares;
∴ he buys 400 Odmans Press shares.

Example 5. A man sells 360 Grocery (10s.) shares paying 12% at 21s., and invests the proceeds in Tin (5s.) shares paying 4½% at 3s. 6d. Find the change in his income.

1st Income: The interest on 1 Grocery (10s.) share is 1/80 of 10s.;
∴ the interest on 360 Grocery (10s.) shares is (360 × 1/80 × 10 × 360)s., =432s.

Selling out: He sells 1 Grocery (10s.) share for 21s. cash;
∴ he sells 360 Grocery (10s.) shares for 21 × 360s. cash.
∴ he invests (21 × 360)s. cash in (5s.) Tin shares at 3½s., paying 4½%.

2nd Income: For each (5s.) Tin share he buys, he pays 3½s. cash and receives 4½ of 5s. interest;
∴ if he invests 2½s. cash, he receives (2½ × 5)s. interest.
∴ if he invests (21 × 360)s. cash, he receives (2½ × 5 × 21 × 360)s. interest, that is 459s. interest.
∴ his income is increased by (459 – 432)s., that is £1 7s.

Note. This example shows that it is not necessary to find the number of shares bought in the second transaction, if we merely wish to know the change of income.

SHARE TRANSACTIONS

Example 6. A man sells 180 Tarmac (£1) shares at 22s. 6d. and then buys 200 Hotel (10s.) shares at 18s. 8d. Brokerage is charged at the rate of 2d. per share on the sale and at 1d. per share on the reinvestment. Find the cash balance due to him.

1 Tarmac share is sold for 22s. 6d., but the brokerage charge is 2d.,∴ net receipt from sale of 1 Tarmac share is 22s. 4d.
∴ net receipt from sale of 180 shares is (22s. 4d.) × 180, =£201.

1 Hotel share costs 18s. 8d., but there is also a brokerage charge of 1d.
∴ net cost of 1 Hotel share is 18s. 9d.
∴ net cost of 200 Hotel shares is (18s. 9d.) × 200, =£187 10s.
∴ cash balance due to man is £201 – £187 10s., =£13 10s.

Note. When shares are sold and the proceeds are reinvested, the brokerage on the reinvestment is charged at half the ordinary rate.

EXERCISE 173

1. A man sells 700 Ciro Pearls (5s.) shares at 4s. 6d. and invests the proceeds in Gamage (10s.) shares at 17s. 6d. How many Gamage shares does he buy?

2. A man sells 450 Gaumont British (10s.) shares at 13s. 4d. and invests the proceeds in Sears (5s.) shares at 15s. How many Sears shares does he buy?

3. A man sells 550 Humber (10s.) shares at 16s. and with the proceeds can just buy 600 Celanese (10s.) shares. At what price do the Celanese shares stand?

4. A man sells 150 Ford Motor (£1) shares at 29s. 4d. and with the proceeds can just buy 1600 Maypole (2s.) shares. At what price do the Maypole shares stand?

5. A man bought 450 (10s.) shares at 6s. 6d. and sold them when they had risen to 8s. 4d. How much did he gain?

6. A man bought 320 (£1) shares at 1½ and sold them when they had fallen to 9/16. How much did he lose?

7. A man invested £330 in Bradford (£1) shares at 13s. 9d. and sold them at 15s. 3d. How many shares did he buy and how much profit did he make?

8. A man invested £765 in Geevor (5s.) shares at 14s. 2d.; he received one dividend of 20%, and then sold the shares at 13s. 9d. How much did he gain or lose?
9. A man sold 500 (£1) shares, paying 5% at 18s. and invested the proceeds in (10s.) shares, paying 7½%, at 12s. 6d. How many (10s.) shares did he buy and what was the change of income?

[10] A man sold 420 (£5) shares, paying 7%, at 8½ and invested the proceeds in (10s.) shares, paying 9¾%, at 24s. 6d. How many (10s.) shares did he buy and what was the change of income?

11. A man invested £900 in (£1) shares, paying 10¾%, at 27s. 6d.; he sold the shares at 32s. and invested the proceeds in (10s.) shares, paying 5½%, at 9s. How many (10s.) shares did he buy and what was the change of income?

*12. A man bought some (£1) shares at 15s. 7½d., and after 4 months sells them at 16s. 1d. Express the profit from the transaction as a rate per cent. per annum of the money invested.

*13. A man bought 25 (£1) shares at 4½; he received a dividend of 9s. 6d. per share and a bonus of one additional share (on which no dividend was paid) for every five shares held by him. He then sold all his shares at 4½. Find the profit (including the dividend) he made by the transaction and express it as a percentage of the money he invested, to 2 figures.

*14. The year's profits of a company are £35,250. From this amount, interest is first paid at 5½% on 150,000 (£1) preference shares, and then £10,000 is set aside as a reserve. The remainder is distributed among the 200,000 (£1) ordinary shares of the company. What dividend is paid on the ordinary shares?

*15. A company has a nominal capital of 12,000 (£10) shares, all issued. Its profits for the year are £15,000, of which £6000 is set aside as a reserve and the remainder is distributed among the shareholders. What dividend is paid on the shares? What percentage will a man obtain on his money who had bought 30 (£10) shares for £450?

Find, allowing for brokerage, (i) the cost of buying, (ii) the proceeds from selling:

16. 150 (£1) shares at 18s. 6d., brokerage 1d. per share.

[17] 240 (10s.) shares at 8s. 3d., brokerage 1d. per share.

18. 1800 (2s.) shares quoted at 1s. 7d. to 1s. 8d., brokerage 1d. per share.

[19] 720 (5s.) shares quoted at 11s. 3d. to 11s. 6d., brokerage 1d. per share.

20. A man sells 360 (10s.) shares at 11s. 6d. and buys 150 (£1) shares at 27s. 4d., brokerage being 1d. per share on each transaction. How much cash does he have to pay to balance the account?

[21] A man sells 120 (£1) shares at 28s. 2d. and buys 150 (£1) shares at 22s. 3d., brokerage being 2d. per share on the sale and 1d. per share on the reinvestment. What cash balance is due to the man?

**Stocks.** It is often necessary for Governments, City Corporations, Railways, Public Utility concerns, etc., to borrow large sums of money either to repay old loans or to develop new enterprises, and such sums are usually raised by an Issue of Stock redeemable within a fixed period and bearing interest at a fixed rate per annum. For example: In April 1934 the Liverpool Corporation raised a loan by an Issue of £4,000,000 Stock. This Stock could be bought in units of £100, and entitled the holders to £3 per cent. interest a year (payable half-yearly) on the amount of their holding. The Corporation promised to redeem the loan at par in 1964, and claimed the right to redeem at par any time after 1954 if they should wish to do so.

The price at which each unit of £100 Stock was offered for sale was, however, £96 10s. cash. These facts are expressed shortly by the phrase:

Liverpool 3% Stock, 1954/1964 at 96½.

The whole Loan was called an Issue of £4,000,000 Stock, for which the public paid £3,860,000 cash (i.e. £96 10s. for every £100 Stock), and will eventually receive in repayment £4,000,000 cash.

One important difference between Stock and Shares is that whereas Shares must always be bought and sold in whole numbers, Stock, once it has been issued, can in many cases be bought and sold in fractional amounts. In such cases, half of £100 Stock is called £50 Stock and costs half the price of £100 Stock; one-third of £100 Stock is called £33 6s. 8d. Stock and costs one-third of the price of £100 Stock, and so on.

In most respects calculations involving £100 Stock are the same as for 100 (£1) Shares. There is no more difficulty in working out questions about Stocks than there is about Shares, provided only the meaning of the description of a stock is clearly understood.
The phrase "Liverpool 3½% stock at 96½" means

(i) £100 Liverpool stock costs £96½ cash;  
(ii) The holder of £100 Liverpool stock receives £3 cash, yearly interest;  
(iii) £96½ cash invested in Liverpool stock yields £3 cash, yearly interest;  

or we may say that, for the Liverpool 3½% stock,

(i) The cash value of £100 stock is £96½;  
(ii) The yearly interest on £100 stock is £3 cash;  
(iii) An investment of £96½ cash yields £3 cash yearly interest.

Further, the phrase "£500 Liverpool 3½% stock" means £500 stock of the Liverpool 3½% loan; its value is not £500 cash, but, if the stock stands at 96½, is £(96½ × 5) cash.

It is important to insert the word stock where it is implied, and the word cash where money is involved, although the form of a statement often makes the distinction between cash and stock clear. Thus, in the statement "a man invests £300 in a 5% stock," the word invests makes it obvious that the £300 is cash and not stock; and in the statement that "a man sells £400 of a 3½% stock," the word sells makes it obvious that the £400 is stock and not cash. The phrase "£600 Belgium 7½% stock," where the £600 is stock and not cash, is often expressed in the form "£600 of a 7½% stock," and it must be realised that in this form of the phrase the £600 is stock, not cash.

The procedure for buying and selling stock is exactly the same as for shares, and the charges on the transaction, viz. the broker's commission called brokerage, the Government tax, stamps, and transfer expenses, operate as for shares.

If the price of a stock is 85, and if the brokerage is "½ per cent.," a man who buys £100 stock pays £85 cash for the stock and £4 for the broker's commission, and therefore £100 stock costs him altogether £89½ cash; while a man who sells £100 stock obtains £85 cash for the stock, but is charged £4 for brokerage, and therefore receives only £81½ cash from the sale of £100 stock.

A quotation such as 41½–41⅞ for a stock means that a man who is buying £100 stock has to pay £41½ cash for it, but that a man who is selling £100 stock receives only £41⅞ cash for it. If brokerage is ½ per cent., a buyer pays altogether £(41½ + ½) cash for £100 stock, and a seller receives altogether £(41⅞ – ½) cash for £100 stock, disregarding stamp duty, etc.

Example 7. Find the cost of £1500 War Loan 3½% stock at 104.  
£100 War Loan stock costs £104 cash;  
∴ £1500 War Loan stock costs £(104 × 15) cash = £1560 cash.  
(Note. War Loan 3½% stock is the name of the stock. In order to find out how much £1500 stock costs, it is unnecessary to be told that it is a 3½% stock; that fact merely enables us to find the income obtained from any amount of the stock.

Example 8. A man invests £450 in Indian 2½% stock at 69. Find, to the nearest penny, how much stock he buys and the income from it.

(i) £69 cash buys £100 stock,  
∴ £450 cash buys £(100 × 450069) stock, = £652-174 stock;  
∴ he buys £652 3s. 6d. Indian 2½% stock.

(ii) £100 stock yields £2⅞ cash, income,  
∴ £652-174 stock yields £(2⅞ × 652174) income;  
∴ the income = £16 204, cash, = £16 6s. 1d., to nearest penny.  
(Alternatively, the income may be found as follows:—)  
£69 cash buys £100 stock on which the income is £2⅞ cash;  
∴ £69 cash invested yields £2⅞ cash, income,  
∴ £450 cash invested yields £(2⅞ × 450) income;  
∴ the income = £210 = £16 6s. 1d., to nearest penny.

This example refers to a stock of which it is possible to buy or sell any fraction of £100 stock; the amount purchased was £652-174 stock, approximately; but this is expressed in the form £652 3s. 6d. stock, obtained by reducing £652-174 to shillings and pence.

Example 9. A man holds £87 13s. 10d. Local Loans 3½% stock. What is his net income from it, after income tax at 5s. in the £ has been deducted. If he sells the stock at 91, what does he receive?

(i) 13s. 10d. = £0-6917...;  
∴ he holds £87-6917 stock.  
£100 stock yields £3 cash, gross income; but ½ of £3 is deducted for income tax;  
∴ £100 stock yields £2 ½ of £3 cash, net income;  
∴ £87-6917 stock yields £(2 ½ × 87-6917) cash, net income;  
∴ his net income = £1 973, to 3 places of decimals;  
= £1 973, to nearest penny.  

(ii) £100 stock is sold for £91 cash,  
∴ £87-6917 stock is sold for £(91 × £0-6917) cash,  
∴ he receives £79-799 cash, = £79 16s. 6d. cash, to nearest penny.
SHARES AND STOCKS

Example 10. A man buys Japan 5% stock at 80. Find the yield on his investment.

£80 cash buys £100 stock on which the income is £5 cash.
\[ \therefore \text{ £80 cash invested yields £5 cash, income;} \]
\[ \therefore \text{ £100 cash invested yields } (\frac{5 \times \text{80}}{\text{100}}) \text{ cash, } = \text{ £6} \frac{1}{2} \text{ cash, income.} \]
\[ \therefore \text{ the yield on the investment is } 6\frac{1}{2} \text{ per cent.} \]

Note. The yield of a stock is often expressed in the form, £ s. d. per cent.; here it could be written 6 5s. per cent.

Example 11. By investing £345 in a 4½% stock, a man obtains an income of £15. Find the price of the stock (that is, the cost of £100 stock).

\[ \frac{4\frac{1}{2}}{100} \text{ is the income on £100 stock,} \]
\[ \therefore £15 \text{ is the income on } \left( \frac{100 \times 15}{4\frac{1}{2}} \right) \text{ stock; } \]
\[ \therefore \left( \frac{100 \times 15}{4\frac{1}{2}} \right) \text{ stock costs £345 cash,} \]
\[ \therefore £100 \text{ stock costs } \left( \frac{345 \times 4\frac{1}{2}}{15} \right) \text{ cash, } = £103\frac{1}{2} \text{ cash.} \]
\[ \therefore \text{ the price of the stock is } 103\frac{1}{2}. \]

EXERCISE 174 (Oral)

Write down the cost of and the income derived from
1. £300 of 4% stock at 80.
2. £200 of 6% stock at 120.

How much stock can be bought, and what is the income obtained, by investing
8. £240 in 7% stock at 120?
4. £300 in 3% stock at 60?

Write down the amount of the proceeds from selling
5. £500 of 4% stock at 90.
6. £300 of 5½% stock at 108.

Write down the prices of the following stocks:
7. £400 of 3% stock costs £320.
8. £1000 of 6% stock costs £1100.

Write down the yield obtained by investing in
9. 3% stock at 50.
10. 6% stock at 120.

STOCKS

EXERCISE 175

[Give answers correct to the nearest penny]

Find the cost of and the income derived from the following:
1. £300 Consols 4% stock at 112.
2. £250 Portugal 3½% stock at 72.
3. £825 Italian 3¾% stock at 36.
4. £560 New Zealand 6% stock at 107.
5. £145 7s. 6d. Indian 3% stock at 80.

How much stock can be bought, and what is the net income obtained, after deduction of income tax at 4s. in the £, by investing:
6. £1000 in Nigeria 6% stock at 125?
7. £600 in Irish Land 3% stock at 90?
8. £1400 in Belgium 7½% stock at 112?
9. £285 in Port of London 3½% stock at 89?
10. £420 in Canada 2¼% stock at 92½?

Find the proceeds obtained by selling:
11. £750 Japan 6½% stock at 90.
12. £1400 Peru 6½% stock at 18.
13. £427 10s. Consols 2¼% stock at 80.
14. £254 13s. 6d. Conversion 4½% stock at 111.

Find, in the form £ s. d. per cent., the yield from the following stocks:
15. German 7½% stock at 75. [16] Italian 3½% stock at 35.
17. Consols 2½% stock at 80.
18. London County 4½% stock at 111.
19. Find the price of a 5% stock if it yields 4½% on an investment.
20. Find the price of a 4½% stock if it yields 7½% on an investment.
21. Find the price, to the nearest £, of a 4½% stock if it yields £3 11s. per cent.
22. By investing £456 in a 3½% stock a man obtained an income of £19. Find the price of the stock.
SHARES AND STOCKS

[23] By investing £2704 in a 2½% stock a man obtained an income of £104. Find the price of the stock.

24. Find the total cost of £720 Chinese 5% stock at 75 and £675 Egyptian 4% stock at 108. What is the total income derived from this investment?

25. Which investment gives the higher yield: 5% New Zealand stock at 115 or 6½% Nigeria stock at 125?

*26. What sum must be invested in a 4% stock at 114 to give a net income of £120 after income tax at 5s. in the £ has been deducted?

*27. A man invests a certain sum of money in a 3% stock at 75, twice that sum of money in a 3½% stock at 80, and three times that sum of money in a 5% stock at 90. Find the average yield per cent on the whole investment, to 3 figures.

Find, allowing for brokerage, (i) the cost of buying, (ii) the proceeds from selling:

28. £1600 stock at 86, brokerage ½ per cent.

29. £2800 stock at 53½, brokerage ½ per cent.

30. £580 stock quoted at 58½-59, brokerage ½ per cent.

31. £2500 stock quoted at 109½-110½, brokerage ½ per cent.

32. £3200 stock quoted at 172½-181½, brokerage ½ per cent.

*33. A man paid £306 for £400 stock. At what price did the stock stand if brokerage was ½ per cent?

*34. A man received £837 from the sale of £720 stock. At what price did the stock stand if brokerage was ½ per cent?

Miscellaneous Transactions: Sales and Reinvestments

Example 12. A man invests £396 in a 4% stock at 82 and sells out when the stock has risen to 88. Find his profit, if brokerage is charged at the rate of ½ per cent. on each transaction.

Allowing for brokerage, he pays £224 cash for each £100 stock he buys and he receives £274 cash for each £100 stock he sells; he therefore gains £(£274 – £224) cash, =£50 cash, on each of the shares he invests.

If he invests £224 cash, he gains £50 cash;

∴ if he invests £396 cash, he gains £50 × (274 / 224) = £76.25 cash.

∴ his profit = £(76.25 × £100) = £7625 cash.

MISCELLANEOUS EXAMPLES

Example 13. A man sells £3150 of 7½% stock at 112, and invests the proceeds in 3½% stock at 72. Find the change in his income.

1st Income: In his first holding, £100 stock yields £7 income,
7½% stock yields £(7 × (100 / 107.5)) income, =£1.68 income.

Selling out: He sells £100 stock for £112 cash,
∴ he sells £3150 stock for £3150 × (100 / 112) cash;
∴ he invests £(3150 × (100 / 112)) cash in 3½% stock at 72.

2nd Income: £72 cash invested yields £3 income,
∴ £(3150 × (100 / 112)) cash invested yields £(3 × (3150 × (100 / 112))) income, that is £141.11 income.
∴ his loss of income = £3150 – £141.11 =£2910.89.

Example 14. A man held £4200 of 3½% stock. He sold out at 76 and invested the proceeds in a 4½% stock, and thereby increased his income by £7. Find the price of the 4½% stock.

In the 3½% stock, £100 stock yields £3 income,
∴ £4200 stock yields £(3 × 42) income, =£126 income.
∴ his income from the 4½% stock is £(126 + 7), =£133.
∴ £100 of 3½% stock is sold for £76 cash.
∴ £4200 of 3½% stock is sold for £76 × 42 cash.
∴ he obtains an income of £133 by investing £(76 × 42) cash.
∴ he obtains an income of £4½ by investing £[(76 × 42) × (4½ / 133)] cash.

But £4½ is the income on £100 stock,
∴ £100 stock costs £(76 × 42 × (4½ / 133)) cash, =£108 cash.
∴ the price of the 4½% stock is 108.

MISCELLANEOUS EXAMPLES

EXERCISE 176

1. A man sells £840 Greek 7½% stock at 39 and invests the proceeds in Siamese 6½% stock at 105. How much Siamese stock does he buy?

2. A man sells £552 L.M.S. 4½% preference at 80 and invests the proceeds in Kenya 5% stock at 115. How much Kenya stock does he buy?
SHARES AND STOCKS

3. A man invests £180 in Roumanian 4% stock at 24 and sells out when it has fallen to 16s. What does he lose?

4. A man invests £315 in Polish 7% stock at 84 and sells out when it has risen to 93s. What does he gain?

5. A man has £25,200 on deposit at a bank at 2½%. If he withdraws this money and invests it in 4% stock at 84, find the change of income.

6. A man invests £8190 in 3% stock at 91. He sells out £6000 stock at 93½ and the rest when it has fallen to 85. How much does he gain or lose?

7. A man sold £20,000 of a 4% Canadian stock at 90 and invested the proceeds in 7% Belgian stock at 108. Find the change of income.

8. A man invested £2000 in a 4% stock at 80. Later, he sold out at 84 and invested the proceeds in a 5½% stock at 105. Find the change of income.

9. A man sold £2500 of 2½% stock at 56 and invested the proceeds in 5% stock at 98. Find the change of income, to the nearest penny.

10. A man invested £500 in a 6% stock at 80. When it has risen to 104, he sells out and reinvests in a 4% stock at 90. Find the change of income.

11. A man invested £10,000 in 3% stock at 75. When the stock has risen to 78 he sells out and invests in bank shares on which a dividend of 6% per share is paid. His income is unchanged; find the price of a bank share.

12. A man’s income from a 4½% stock is £130 10s. How much money will he obtain if he sells his holding of this stock at 92?

13. A man invests £3500 in 3½% stock at 88, and when the stock has risen to 90 he sells out enough stock to buy a car. His income is thereby diminished by 10 guineas a year. What did the car cost? How much stock does he still hold?

14. A man sells his holding of 4½% Brazil stock at 88½, and after buying 200 (10s) shares at 34s, 6d. has £10 in hand. If the dividend on the shares is 20%, find the change of income.

15. The difference of income derived from investing a certain sum in 3% stock at 63 and the same sum in 4¼% stock at 105 is £22 10s. What is the sum?

16. A man received a half-year’s dividend of £12 10s. 3d. from his holding of a 6½% stock. If income-tax at 6s. in the £ was deducted from the dividend before it was paid to him, find how much of the 6½% stock he held.

17. A man sells some 4% stock at 84 and invests the proceeds in 5% stock at 96, thereby increasing his yearly income by £3. How much 4% stock did he sell?

18. A man sold £5000 of 3½% stock at 91 and invested the proceeds in a 4½% stock, thereby increasing his income by £16 13s. 4d. Find the price of the 4½% stock.

19. A man has an income of £52 10s. from a 3½% stock. He sells his holding at 80 and invests the proceeds in a 5½% stock, thereby increasing his income by £27 10s. Find the price of the 5½% stock.

20. A man invested £2220 in a 4½% stock at 92½, and later sold out at a profit of £200. At what price did he sell the stock?

21. By selling £3500 of 3½% stock at 73 and investing the proceeds in a 4% stock, a man increased his income by £10 a year. If brokerage is reckoned at ½ per cent. on each transaction, find the price of the 4% stock.

22. A man bought £100 of 3½% stock at 79½, and £100 of 4% stock at 88½, and £200 of 6% stock. Find the price of the 6% stock if the average yield on his investment was 5%.

23. A man had a net income of £340 from 5½% War Bonds, after deduction of income tax at 3s. in the £. He then sold one-half of his War Bonds at 95 and invested the proceeds in 7¼% stock at 114. What is his new net income after deducting income tax at 3s. in the £?

24. A man invests £30,155, partly in 3½% stock at 86 and the rest in 4½% stock at 99. He divides the money so as to obtain the same income from each stock. Find the total income.

CHAPTER XXV

MISCELLANEOUS PROBLEMS

Change of Units. The statement that the speed of a car is 40 mi. an hour does not mean that it will actually travel 40 mi. in the next hour, or 20 mi. in the next half-hour, etc., but that if it should continue to move at the same rate as at present it would then travel 40 mi. in 1 hr.
Example 1. Express a speed of 30 mi. an hour in feet per second.

30 mi. = (30 × 1760 × 3) ft.; 1 hr. = (60 × 60) sec.;
∴ a speed of 30 m.p.h. = a speed of \( \frac{80 \times 1760 \times 3}{60 \times 60} \) ft. per sec.
= a speed of 44 ft. per sec.

Note. This result may be used to convert any speed given in m.p.h. to the equivalent speed in ft. per sec.; thus
17 m.p.h. = (\( \frac{17}{44} \)) of 44 ft. per sec.

Nautical speeds are measured in knots; a speed of 1 knot means a speed of 1 nautical mile per hour; a nautical mile is \( \frac{1}{60} \) of a degree of latitude, and near the British Isles is taken as 6080 ft. If, however, the speed of a boat is given in "miles per hour," the word mile is used as an abbreviation for "statute mile," not "nautical mile" unless otherwise stated.

Example 2. Given that 1 m. = 39-37 in. and 1 gall. = 277-3 cu. in., express 1 litre in pints to 3 figures.

1 litre = 1000 c.c. and (100)^3 c.c. = 1 cu. m. = (39-37)^3 cu. in.;
∴ 1 litre = \( \frac{39-37^3}{1000} \) cu. in.;
but 1 gall. = 277-3 cu. in.,
∴ 1 litre = \( \frac{39-37^3}{1000} \times \frac{277-3}{5} \) pts.
= 0-2437 pt., to 3 figures.

N.B.—Do not perform any numerical calculation until it is necessary to do so.

Example 3. Find the cost per yard, to the nearest penny, if 1 metre of cloth costs 18 fr. 50 c., given 1 m. = 39-37 in., £1 = 78 fr. 40 c.

[First write down the given statement.]

1 metre costs 18-5 fr.

[Next change to British units.]

39-37
36 yd. cost \( \frac{18-5}{784} \)
∴ 1 yd. costs \( \frac{18-5}{784} \) of 36

Rough Estimate: \( \frac{20}{80} \), that is 5\%., therefore 4-figure logarithms can be used.
∴ 1 yd. costs £0-2158, that is 4s. 4d., to nearest penny.

CHANGE OF UNITS

EXERCISE 177

[Give answers to 3 figures]

1. Express a speed of 23 m.p.h. in ft. per sec.

2. Express a speed of 17 ft. per sec. in m.p.h.

3. Express a speed of 18 knots in m.p.h., taking 1 nautical mile as 6080 ft.

4. Express a speed of 45 m.p.h. in metres per second. [1 in. = 2-540 cm.]

5. Taking 1 cu. ft. = 6-228 gall., express 1 pt. in cu. in.

6. Taking 1 gal. = 277-3 cu. in. and 1 ft. = 0-3048 m., express 1 gal. in litres.

7. A metal weighs 524 lb. per cu. ft., find its weight in gm. per c.c. [1 oz. = 28-35 gm.; 1 in. = 2-540 cm.]

8. A liquid weighs 0-726 kg. per cu. dm., find its weight in oz. per cu. in. (Use the data in No. 7.)

9. Find the cost per yard, to the nearest penny, if 1 metre of cloth costs 21-75 fr., if £1 = 76-25 fr., given 1 m. = 39-37 in.

10. Express in francs per kg. the price of butter costing 1s. 7\% d. per lb., if £1 = 75-4 fr., given 1 kg. = 2-05 lb.

11. Find the cost per pint, to the nearest penny, if 1 litre costs 27-5 fr., if £1 = 72-75 fr., given 1 gall. = 4-546 litres.

12. Find the cost per sq. ft., to the nearest penny, if 1 sq. m. costs 128-5 fr., if £1 = 74-35 fr., given 1 m. = 39-37 in.

13. On a day when £1 is worth 75-85 fr. and is worth 36-35 pesetas, express the value of 350 pesetas in francs.

14. On a day when £1 is worth 77-85 francs and is worth 57-55 lire, express the value of 100 francs in lire.

15. Express a price of 3s. 6d. per sq. ft. in francs per sq. dm., when £1 = 77-85 francs. [1 ft. = 0-3048 m.]

16. Taking 1 sq. yd. = 0-8361 sq. m. and 1 gall. = 277-3 cu. in., express 1 gal. in litres.

17. A liquid weighs 7-63 lb. per gallon; find its weight in kg. per litre. [1 lb. = 453-6 gm.; 1 cu. ft. = 6-228 gall.; 1 in. = 2-540 cm.]
*18. Express in £ per acre the price of land which costs 24,650 francs per Hectare when £1 = 77.85 francs. [1 Ha. = 100 ares = 100 sq. Dam.; 1 km. = 0.6214 mi.]

*19. 1 c.c. of water weighs 1 gm.; express the weight of 1 gallon of water in lb., taking 1 cm. = 0.3937 in., 1 cu. ft. = 6.228 gal., 1 lb. = 0.4536 Kg.

*20. The pressure on a containing wall is 1.275 tons per sq. ft.; express it in lb. per sq. in.

*21. Express in Kg. per sq. cm. a pressure of 36.85 lb. per sq. in. [1 lb. = 0.4536 Kg., 1 in. = 2.540 cm.]

*22. On April 17, 1934, the London exchange on Paris was 78.25 francs to £1 and the London exchange on New York was 5.15 dollars to £1, and the Paris exchange on New York was 15.08 francs to the dollar. A man in London has to pay 100,000 dollars in New York; what is the difference in £ between paying it direct and paying it through Paris?

*23. A man in London has to pay 10,000 pesetas in Madrid. He can buy pesetas in London at 37.72 ptas. to the £, or he can buy francs in London at 78.25 francs to the £ and pesetas in Paris at 45.35 ptas. per 100 francs. What is the difference of cost in £ between paying direct and paying through Paris?

**Relative Velocity.** If a motorist travelling at 42 m.p.h. is overtaking a cyclist travelling at 12 m.p.h. in the same direction, the motorist gains on the cyclist at the rate of (42 - 12) m.p.h., that is at 30 m.p.h. Therefore if the motorist first sees the cyclist when he is 1 mi. away, it will take him \( \frac{1}{30} \) hr. to catch up the cyclist. We say that the velocity of the motorist relative to the cyclist is 30 m.p.h.

If, however, the motorist and cyclist are approaching one another at these respective speeds, the distance between them decreases at the rate of (42 + 12) m.p.h., that is at 54 m.p.h. Therefore if, as before, the motorist first sees the cyclist when he is 1 mi. away, it will be only \( \frac{1}{54} \) hr. before they pass one another; and in this case we say that the velocity of the motorist relative to the cyclist is 54 m.p.h.

Thus, if two people are travelling in the same direction, the velocity of one relative to the other is equal to the difference of their actual velocities; but if they are travelling towards one another, the velocity of one relative to the other is equal to the sum of their actual velocities.

**Example 4.** A train 140 yd. long takes 8 sec. to pass completely over a bridge 36 yd. long. Find the speed of the train.

\[ \begin{array}{c}
\text{36 yd.} \\
\text{140 yd.}
\end{array} \]

The distance of the point E where the engine reaches the bridge from the position E' of the engine when the train is just clear of the bridge is (36 + 140) yd., that is 176 yd.

\( \therefore \) in 8 sec. the engine travels 176 yd.

\( \therefore \) in 1 hr. the engine travels \( \frac{176 \times 60 \times 60}{8 \times 60} \) mi. = 45 mi.

\( \therefore \) the train is travelling at 45 m.p.h.

**Example 5.** A train 120 yd. long, travelling at 45 m.p.h., overtakes another train travelling in the same direction at 36 m.p.h. and passes it completely in 50 sec. Find the length of the second train.

Find also the time they would have taken to pass one another if they had been travelling at these speeds in opposite directions.

(i) The engine A of the first train gains on the engine B of the second train a distance equal to the sum of the lengths of the two trains in passing it completely. [Draw a diagram as in the previous example to illustrate this fact.] From the data it takes 50 sec. to gain this distance.

A gains on B at the rate of (45 - 36) m.p.h.;

\( \therefore \) in 1 hr. A gains on B 9 mi.

\( \therefore \) in 50 sec. A gains on B \( \frac{9 \times 1760 \times 60}{50 \times 60} \) yd. = 220 yd.

\( \therefore \) the sum of the lengths of the two trains is 220 yd.; but the length of the first train is 120 yd.,

\( \therefore \) the length of the second train is 100 yd.

(ii) If the trains are travelling in opposite directions, the velocity of one relative to the other is (45 + 36) m.p.h., that is 81 m.p.h.; therefore the distance between the engines A and B decrease at the rate of 81 mi. per hour;

\( \therefore \) the distance decreases by 220 yd. in \( \frac{81 \times 1760}{50 \times 60} \) sec. = 50 sec.

\( \therefore \) the time the trains take to pass one another is \( \frac{50}{5} \) sec.

Alternatively, we may argue more shortly as follows:

When the relative velocity is 9 m.p.h., the time of passing is 50 sec.

\( \therefore \) when the relative velocity is 81 m.p.h., the time of passing is \( \frac{50}{9} \) sec.
Example 6. A steamier can travel at 9 m.p.h. upstream and at 12 m.p.h. downstream. At what speed is the stream running?

If the steamier can travel at $u$ m.p.h. in still water, and if the stream runs at $v$ m.p.h., the speed of the steamier, relative to the land, is $(u + v)$ m.p.h. upstream and is $(u - v)$ m.p.h. downstream.

$. \therefore u - v = 9$ and $u + v = 12$;
$. \therefore 2u - 12 = 9$;
$. \therefore 2u = 21$;
$. \therefore u = 10.5$;
$. \therefore$ the steam runs at $10.5$ m.p.h.

EXERCISE 178

1. A man cycles 8 m.p.h. on the level, 6 m.p.h. uphill, and 12 m.p.h. downhill. If he first rides 3 mi. uphill, then 6 mi. along the level and ends with 2 mi. downhill, find his average speed. Find also his average speed on the return journey.

2. A launch travels $7\frac{1}{2}$ m.p.h. upstream and $10\frac{1}{2}$ m.p.h. downstream. How long does it take to go (i) 21 mi. upstream and back, (ii) 40 mi. downstream and back?

3. A train is timed to make a journey of 63 mi. at 45 m.p.h. It starts 4 min. late and runs at its proper speed for 30 min. At what rate must it do the rest of the journey to arrive punctually?

4. How long will a train 160 yd. long, travelling at 60 m.p.h., take to pass a train 148 yd. long travelling at 25 m.p.h. in the same direction?

5. A train 88 yd. long moving at 50 m.p.h. overtakes a second train moving at 35 m.p.h. in the same direction and passes it completely in 22$\frac{1}{2}$ sec. Find the length of the second train. Find also how long the first train takes to pass a passenger in the second train.

6. A man in a train travelling at 30 m.p.h. notices that a train going in the opposite direction passes him in 9 sec. If the length of this train is 220 yd., find its speed.

7. A man standing on a platform 220 yd. long notices that a train passes him in 6 sec. and passes completely through the station in 21 sec. Find the length of the train and its speed in m.p.h.

8. A dog pursues at 18 m.p.h. a hare which runs at 14 m.p.h.; the hare has 44 yd. start; after what time will it be caught?

9. Two cyclists A, B ride together; A stops for 5 min. while B rides on at 10 m.p.h. If A overtakes B after another 10 min., at what speed does he ride?

10. A can row 1 mi. upstream in 20 min. and back to his starting-point in 12 min. How long would he take to row 1 mi. in still water?

11. In a paper chase, the hares had 20 min. start and ran the whole distance at 9 m.p.h. One of the hounds who ran the same way at 10 m.p.h. arrived 15 min. after the hares. How long was the run?

12. A liner, 814 ft. long, travelling at 20 m.p.h., is overtaken by a torpedo-boat 154 ft. long. A passenger from a porthole on the liner notices that the torpedo-boat takes 7 sec. to pass him. How long would the torpedo-boat take to pass the liner completely if going in the opposite direction?

13. A man running at 6 m.p.h. takes 40 sec. to get from the rear to the front of a column of men marching at $\frac{3}{4}$ m.p.h. What is the length of the column?

14. A steamer is travelling directly away from a fort at 20 m.p.h.; a gun is fired from the fort at intervals of 1 min. At what intervals are the reports of the gun heard on the steamer, if sound travels at 1100 ft. per sec.?

15. A car travelling at 55 m.p.h. overtook at 3 p.m. a man walking the same way at 4 m.p.h., and met at 3.5 p.m. another man walking in the opposite direction at $4\frac{1}{2}$ m.p.h. At what time will the two men meet?

Problems on Clocks and Races

Example 7. In a race of 100 yd., A beats B by 10 yd., and beats C by 13 yd. By how much will B beat C in a race of 120 yd., assuming that A, B, C all run at constant speeds throughout?

In the time that A takes to run 100 yd.,
$. \therefore$ B runs 90 yd. and C runs 87 yd.;
$. \therefore$ when B has run 90 yd., C has run 87 yd.,
$. \therefore$ when B has run 120 yd., C has run $\frac{87}{90} \times 120$ yd., =116 yd.
$. \therefore$ B beats C by 4 yd. in a race of 120 yd.
Example 8. At what times between 4 o'clock and 5 o'clock are the hands of a clock at right angles?

Imagine that the short hour-hand is prolonged so as to meet the circle round the edge of the clock which is divided into 60 equal arcs or "minute-spaces."

The hands are at right angles when the distance between their tips measured along the circle is 15 minute-spaces.

At 4 o'clock, the tip of the minute-hand is 20 minute-spaces behind the tip of the prolonged hour-hand; therefore it will be 15 minute-spaces behind when it has gained 5 minute-spaces on the tip of the prolonged hour-hand, and will be 15 minute-spaces in front when it has gained 35 minute-spaces.

In 1 hr, the tip of the minute-hand travels 60 minute-spaces and the tip of the prolonged hour-hand travels 5 minute-spaces;

\[ \therefore \text{the minute-hand gains } 55 \text{ minute-spaces in 60 min.} \]
\[ \therefore \text{the minute-hand gains } 5 \text{ minute-spaces in } \frac{5}{11} \text{ min.} = 5 \frac{5}{11} \text{ min.,} \]
and the minute-hand gains 35 minute-spaces in \( \frac{60}{2} + \frac{5}{11} \) min. = \( 38 \frac{6}{11} \) min., that is in \( 38 \frac{6}{11} \) min.

\[ \therefore \text{the hands are at right angles at } 5 \frac{5}{11} \text{ min. past 4 and at } 38 \frac{6}{11} \text{ min. past 4.} \]

EXERCISE 179

1. At what times are the hands of a clock at right angles between (i) 5 p.m. and 6 p.m.; (ii) 7 p.m. and 8 p.m.?
2. At what time are the hands of a clock coincident between 3 p.m. and 4 p.m.?
3. At what time are the hands of a clock opposite one another between 9 p.m. and 10 p.m.?
4. At what time do the hands of a clock contain an angle of 60° between 6 p.m. and 7 p.m.?
5. A clock is set right at 1 p.m. If it gains 1 min. an hour, what is the true time when the clock indicates 6 p.m. the same day?
6. A clock is set right at 9 a.m. If it gains 30 sec. an hour, what is the true time when the clock indicates 9 a.m. the next morning?
7. At 7 a.m. the time by my watch is 6:59, and at 7 p.m. the same day my watch reads 2 min. past seven. What is the right time when my watch shows 7 a.m. the next morning?
8. Two clocks are set right at midday on Monday; one loses 20 sec. an hour, the other gains 4 min. a day. What is the right time when one clock is 9 min. ahead of the other?

MISCELLANEOUS EXAMPLES

EXERCISE 180

1. Some Christmas cards are bought at 10d. a dozen. One half of them are sold at 2d. each, one quarter of them at 1d. each, and the rest are wasted. Find the gain per cent.
2. By making 65 out in his last 100 test-match innings, Hobbs brought his average for 58 completed test-match innings to 57\frac{1}{4}. What would he have had to make to bring it up to 60?
3. A commodity is catalogued at £2 17s. 6d. per cwt.; if a discount of \( \frac{1}{7} \) is allowed, find the cost per lb., correct to \( \frac{1}{10} \)d.
4. The sides of a rectangle are measured as 7-4 cm. and 6-2 cm., correct to the nearest mm. Between what limits does the area of the rectangle lie? To how many figures is it justifiable to give the area? Find, to 2 figures, the greatest possible error in taking the area as \( 7 \times 6 \) cm.**
MISCELLANEOUS PROBLEMS

5. A telegraph pole is 18 ft. high and 1 ft. in diameter. Find its weight, correct to the nearest lb., if the wood weighs 55 lb. per cu. ft.

6. A French car uses 12 litres of petrol in running 100 km. How many miles, to 3 figures, does it run on a gallon of petrol?  [1 ml. = 1⁄800 km.; 1 gal. = 4.546 litres.]

7. A man buys shirts at 4s. 6d. each and sells them at 6s. 3d. each. Find his gain per cent. What would be his gain per cent. in a sale when he allows 10% discount?

8. Find in yards the length of the diagonal of a square field of area 20 ac.

9. A boy runs 55 yd. round a circular track, one-quarter of a mile in circumference. Through what angle has he turned?

10. Electricity is said to be consumed at the rate of 1 watt, if 1000 of a “unit” is used per hour. For a small electric fan, the rate of consumption is 70 watts and for a larger fan is 120 watts. Find, to the nearest penny, the cost of running one fan of each kind for 28 hr. a week for 30 weeks, if electricity costs 13d. per unit.

11. The side walls of a building, 80 ft. long, are 40 ft. high, and the roof-rails which form a right angle at the ridge are 15 ft., 20 ft. long. Find (i) the width, (ii) the volume of the building.

12. The gear of a bicycle is 70, that is for each revolution of the pedals the bicycle travels a distance equal to the circumference of a circle of diameter 70 in. If the pedals revolve 18 times in 25 sec., find the speed of the bicycle in miles per hour. [Take \( \pi = \frac{22}{7} \).]

13. A tap discharges water at 3 cu. ft. per minute into a cylindrical tank, diameter 5 ft.; find in inches per minute, to 3 figures, the rate at which the water-level rises.

14. Calculate, to 3 figures, (i) the number of tons produced per miner, (ii) the value in £ of the coal produced per miner, in the year 1929 from the following data:

<table>
<thead>
<tr>
<th>Miners</th>
<th>Tons of coal produced</th>
<th>Total value</th>
</tr>
</thead>
<tbody>
<tr>
<td>985,422</td>
<td>257,907,000</td>
<td>£173,233,000</td>
</tr>
</tbody>
</table>

15. A tradesman marks his goods so that his profit is 40% of the marked price. How much per cent. of the marked price can he deduct for cash so as to gain 40% of the cost price?

16. Each member of a club pays £3 a year subscription, and the expenses of the club are £240 a year. If the expenses increase by 35% and if the number of members is increased to 100, find, to the nearest shilling, what increase must be made in the subscription to cover expenses.

17. A merchant bought cheese at 6 guineas per cwt. and sold it at 1s. 9d. per lb. Find his gain per cent. if there was a loss in weight of 4 lb. per cwt. in retailing it.

18. A cricket ground, 120 yd. by \( \frac{1}{2} \) mi., is mowed by 3 machines, A, B, C; A and B run at 3 m.p.h. and cut widths of 1 ft. and 3 ft. respectively; C runs at 5 m.p.h. and cuts a width of 3 ft. All start together, but when half the ground is done, B breaks down and the work is finished by A and C. Find the total time taken.

19. A figure is drawn on a rectangular sheet, 24 cm. by 15 cm., of stiff paper, which is found to weight 4.72 gm. When the figure has been cut out, the rest of the paper is found to weigh 3.65 gm. Find the area of the figure, to the nearest sq. cm.

20. A submarine can travel at 21 knots on the surface and at 15 knots when submerged. How many hours will it take for a passage of 252 nautical miles if it is submerged (i) for \( \frac{1}{4} \) of the distance, (ii) for \( \frac{1}{3} \) of the time?

21. The cross-section of a railway tunnel 100 yd. long is a square with a semicircle on the top edge as diameter. The tunnel is 17 ft. high at the centre; find, to 3 figures, the number of tons of material excavated at 2 tons per cu. yd.

22. The cost of fuel per week of 54 hr. for engines of various horse-powers in a factory is as follows:

<table>
<thead>
<tr>
<th>Horse-power</th>
<th>10</th>
<th>20</th>
<th>50</th>
<th>80</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>4s. 11d.</td>
<td>9s. 3d.</td>
<td>21s. 9d.</td>
<td>31s. 8d.</td>
<td>37s. 6d.</td>
</tr>
</tbody>
</table>

Represent these facts by a graph and estimate the cost per week for an engine of 30 H.P., and the horse-power of an engine which costs 29s. 3d. per week.

23. In an examination the marks of the boys A, B, C, who were top respectively in the three papers, were : Arithmetic, A 80, B 77, C 62; Algebra, A 49, B 80, C 73; Geometry, A 80, B 64, C 86. The marks are scaled so that the top boy in each subject obtains 100, other marks being calculated to the nearest integer. Find the scaled totals of A, B, C.

24. A piece of paper 1 mi. long, \( \frac{1}{8} \) in. thick, is rolled into the form of a solid cylinder. Find its diameter, correct to \( \frac{1}{8} \) in.

25. A shopkeeper marks his goods at 45% above cost, but allows a discount of 10% on the marked prices. What is his net gain per cent.?
378 MISCELLANEOUS PROBLEMS

*26. Electric power may be paid for either (i) at the rate of 2·35d. per unit for heating and 4·4d. per unit for lighting, or (ii) at 0·65d. per unit for all power, together with a quarterly payment of 2/3% of the rent of the house. If the quarterly consumption is 261 units for heating and 94 units for lighting, and if the rent is £37, find the cost of each method of payment per quarter, to the nearest penny.

27. In a certain year, income tax was levied as follows: the first £120 of a man's income was free of tax, the next £260 of his income was taxed at 2s. 3d. in the £, and the remainder at 4s. 6d. in the £. What was a man's income who paid £87 15s. in tax?

*28. A policeman, by observing the time a car takes to travel a measured distance, calculates the average speed of the car to be 68 m.p.h. If there may be an error of 5% in the distance and 10% in the time, find, to 2 figures, the limits between which the average speed lies.

29. Water is drawn off from a tank at 20 gall. per minute through a pipe of diameter 1\(\frac{1}{2}\) in. Find the rate of flow, in ft. per second correct to 2 figures, of the water in the pipe, if the pipe remains full. [1 gallon = 277·3 cu. in.]

30. A sells goods to B at a profit of 10\%; B sells them to C for £2 7s. 8d., making a profit of 4\%. What did the goods cost A?

31. The £10 shares of a company stand at 8 and pay a dividend of 5 \% per cent. What yield per cent. is obtained by investing in these shares?

32. A man borrows £3000 at 5\% p.a. compound interest, reckoned yearly. He pays back £1000 at the end of the first year and £1000 at the end of the second year. How much must he pay at the end of the third year to clear off the debt?

*33. A train X travelling at 31 m.p.h. passes a station at 2 p.m.; a train Y travelling the same way at 56 m.p.h. passes this station at 2·23 p.m. At what time will Y overtake X?

*34. A tyre pressure-gauge is graduated in kg. per sq. cm. A tyre is to be inflated to a pressure of 30 lb. per sq. in.; find, to 3 figures, the reading on the gauge for this pressure. [1 lb. = 453·6 gm., 1 cm. = 0·3937 in.]

35. The eight edges of a pyramid on a square base are each 2 in. long. Find, to 3 figures, (i) the height, (ii) the volume, (iii) the total area of the surface of the pyramid.

379 MISCELLANEOUS EXAMPLES

38. A and B start a business, A contributing £1200 and B £400. After 6 months B contributes an additional £1000. If the profits at the end of the year are £140, what should A receive?

39. A goods train starts from X at 11·30 a.m., travelling at 20 m.p.h., stops for 15 min. at Y, 10 mi. from X, and then proceeds at 20 m.p.h. A passenger train leaves X at 12·10 p.m. and travels at 40 m.p.h., without stopping at Y. Find from a graph when and where it passes the goods train.

*41. A man goes from Winchester to London to buy an article which costs 10\% less in London than in Winchester. His expenses are 15\%, but he makes a clear profit of 24\%. by doing so. What did the article cost in Winchester?

42. A bath has two taps A, B and an outlet C. With A turned on, it fills in 15 min.; with A and B turned on, it fills in 10 min.; with B turned on and C opened, the water-level remains stationary. How long will it take to fill the bath with A turned on and C opened?

*43. A bicycle pump is 8 in. in diameter and has a stroke of length 14 in. It takes 25 strokes to pump up a flat tyre; what volume of uncompressed air has been put in? Find the volume of the interior of the tyre when pumped up, if the air which has been put in is compressed in the ratio 7:2. Give answers to 2 figures.

44. A lead sphere weighs \(\frac{1}{2}\) lb.; find its diameter in in. to 2 figures, if 1 cu. ft. of lead weighs 708 lb.

45. A man buys 50 ordinary shares at £4 2s. 6d. per share. He receives a dividend of 8s. 6d. per share, and a bonus of one additional share for every 5 shares held by him. He then sells all his shares at £3 16s. 4d. Find his total gain per cent. on his outlay.

46. How much money must be invested in a 4\% stock at 115 to yield a net income of £60, after tax at 5s. in the £ has been deducted?

*48. Shop A sells goods at the prices marked on them. Shop B sells at 5\% less than the prices marked. Actually B sells for £98 goods that are sold in A for £100. What price does B mark on an article that is sold in A for £9 10s.?
47. A man buys a Savings Certificate for 16s. and after 3 years receives 18s. 3d. for it. At what rate per cent. per annum is compound interest reckoned?

48. A faulty speedometer records the speed of a car, running at \( v \) m.p.h., as \( (a + bv) \) m.p.h., where \( a, b \) are fixed numbers. The speedometer gives the correct reading when the car is running at 45 m.p.h., but registers 60 m.p.h. when the car’s speed is \( 57\frac{1}{2} \) m.p.h. What is the reading on the speedometer when the car is running at 60 m.p.h.? What is the speed of the car when the speedometer registers 35 m.p.h.?

49. For a piece of furniture a firm pays sums for material and labour in the ratio 6 : 5, and sells it at a profit of 25%. If I make the same article for myself, the material costs me twice as much as the firm paid, but by avoiding all other expenses I save £1 15s. At what price did the firm sell the article?

50. In ascending a mountain the difference in height between \( X \) and \( Y \) is 65620 \((\log H - \log h)\) ft, where \( H, h \) are the barometric readings at \( X, Y \). If the readings at \( X \) and \( Y \) are 29.85 in., 19.35 in., find the height of \( Y \) above \( X \), and the reading at a place half-way up from \( X \) to \( Y \).

**TESTS IN COMPUTATION**

**TESTS 33-40 (Ch. XIII-XX)**

**Test 33**

1. If 84% of a sum of money is £9324, what is 116% of the same sum?

2. The population of New York increased from 5,620,048 in 1920 to 6,930,446 in 1930. Find the increase per cent. correct to 3 figures.

3. (i) Find the square root of 0.094864.

(ii) Use tables to evaluate \( \sqrt{2.874 + 1.692} \), correct to 3 figures.

4. Use tables to evaluate, correct to 3 figures:

   (i) \( \frac{18.92 \times 7.064}{56.37 \div 0.080} \)

   (ii) \( 0.7246^2 \)

   (iii) \( 0.8103^2 \)

5. The base of a triangle of area 13.7 sq. yd. is 9.85 yd.; find its height in yards, correct to 3 figures.

6. Find, correct to 3 figures, the length of the circumference of a circle of area 15 sq. in.

**Test 34**

1. Find the value of \( \frac{17^2 + 8}{3^2 - 8} \).

2. The area of the country is 7,638,000 sq. mi. Find, correct to 3 figures, the average population per sq. mile.

3. (i) Find the square root of 16.5649.

(ii) Use tables to evaluate \( \frac{1}{4.72} + \frac{1}{7.36} \), correct to 3 figures.

4. Use tables to evaluate, correct to 3 figures:

   (i) \( \frac{0.9172 \times 0.0604}{10.79 \times 0.5147} \)

   (ii) \( \sqrt{0.6218} \).
5. A pond of area $\frac{3}{4}$ ac. is frozen over with ice $2\frac{1}{2}$ in. thick. Find in tons the weight of the ice, correct to 2 figures, given that 1 cu. ft. of ice weighs 57-3 lb.

6. Find, correct to $1\frac{1}{2}$ in., the radius of a sphere whose volume is 30 cu. in.

Test 35

1. (i) Find the square root of 101-6064.
   (ii) Use tables to evaluate $\sqrt[3]{7}$, correct to 3 figures.

2. Find, correct to 3 figures, by how much per cent. $0-3817$ exceeds $0-3294$.

3. Use tables to evaluate, correct to 3 figures:
   (i) $\frac{9\frac{1}{2} \times 0.7071}{0.3174^2 \times 0.17}$
   (ii) $0.5076^3 \sqrt[3]{0.9136}$

4. Find in yards, correct to 3 figures, the diameter of a circular pond of area $\frac{3}{4}$ ac.

5. What sum, correct to the nearest penny, will amount to £100 in 10 months at $7\%$ p.a. simple interest?

6. The base of a right pyramid is a square of side 6 in., and each edge is 7 in. long. Find, correct to 3 figures, (i) the height in inches, (ii) the volume in cu. in., of the pyramid.

Test 38

1. (i) Calculate the square root of 0.0925, correct to 3 figures.
   (ii) Use tables to evaluate $\sqrt[4]{0.9} - \sqrt[3]{0.9}$, correct to 3 figures.

2. Taking 1 cu. yd. equal to 0.7645 cu. m., express 1 m. in yards, correct to 3 figures.

3. The value of some goods rises in the ratio 20:27. If the original value was £17 4s. 8d., find the new value correct to the nearest penny.

4. Use tables to evaluate, correct to 3 figures:
   (i) $32 \times \sqrt{0.8847}
   (ii) (0.865)^3$

5. The perimeter of a semicircular plate is 1 ft. Find, correct to 3 figures, (i) its diameter in inches, (ii) its area in sq. in.

6. Find, correct to $1\frac{1}{4}$ in., the radius of the base of a circular cone of volume 10 cu. in., if its height is 4 in.
Test 39

1. Arrange in ascending order of magnitude:
   \[ \frac{1}{2}, \frac{2}{3}, \sqrt{2}. \]

2. In Great Britain there were 623,231 deaths in 1929, and 536,860 deaths in 1930. Find the decrease per cent. in the number of deaths, correct to 3 figures.

3. Use tables to evaluate, correct to 3 figures:
   \[ \log \left( \frac{0.67284}{0.66195} \times 4\frac{3}{4} \right); \quad \log \left( \frac{305 - 273}{6.07} \right). \]

4. Find, correct to the nearest penny, the true discount on a bill for £76 15s. due in 4 mo., reckoning interest at 4\% p.a.

5. Find, correct to 3 figures, the area in sq. ft. of a path 5 ft. wide surrounding a circular grass plot of diameter 80 ft.

6. The base of a conical tent is 13 ft. in diameter and its height is 9 ft. Find, correct to 3 figures, (i) the volume of the tent in cu. ft., (ii) the area of canvas used for making it, in sq. ft.

Test 40

1. Express 7 hr. 28 min. 40 sec. as a percentage of 24 hr., correct to 3 figures.

2. (i) \( \pi = 3.14159 \ldots \), use tables to find the value of \( \sqrt[3]{2} \), correct to 3 figures.
   (ii) Find in inches, correct to \( \frac{3}{4} \) in., the radius of a circle of area of 1 sq. ft.

3. Use tables to evaluate, correct to 3 figures:
   \[ \left( \frac{239.34 \times \sqrt{17}}{10} \right); \quad 0.034. \]

4. Find, correct to 3 figures, the volume in cu. in. of the metal in a tube 2\frac{1}{2} ft. long, internal diameter \( \frac{7}{8} \) in., if the metal is 0.3 in. thick.

5. Find, correct to 3 figures, the capacity in gallons of a cylindrical tank of diameter 2 ft. 9 in. and height 5 ft. 6 in. [1 gall. = 0.1604 cu. ft.]

6. A hollow spherical metal shell has an external diameter of 5 in. and is \( \frac{1}{4} \) in. thick. Find, correct to 3 figures, (i) the volume in cu. in. of metal used in making it, (ii) the weight of the shell in oz. if the metal weighs 524 lb. per cu. ft.

Test 41

1. Find, correct to 2 figures, the error per cent. in taking \( \frac{\sqrt{2}}{4} \) as 1 per cent. greater than \( \frac{2}{\sqrt{3}} \).

2. A field, 285 yd. long, 155 yd. wide, is rented at £4 10s. a year. Find, to the nearest penny, the annual rent per acre.

3. Use tables to find, correct to 3 figures:
   (i) the value of \( x \) if \( 2^x = 3 \); (ii) the value of \( \sqrt[3]{0.728 \times \frac{3}{8}} \).

4. Find, correct to 2 figures, the weight in kg. of a rectangular wooden beam 4·72 m. long, 0·38 m. wide, 0·26 m. thick, if the specific gravity of the wood is 0·65.

5. Find, correct to the nearest penny, the compound interest on £283 for 3 yr. at 4\% p.a.

6. Find the income obtained from investing £1680 in 3\% stock at 96.

Test 42

1. To how many places of decimals is \( \sqrt[4]{19} \) an accurate approximation for \( \sqrt[14]{2} \)?

2. A merchant blends 350 lb. of tea costing Is. 5d. per lb. with 140 lb. costing Is. 8\frac{1}{4}d. per lb. At what price per lb. must he sell the mixture to gain 25%.

3. Use tables to find, correct to 3 figures:
   (i) the value of \( r \) if \( 2^r \times 2^{0.5} = 2^1 \);
   (ii) the value of \( n \) if \( (1.05)^n = 1.35 \).

4. If American dollars are quoted at 4·865 to the £ and French francs at 74·35 to the £, express the value of 100 French francs in dollars, correct to 3 figures.

5. An ordinary brick measures \( 8\frac{3}{4} \) in. by \( 4\frac{1}{2} \) in. by \( 2\frac{3}{4} \) in., and 500 of these bricks weigh 1 ton 11 cwt. Find the sp. gr. of the material, correct to 2 figures. [1 cu. ft. of water weighs 62·3 lb.]

6. Find the income from, and the cost of, £1650 of 4\% stock at 83\frac{3}{4}.
TESTS IN COMPUTATION

Test 43

1. A bankrupt is only able to pay 67·3% of his debts. How much in the £ does he pay? Answer to nearest farthing.

2. On a map, a road 850 yd. long is represented by a line of length 1·93 in. What area on the map, correct to 3 figures, represents 1½ acres?

3. Use tables to find, correct to 3 figures:
   (i) the value of \( (0·4615)^8 \times \sqrt{(5·138)^2 - (3·816)^2} \);
   (ii) the value of \( \pi r^2 \) if \( 2\pi r = 439·5 \).

4. A grocer mixes 150 lb. of tea which costs 1s. 4d. per lb. with 360 lb. of tea which costs 1s. 7d. per lb., and sells the mixture at 1s. 9d. per lb. Find his gain per cent.

5. Two solid lead spheres of radii 3 in., 4 in., are melted together and recast as a single sphere; find its radius, correct to 3 figures.

6. 740 (£1) shares are bought at 23s. 6d. What is the purchase price? What is the yield per cent., correct to 3 figures, on the money invested if a dividend of 1s. 9d. per share is paid?

Test 44

1. A man is paid at the rate of £470 a year (365 days); how much, to the nearest shilling, is due to him after 23 days?

2. Taking 1 gal. = 277·3 cu. in. and 1 ft. = 0·3048 m., express 1 litre in gallons, correct to 3 figures.

3. Use tables to find, correct to 3 figures:
   (i) the value of \( n \) if \( 4·18 \times (3·4)^n = 500 \);
   (ii) the value of \( \sqrt{(0·097)^4 + (0·28)^2} \).

4. The internal and external diameters of a flat ring are 8·1 cm. and 9·7 cm., and the thickness is 1·24 cm. (i) Find the volume, correct to 3 figures. (ii) If the weight is 27·3 gm., find the specific gravity of the substance, correct to 2 figures.

5. Find, to the nearest penny, the compound interest on £126 for 1½ yr. at 7½% p.a., the interest being credited half-yearly.

6. In 1924 Leeds Corporation paid a dividend of 2½% on £750,000 stock and a dividend of 3½% on £2,000,000 stock. What was the total amount paid out to stockholders if income tax was deducted at the rate of 4s. 6d. in the £?

TESTS IN COMPUTATION

Test 45

1. Prove that \( \sqrt{101} \) may be taken as \( 10\frac{10}{9} \), correct to 5 places of decimals.

2. Brass is composed by weight of 167 parts of copper, 27 parts of tin, and 6 parts of zinc. If the copper is worth £27 per ton, the tin £2·33 per ton and the zinc £22 per ton, find the value of the brass per ton, correct to 3 figures.

3. Use tables to evaluate, correct to 3 figures:
   (i) \( (0·4873)^{-0·8} \);
   (ii) \( \sqrt{(0·7245)^2 + (0·6083)^2} \).

4. The present population of a district is 164,500. If it has increased at the rate of 14 per 1000 per year for the last 3 yr., find, correct to 3 figures, what it was 3 yr. ago.

5. The base of a solid metal right pyramid is a square of side 12 cm., and the height of the pyramid is 9 cm. Find, correct to 3 figures, (i) the weight in kg. if the specific gravity of the metal is 8·52, (ii) the total area of the surface in sq. cm.

6. Find the yield, in £ s. d. per cent., correct to the nearest penny, from an investment in a 4½% stock at 93.

Test 46

1. A bankrupt is only able to pay at the rate of 7s. 4d. in the £. What percentage of his debts, correct to 3 figures, remains unpaid?

2. A metal weighs 573·4 lb. per cu. ft. Find, correct to 3 figures, its specific gravity (i) by taking 1 cu. ft. of water to weigh 62·3 lb., (ii) by taking 1 oz. = 28·35 gm. and 1 in. = 2·540 cm.

3. £165 was borrowed on June 1 and repaid with interest on August 10 of the same year (365 days). Reckoning simple interest at 5½% p.a., find the amount repaid correct to the nearest penny.

4. Find, correct to 3 figures:
   (i) the value of \( \frac{1}{2} \pi r^2 \) if \( 4\pi r = 1 \);
   (ii) the value of \( \frac{2}{3} \) of \( (15·37)^{0·4} \times (0·836)^{-0·2} \).

5. A closed hollow metal cylinder has an external diameter of 5 in. and external height of 8 in.; the metal is 0·2 in. thick. Find, correct to 2 figures, (i) the volume of the metal, (ii) the weight of the cylinder if the metal weighs 5·45 oz. per cu. in.
6. In 1934 Derwent Valley Water Board paid a dividend of 5% on £1,250,000 stock and a dividend of 3¼% on £1,000,000 stock. What was the total amount paid out to the stockholders if income tax was deducted at the rate of 4s. 6d. in the £?

Test 47
1. Find, correct to 1 figure, the error per cent. in taking \( \frac{70}{r} \) as the value of \( \log 2 + \log \left(1 + \frac{r}{100}\right) \) when \( r = 6\frac{1}{8} \).

2. Express, correct to 3 figures, in lb. per sq. in. a pressure of 7.36 kg. per sq. cm., given 1 kg. = 2.205 lb., 1 cm. = 0.3937 in.

3. Find, correct to 2 figures, the value of \( e^{2n\mu \pi} \), where \( e = 2.718, n = 1\frac{1}{8}, \mu = 0.645, \pi = 3.142 \).

4. Two solid lead spheres of radii 6 in., 1 ft., are melted together and recast as a solid circular cylinder of height 1 ft. 6 in. Find in inches the radius of the cylinder. Prove that the total surface of the two spheres is equal to that of the cylinder.

5. Find, correct to the nearest shilling, the sum of money which will amount to £350 in 3 yr. at 4% p.a. compound interest.

6. What yield in £ s. d. per cent., to the nearest penny, did a man who bought Poole Corporation 4½% stock at 115, obtain on his money?
   If he invested £5520 in this stock and sold out when the stock rose to 117½, find his profit.

Test 48
1. If pesetas are quoted at 35.85 to the £ and lire at 57.62 to the £, express the value of 100 lire in pesetas, correct to 3 figures.

2. A liquid weighs 8.275 lb. per gallon; find, correct to 3 figures, its weight in kg. per litre. [1 lb. = 453.6 gm.; 1 cu. ft. = 622.8 gal.; 1 in. = 2.54 cm.]

3. Find, correct to 3 figures:
   (i) the value of \( x \) if \( \log x = 1\frac{1}{4} \); and if \( \log x = \frac{1}{4} \);
   (ii) the value of \( \frac{\sqrt{2}}{35} \), \( (742)^{3/4} \times 753 \).

4. A sells an article to B at a profit of 20%; B sells it to C at a profit of 10%; C sells it to D at a profit of 50%. If D pays £13 for it, at what price did A sell it?

5. The volume of a circular cone, height 8 in., is 65 cu. in.; find, correct to 3 figures, (i) the diameter of the base, (ii) the length of the slant height of the cone.

6. Find the change of income if a man sells £2600 of 3¼% stock at 84 and invests the proceeds in 4½% stock at 91.

REVISION PAPERS

PAPERS 49-56 (Ch. I-XX)

Paper 49
1. (i) Express in prime factors the L.C.M. of 385, 231, 165, 105.
   (ii) A man sells for a guinea what costs him 17s. 6d.; find his gain per cent.

2. Use logarithms to evaluate, correct to 3 figures:
   (i) \( \frac{2079 \times 0.0804}{0.09072} \);
   (ii) \( \frac{1}{\sqrt{0.8514}} \).

3. The running record for 440 yd. is 47 sec.; express this speed in miles per hour, correct to 3 figures.

4. Find, correct to \( \sqrt[3]{5} \) yd., the radius of a circular lake of area 5 ac.

5. A swimming-bath is 30 ft. long, 18 ft. wide; it is 4 ft. deep at one end and 8 ft. deep at the other end, and the floor slopes evenly. Find how many gallons of water are required to fill the bath. [1 gal. = 0.1604 cu. ft.]

6. Find in sq. in. to 3 figures, the area of the curved surface of a cone 1 ft. high, base-diameter 10 in.

Paper 50
1. (i) Express £8 3s. 4d. as a fraction of £9 12s. 6d.
   (ii) Divide £1 into 3 parts in the ratios 3: 4: 5.

2. Use logarithms to evaluate, correct to 3 figures:
   (i) \( \sqrt[3]{9347 \times 10.73} \);
   (ii) \( \sqrt[3]{5509 \times 0.173} \).

3. In 1930 the number of deaths due to accidents in England and Wales was as follows: males, 10,922; females, 4838. What percentage, to 3 figures, of the total were males?
4. A tankard is a cylinder of internal diameter 4.3 in. Find to what depth in inches, to 3 figures, a quart of liquid will fill it. [1 gall. = 277.3 cu. in.]

5. Find, correct to \( \frac{1}{2} \) sq. in., the area of an equilateral triangle whose sides are each 6 in. long.

6. Find, correct to \( \frac{1}{8} \) in., the radius of the base of a circular cone, 9 in. high, if its volume is 345 cu. in.

**Paper 51**

1. (i) Divide £35 9s. 6d. by 2\( \frac{1}{2} \).

(ii) B exceeds A by 16\% ; C exceeds B by 25\%; find the ratio of A to C.

2. Use logarithms to evaluate, correct to 3 figures:
   
   \[
   \begin{align*}
   (i) \quad & \frac{47.326 \times (8.7112)^2}{698.43} ; \\
   (ii) \quad & \sqrt[3]{0.6394^{1/4}}
   \end{align*}
   \]

3. The areas in sq. mi. of the Pacific, Atlantic, Indian, Arctic oceans are respectively 63,986,000; 30,000,000; 28,350,000; 5,541,600. These form together 96.51 per cent. of the total sea area. Find the total sea area, correct to 3 figures.

4. Find, correct to the nearest penny, the simple interest on £428 for 35 days at 4\%\%\% p.a. [Take 1 year = 365 days.]

5. A pump delivers 165 gal. of water per minute through a pipe 3 in. in diameter. Find in feet per second, correct to 3 figures, the rate of flow of water in the pipe, if kept full. [1 gal. = 277.3 cu. in.]

6. Find, correct to \( \frac{1}{2} \) in., the diameter of a sphere whose volume is 100 cu. in. Find also the area of the surface of the sphere, to the nearest sq. in.

**Paper 52**

1. (i) Simplify \( (10\frac{3}{4} \times 1\frac{3}{4} - 10\frac{3}{4}) ÷ (1\frac{3}{4} - 3\frac{1}{4} + 2\frac{1}{4}) \).

(ii) Calculate the square root of 0.09653449.

2. Use logarithms to evaluate, correct to 3 figures:
   
   \[
   \begin{align*}
   (i) \quad & 0.386 \times 2^{1/3} ; \\
   (ii) \quad & \sqrt[3]{9.7162} \\
   & \frac{20.07 \times 1.01}{2^{1/3}} \quad ; \\
   & \frac{2}{(0.809)^4}
   \end{align*}
   \]

3. In 1931 the population of (Greater) London was 8,202,818 and of New York 6,981,927. By how much \%, correct to 3 figures, did the population of London exceed that of New York?

4. A bankrupt's assets are £3526 and his liabilities are £5693. How much can he pay in £, to the nearest penny?

5. A pianola record is made of paper \( \frac{1}{8} \) in. thick; it is 22 yd. long and is rolled up tightly on a cylindrical roller \( \frac{1}{2} \) in. in diameter. Find the radius of the complete roll, to the nearest \( \frac{1}{8} \) in.

6. The height of an open cylindrical vessel equals the diameter of its base, both measured internally. Its capacity is 3 qt. Find its height in inches, correct to 3 figures. [1 gal. = 277.3 cu. in.]

**Paper 53**

1. (i) Find the whole numbers between 1550 and 1800 which are perfect squares.

(ii) Calculate the square root of 0.01459616.

2. Use logarithms to evaluate, correct to 3 figures:
   
   \[
   \begin{align*}
   (i) \quad & \frac{0.086}{(0.9134)^2} ; \\
   (ii) \quad & (0.6135)^{0.77}
   \end{align*}
   \]

3. In 1930 there were 455,427 deaths in England and Wales, and this was 1.414 per cent. of the population. Find the population, correct to 3 figures. The births were 1,629 per cent. of the population, find the number of births, correct to 3 figures.

4. By selling a commodity at 3s. 4\( \frac{3}{4} \)d. per lb., a merchant gains 17\%. What would be his profit per cent. if he increased the price by 2\( \frac{1}{4} \)d. per lb.?

5. A swimming-bath is emptied by three equal pipes, each of diameter 2 in. It is desired to replace them by a single pipe which will empty the bath in the same time as the three pipes, for the same rate of flow. Find the diameter of the single pipe, correct to \( \frac{1}{8} \) in.

6. The cross-section of a bar, 6 ft. long, is a trapezium ABCE (parallel sides AB and EC), surmounted by an isosceles triangle DEC (DE = DC). The distances of D and A from EC are \( \frac{1}{2} \) in., \( \frac{3}{4} \) in. respectively; AB = 4 in., EC = 6 in. Find the volume of the bar in cu. ft.
Paper 54

1. (i) Simplify \(\frac{0.05 \times 0.012}{(0.3)^2} \div 0.25\). (ii) Find \(7\frac{3}{4}\%\) of £45.

2. Use logarithms to evaluate, correct to 3 figures:
   (i) \(\pi (17.37^2 - 4.08^2)\); (ii) \(0.3718^{-0.4}\).

3. Find in lb., correct to 3 figures, the weight of 1 cu. in. of mercury, if 1 c.c. of mercury weighs 13.60 gm. [1 in. = 2.540 cm.; 1 lb. = 453.6 gm.]
   Find also the volume of 1 cwt. of mercury in cu. in., correct to 3 figures.

4. What sum of money will yield £162 interest in 2\(\frac{3}{4}\) years at 4\% p.a. simple interest?

5. A cylindrical tank holds 150 litres; its internal height is 65 cm., find its internal diameter in cm., correct to 3 figures.

6. How many spherical shot, each of diameter \(\frac{1}{2}\) in., can be cast from \(\frac{1}{2}\) cu. ft. of lead? Answer correct to 3 figures.

Paper 55

1. (i) Divide 8 tons 5 cwt. 3 qr. of coal equally among 17 pensioners.
   (ii) A penny weighs \(\frac{1}{8}\) oz. and is 1.2 in. in diameter. Find in miles, to 2 figures, the distance 1 ton of pennies placed edge to edge would stretch.

2. Use logarithms to evaluate, correct to 3 figures:
   (i) \(\sqrt[6]{\frac{3.2436 \times (5.321)^2}{19.836}}\); (ii) \(\frac{\log 56.8 - \log 48.5}{\log 32.6}\).

3. A bicycle is geared up to \(x\) cm. when, for every revolution of a pedal, the bicycle moves forward \(\pi x\) cm. Find, correct to 3 figures, the number of revolutions made by a pedal while a bicycle geared up to 180 cm. is travelling \(\frac{3}{4}\) mi. [1 mi. = 1.609 km.]

4. Find the principal which will amount in 4 months to £228 at 4\% p.a.

5. A rectangular block of iron has a square base and is 2.6 in. high; it weighs 65 lb. Find, to 1\(\frac{3}{10}\) in., the side of the base, if 1 cu. ft. of iron weighs 444 lb.

Paper 56

1. (i) Express, correct to 3 figures, 14s. 5d. as a decimal of 18s. 9d.
   (ii) When the price of an article is reduced in the ratio 7:5, the reduction in price is 3s. 6d.; find the reduced price.

2. (i) Use logarithms to evaluate, correct to 3 figures:
   \(0.000482 \times (42.7^3) = \sqrt{8.71 \times 0.46}\).
   (ii) Find \(n\) if \(724 \times (0.273)^n = 5.18\).

3. Find in cu. in., to 3 figures, the volume of a lump of lead weighing 20 kg., given that 1 c.c. of lead weighs 11.37 gm. [1 m. = 39.37 in.]

4. A, B, C, D form a business with a total capital of £11,900. At the end of the year, their shares of the profits are £319, £446 12s., £638, £765 12s., respectively. What amount of capital did each put into the business?

5. The base of a right pyramid is a rectangle 6 in. by 4 in., and each edge of the pyramid is 7 in. long. Find, correct to 3 figures, (i) the volume, (ii) the total area of the surface of the pyramid.

6. A hollow sphere whose radius is 3 in. weighs 28 lb. and is made of metal weighing 448 lb. per cu. ft. Find, correct to 2 figures, the thickness of the metal, assuming it to be uniform.

PAPERS 57-64 (Ch. I-XXV)

Paper 57

1. (i) Taking 1 lb. = 0.4536 kg., express 9\(\frac{1}{2}\) oz. in grams, correct to 3 figures.
   (ii) By what percentage does a speed of 30 mi. an hour exceed a speed of 11 yd. per second?

2. Evaluate, correct to 3 figures:
   (i) \(\sqrt{\frac{(x - b)(x - c)}{x(x - a)}}\) if \(a = 4\), \(b = 5\), \(c = 6\), \(s = \frac{1}{2}(a + b + c)\);
   (ii) \(12 \div (0.7)^{1.4}\).
3. Find, correct to the nearest penny, the compound interest on £215 for 3 years at 4½% p.a.

4. Two tins have the same volume: one is rectangular, 4·32 in. long, 1·72 in. broad, 0·76 in. high; the other is cylindrical, with base-radius 1·31 in.; find its height, correct to 3 figures.

5. 3·68 c.c. of a liquid of specific gravity 0·92 are mixed with 5·14 c.c. of another liquid of specific gravity 0·78. Find, correct to 2 figures, the specific gravity of the mixture (assuming no chemical reaction).

6. A man invested £630 in 4½% stock at 84. What income did he obtain? Later on, he sold out and made a capital profit of £45. At what price did he sell?

Paper 58

1. (i) By what percentage does $$\frac{4}{9}$$ exceed $$\frac{2}{3}$$?
(ii) Find, correct to 1 figure, the difference between 7$$\frac{2}{3}$$ and $$\sqrt[3]{59}$$.

2. Find:
   (i) the value of $$1 + \frac{5}{2} \times \{(0·867)^2 + (0·405)^2\}$$, correct to 3 figures;
   (ii) the least integral value of $$n$$ for which $$(\frac{2}{3})^n$$ is less than 0·001.

3. If £100 is allowed to accumulate at 5½% p.a. compound interest, the amount, to the nearest £, is as follows:

<table>
<thead>
<tr>
<th>Number of years</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>12</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amount in £</td>
<td>100</td>
<td>128</td>
<td>163</td>
<td>180</td>
<td>208</td>
</tr>
</tbody>
</table>

Represent these facts by a graph, and draw with the same scale and axes a graph showing the amount of £100 at 5½% p.a. simple interest. Find from the graph the number of years after which the amount of £100 at 5½% p.a. compound interest (i) first exceeds £140, (ii) first exceeds the amount of £100 at 5½% p.a. simple interest by £20.

4. The diameter of the base of a solid cone of height 6 in. is 9 in. Find, correct to 3 figures, (i) the volume, (ii) the total area of the surface of the cone.

Paper 59

1. (i) Evaluate $$\frac{(2·4)^3 \times (1·8)^2 \times 125}{(10·8)^2 \times 90 \times 0·25}$$.
(ii) Evaluate, correct to 3 figures, $$\sqrt[3]{0·06157} \div \sqrt[3]{0·4183}$$.

2. How many seconds will a train 480 ft. long, travelling at 45 mi. an hour, take to pass completely through a station 170 yd. long?

3. A town of 52,270 inhabitants is supplied with water from a reservoir of surface area 7·63 ac. The consumption of water per day is 185 gall. per 100 inhabitants. Find, correct to 2 figures, the fall in inches of the water-level per day in the reservoir, if no water enters it. [1 cu. ft. = 6·23 gall.]

4. Copper weighs 552 lb. per cu. ft.; find its specific gravity, to 3 figures, by using the relations, 1 kg. = 2·205 lb., 1 m. = 39·37 in.

5. Find, correct to 2 figures, the weight in lb. of iron per cu. ft. if the following rule can be used:

The approximate weight of an iron pipe 1 foot long, r inches thick, and d inches in average diameter (i.e. average of internal and external diameters) is 10 ltd pounds.

6. A man holding £2500 of 3½% Consols sold out at 56 and invested the proceeds in a 5½% stock at 98. Find the increase in his income, to the nearest penny.

Paper 60

1. (i) Taking 1 gall. = 277·3 cu. in., express 1 qt. 1½ pt. in cu. in., correct to 3 figures.
(ii) Find, without using tables, the square root of 1·6, correct to 3 figures.

2. Evaluate, correct to 3 figures:
   (i) $$\frac{1}{(1·56)^2} - \frac{1}{(1·56)^2}$$.
   (ii) 0·064$$\{2·3 \log (72 \div 0·35) + 0·25\}$. 

5. A tradesman marks his goods so that he makes a profit of 15% after giving a discount of $$\frac{2}{9}$$ d. in the shilling off the marked prices. What is the marked price of an article on which his net profit is £1 13s.?

6. A man invests £3564 in a 3½% stock at 88. How much less money need he invest in a 4½% stock at 104 to obtain the same income?
3. On a certain day, 10 kg. of potatoes cost 9 francs in Paris and 14 lb. cost 1s. 6d. in London. If £1 = 78·82 francs, how much cheaper, to the nearest shilling, would it have been to buy one ton of potatoes in Paris instead of in London? [1 lb. = 0·4536 kg.]

4. The cross-section of a bridge is a rectangle 56 ft. long, 21 ft. high, from the lower part of which a semicircle of radius 15 ft. has been removed. The bridge is 16 ft. wide and is made of material weighing 112 lb. per cu. ft. Find in tons, to 3 figures, the weight of the bridge. Find also the cost, to the nearest penny, of painting the curved surface of the under portion of the bridge at 6d. per sq. yd.

5. The composition by weight of a mixture is 70% water, 30% glycerine (specific gravity 1·25). Find the percentage composition by volume of the mixture.

6. By selling a certain amount of a 2⅓% stock at 55⅔ and investing the proceeds in a 6% stock at 87⅔, a man increases his income by £100 a year. How much stock does he sell, assuming that brokerage at ¼ per cent. is charged on each transaction.

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**Paper 61**

1. (i) Express the distance, 122 km., of Ostend from Brussels as a percentage, correct to 3 figures, of the distance, 145 mi., of Cardiff from London. [1 km. = 0·6214 mi.]

(ii) Evaluate, correct to 3 figures, \( \frac{1}{\pi} e^{-h} \), if \( r = 7 \) and \( h = \sqrt{50} \).

2. Two square sheets of metal, one of side 47·3 cm., the other of side 38·6 cm., of the same thickness, were melted down and rolled out into a square sheet of \( \frac{3}{4} \) of this thickness. Find, correct to 3 figures, the length of the side of the final sheet.

3. Find, correct to the nearest penny, the compound interest on £254 for 3 years at 3½% p.a.

4. (i) 20 cu. in. of a substance weigh 3 lb. 7½ oz.; find its specific gravity, to 2 figures. [1 cu. ft. of water weighs 62·3 lb.]

(ii) A metal body weighs 37·3 gm. in air, 28·6 gm. in water, and 21·8 gm. in another liquid. Find, to 2 figures, the specific gravity of (i) the metal, (ii) the other liquid.

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**Paper 62**

5. The cost of manufacturing a certain article can be divided into two parts: the first part (overhead expenses) is the same total sum in the year, whatever number of articles are made; the second part is proportional to the number of articles made. The average cost per article is 6s. when 1000 are made in the year, and is 5s. when 2000 are made in the year. Find the overhead expenses per year, and the average cost per article if 8000 are made in the year.

6. A man buys £300 of 21⅔% stock at 53⅔ and £600 of 3% stock at 98½. What average rate per cent. does he obtain on the money he invests? Later on, he sells his holding in the first stock at 56 and in the second stock at 92, and invests the proceeds in 3⅔% stock at 75. Find the change of income.

---

1. (i) Express, correct to 3 figures, a pressure of 135 lb. per sq. in. in kg. per sq. cm. [1 lb. = 0·4536 kg.; 1 in. = 2·540 cm.]

(ii) Evaluate, correct to 3 figures, \( \sqrt{0·7018} \div (\sqrt{5} + \sqrt{11}) \).

2. When the cost of food increases by 140%, the consumption per head is reduced by 25%. For how many days can 4 persons be fed for the same money that was required previously for feeding 6 persons for 30 days?

3. A solid iron bar, of circular cross-section 4·74 cm. in diameter, is melted down and recast into a hollow pipe of the same length as the bar and of internal radius 3·16 cm.; find, correct to 3 figures, its external radius.

4. Find, to the nearest £, the amount to which £142 accumulates in 30 years at 3⅛% p.a. compound interest. [log 1·03 = 0·0128372.]

5. The specific gravity of gold is 19·3 and of copper is 8·9. An alloy is made of 9 parts gold to 2 parts copper (i) by volume, (ii) by weight. Find the specific gravity of each mixture, correct to 2 figures.

6. Two years ago a man received an income of £51 14s. 7d. from a stock when the dividend was paid at the rate of 3⅜%. Last year he received an income of £87 10s. 10d. from the same stock; at what rate was the dividend paid? What will be obtain if he sells his holding at 93⅔?
Paper 63

1. (i) A locomotive consumes $5\frac{1}{2}$ tons of coal at £1 5s. 7½d. per ton, for a journey of 240 mi. Find the cost per mile, correct to 1½d.

(ii) In what time would a column of 20 battalions, each 320 ft. long, march completely through a defile 1½ mi. long at the ordinary rate of 7½ paces of 2½ ft. each per minute?

2. On a certain day, £1 was worth 4 dollars 77 cents in New York, one dollar was worth 16·07 francs in Paris, and £1 was worth 78·82 francs in London. If a merchant changed 1000 into dollars, then the resulting dollars into francs, and then the resulting francs back into pounds on that day, find to the nearest ten shillings his gain or loss.

3. A cylindrical jar of diameter 11 cm. and of height 11 cm. is half-full of water. A metal sphere of diameter 10·8 cm. is then placed in it. Find in c.c., correct to 2 figures, how much water overflows.

4. A man borrows £575 and is charged interest at 5% each year for the amount owing at the beginning of that year. He repays £200 at the end of the first year, and another £200 at the end of the second year; how much does he still owe at the end of the third year?

5. A body is floating in a liquid of specific gravity 1·24, and the volume of the portion submerged is 52·5 c.c.; find the weight of the body.

6. A man has £3100 of 3½% stock; he sells out at 92½ and invests the proceeds in a 5½% stock, brokerage being charged at ½ per cent. on each transaction. If his income is thereby increased by £31, find the price of the 5½% stock.

Paper 64

1. A man uses 10 lb. 4 oz. of sealing-wax for sealing 5540 envelopes; how much, to the nearest oz., should he use for sealing 7520 envelopes at the same rate? Find, to the nearest penny, the cost of the wax used for 7520 envelopes at 6s. per lb.

2. The rate of exchange varies from 4 dollars 88½ cents to the £ one day to 4 dollars 88½ cents to the £ the next day. How much, to the nearest shilling, will a merchant gain by waiting till the second day to convert 100,000 dollars into English money?
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Find the position of the decimal point by inspection.
APPENDIX TO PART I

CHAPTER I

USE OF PRIME FACTORS

EX. 3 (a). Prime Factors

State which of the following numbers are prime, and express the others in prime factors, using the index notation:

1. 36 2. 38 3. 41 4. 90 5. 96
6. 103 7. 105 8. 112 9. 107 10. 119
16. 343 17. 378 18. 504 19. 627 20. 729
21. 3465 22. 6468 23. 86515 24. 137214

EX. 5 (a). Law of Indices

Express in the index notation:

1. $5^2 \times 5^4$  2. $7^4 \times 7$  3. $b^2 \times b^3$  4. $c^5 \times c^6$
5. $3^6 \div 3^2$  6. $7^4 \div 7$  7. $8^4 \div 8^3$  8. $a^9 \div a^4$
9. $7^4 \times 7 \times 7^2$  10. $3^4 \times 3^4 \div 3$  11. $a^6 \times a^3 \div a^2$
12. $3^7 \div (3 \times 9)$  13. $7^9 \div (7^2 \times 7^3)$  14. $5^2 \div (5^2 \times 5^9)$
15. $2 \times 3 \times 5 \times 3 \times 7 \times 7$. 16. $a \times a \times b \times a \times b$. 17. $a^2 \times c^3 \times a \times c^2$
18. $8 \times 14$  19. $20 \times 25$  20. $3^2 \times 6$. 21. $5^9 \times 20$
22. $12 \times 14 \times 21$. 23. $3^5 \times 15 \times 45$. 24. $2^9 \times 5 \times 20^8$

EX. 6 (a). Powers and Roots

Write down in index form:

1. The square of $2^3 \times 3^4 \times 5^3$.
2. The cube of $2^3 \times 3^8 \times 5^4$.
3. The square root of (i) $3^6$; (ii) $2^4 \times 3^9 \times 5^{18}$.
4. The cube root of (i) $2^9 \times 5^9$; (ii) $2^9 \times 3^{19} \times 7^9$.

COMP.
Write down the values of:
5. \( \sqrt{16 \times 81} \)  
6. \( \sqrt{9 \times 49 \times 144} \)  
7. \( \sqrt[3]{4 \times 18 \times 81} \).

Find the square roots of:
8. 324. 9. 1024. 10. 2025. 11. 3136. 12. 5184.

Find the cube roots of:

Find the least integers by which the following numbers must be multiplied to give perfect squares, and find the square roots of the products obtained:
24. \( 23 \times 3 \). 25. \( 35 \times 5^2 \). 26. 756. 27. 810.
28. Find the least integers by which the following numbers must be multiplied to give perfect cubes, and find the cube roots of the products obtained:
(i) \( 27 \times 3 \times 7^2 \); (ii) 2268.
29. What are the final digits in the squares of the numbers 1, 2, 3, ... 9, 10? Complete the sentence: a perfect square cannot end with a 2 or with ...
30. Find the length of the side of a square whose area is equal to that of a rectangle 189 yd. long, 84 yd. wide.
*31. It costs £528 to fence a square field of area 40 ac. Find the cost of the fencing per foot length.
*32. Find the length of the edge of a cube whose volume is equal to that of a rectangular block, 80 in. by 75 in. by 36 in.

Use the fact that \( a^3 - b^3 = (a + b)(a - b) \) to express in prime factors:
33. \( 31^3 - 13^3 \). 34. \( 75^3 - 57^3 \). *35. \( 86^3 - 36 \).
36. \( 99^3 - 1 \). 37. \( \sqrt{(13^3 - 5^3)} \). 38. \( \sqrt{(25^2 - 7^2)} \). *39. \( 57^2 - 49 \). *40. \( 76^2 - 1 \).

**EX. 8 (a). H.C.F.**

Find by prime factors the H.C.F. of:
1. 112, 420. 2. 462, 588. 3. 98, 75.
4. 603, 1815. 5. 56, 98, 126. 6. 144, 176, 240.

**EX. 10 (a). L.C.M.**

Express in prime factors the L.C.M. of:
1. 8, 14, 42. 2. 8, 12, 45. 3. 16, 26, 36.
4. 27, 48, 78. 5. 14, 28, 30. 6. 42, 63, 105.
7. 38, 44, 57. 8. 36, 48, 300. 9. 57, 84, 91.
10. 35, 75, 98. 11. 18, 28, 49, 84. 12. 32, 54, 72, 108.

**EX. 11 (a). H.C.F. and L.C.M. in Problems**

1. Find the smallest number by which 108 must be multiplied to give a multiple of 80.
2. What is the shortest distance that can be measured exactly both by a 5-ft. pole and by a walking-stick 3 ft. 6 in. long?
3. What is the greatest length which can be divided exactly by 15 or 18 or 24 equal shares, without having fractions of a penny.
4. Find the least sum of money which can be divided either into 15 or 18 or 24 equal shares, without having fractions of a penny.
5. What is the least length of string that can be cut up into an exact number of equal pieces, either 8 in. long or 10 in. long or 18 in. long?
6. Find the smallest number by which 378 must be multiplied to give (i) a perfect square, (ii) a perfect cube.
7. One watch gains 15 sec. a day and another gains 24 sec. a day. They are put right at the same time. What is the least number of days that elapse before both watches are exact numbers of minutes fast?
8. The floor of a room 21 ft. long, 13 ft. 6 in. wide is paved with equal square tiles. Find the largest size of tile and the number required, if only whole tiles are used. What other sizes are possible, if the tiles must be more than 4 in. square.
APPENDIX

9. A rectangular block, 12 ft. 6 in. by 7 ft. 6 in. by 5 ft. 10 in., is built up with equal cubes. What is the largest size that can be used and how many are required?

*10. What is the largest number of 8 digits which is a multiple of 72 and of 96?

*11. What is the least sum of money such that when shared equally either between 9 boys or 10 boys or 15 boys, there is always 2d. left over, if fractions of a penny are not distributed?

*12. Prove that the product of 70 and 126 is equal to the product of their H.C.F. and L.C.M. Prove also that this is true for any pair of numbers. [If the H.C.F. is x, the numbers can be denoted by x × y and x × z, where y and z have no common factor.]

*13. If 36, 48, 300, and a fourth number have 4 as their H.C.F. and $2^6 \times 3^3 \times 5^2$ as their L.C.M., find the fourth number.

14. Find all the divisors of 4049 which give 24 as the remainder.

*15. The numbers 3962, 4085, 4167 when divided by a certain number all leave the same remainder. Find this number and the remainder.

*16. If 150, 190, and 260 are each divided by a certain number, the remainders are 3, 1, 8 respectively. What is the greatest value of this divisor?

CHAPTER II

SIMPLE AREAS AND VOLUMES

EX. 13 (a). Rectangles

Find the areas of the rectangles in Nos. 1–6:

1. 14 in. long, 7 in. high.  
2. 8 ft. square.
3. 4 yd. long, 8 ft. broad.  
4. 1 fur. by 1 ch., in square chains.
5. 1 ft. 3 in. by 2 ft. 6 in., in square inches.
6. 4 yd. 2 ft. by 1 yd. 2 ft., in square feet.
7. Find the area of a sheet of paper 10 in. wide, 1 ft. 8 in. high.
8. Find the area of the floor of a passage 8 yd. long, 7 ft. wide.
9. Find the area of a strip of cloth 5 ft. long, 2 in. wide.
10. How many square inches are there in 3 sq. ft.?

APPENDIX

Find the breadths and perimeters of the rectangles in Nos. 11, 12:

11. Area 54 sq. in., length 9 in.  
12. Area 120 sq. ft., length 24 ft.
13. Find the number of rectangular pieces, 5 in. by 4 in., which can be cut from a rectangle 5 ft. long, 2 ft. wide.

*14. 50 sq. yd. of lint are cut into bandages 4 in. wide; find the total length.

15. Find the total area of the walls of a room, 15 ft. long, 12 ft. wide, 8 ft. high.

*16. Find the cost of a carpet 5 yd. long, 4 yd. wide at 2s. 6d. per square foot.

EX. 14 (a). Figures composed of Rectangles

Find in acres the areas of the rectangular fields, Nos. 1–3:

1. 20 ch. by 12 ch.  
2. 440 yd. by 110 yd.  
3. 2 fur. by 3 fur.
4. A rectangular 20-acre field is 440 yd. long; find its breadth and perimeter.
5. A rectangular enclosure is half a mile long and a quarter of a mile wide; find its area in acres.

Find the areas of the figures, Nos. 6, 7, where all corners are right-angled and dimensions are given in inches.

6.  
7.  

Find the areas of the shaded parts of the figures, Nos. 8, 9, by subtracting one or more areas from another area. All corners are right-angled and dimensions are given in feet.

8.  
9.  

10. In one wall of a room 15 ft. long, 9 ft. high, there is a door 4 ft. wide, 7 ft. high, and the rest of the wall is papered. What area of this wall is papered?
10. How much air-space is there in an empty room 15 ft. long, 12 ft. wide, 8 ft. high?

11. How many cuboids, 2 in. by 3 in. by 4 in., can be packed in a box measuring 2 ft. by 2 ft. by 3 ft. internally?

12. The volume of a rectangular stack of bricks is 540 cu. ft.
   (i) Find the height if the stack is 10 ft. long, 9 ft. wide.
   (ii) Find the area of the base if the stack is 12 ft. high.

**EX. 15 (a). Cuboids**

How many cubic inch blocks are required to build up the following cuboids?

1. 3 in. by 10 in. by 7 in.
2. 1 ft. 6 in. by 1 ft. by 3 in.
3. A 4-in. cube.
5. How many cubic inch blocks can be packed in a rectangular box of internal dimensions 1 ft. 8 in. by 1 ft. 3 in. by 8 in.?

Find the volumes of the following rectangular blocks:

6. 10 in. by 6 in. by 1 ft. 4 in.
7. 2 yd. by 1 yd. by 2 ft.
8. 1 ft. long, 8 in. wide, and 1 in. thick.
9. How many (i) 3-in. cubes, (ii) 4-in. cubes can be packed into a cubical box, 1 ft. high, internal measurement?

*4. The internal measurements of an open wooden box are as follows:
   length 10 in., breadth 8 in., height 7 in.; and the wood
   is half an inch thick. Find the volume of the wood.

*5. A corridor 6 ft. wide, 8 ft. high, runs all round the outside
   of a rectangular enclosure 30 ft. long, 20 ft. wide. Find (i) the
   area of the floor of the corridor, (ii) the volume of the air-space
   in the corridor.

*6. The diagram represents the vertical cross-
   section of a trough, 12 ft. long, open at both
   ends; the corners are right-angled and the
   dimensions are given in inches. Find the
   volume in cubic feet of the wood used in
   making the trough. Also make a freehand sketch of the trough.

7. The diagram represents the upper
   surface of a cross which is 3 in. thick;
   the given dimensions are in inches.
   Find (i) the area of the top surface of the
   cross, (ii) the volume of the solid.
CHAPTER III

EASY UNITARY METHOD

EX. 18 (a). Direct Variation

[Assume that the rates given in this exercise are uniform]

1. If 5 golf balls cost 10s., find the cost of 1 dozen similar golf balls.
2. If 9 yd. of ribbon cost 3s., find the cost of 16 yd.
3. If 6 notebooks cost 2s. 6d., find the cost of 15 notebooks.
4. For 3 shillings I can buy 2 dozen eggs, how many can I buy for 5 shillings?
5. If 10 lb. of lard cost 6s. 8d., find the cost of 14 lb.
6. If 12 copies of a book cost 15s., find the cost of 16 copies.
7. If 9 pears cost 1s. 6d., how many can be bought for 2s. 6d.?
8. Eggs cost half a crown a dozen; find the cost of 8 eggs.
9. If 3 tons of coal cost £10 10s., find the cost of 5 tons.
10. A cook is paid £66 for 12 months; what does she earn in 10 months?
11. If 6 yd. of towelling cost 15s., how much can be bought for 25s.?
12. If 8 in. on a map represent 10 mi., what distance is represented by 20 in. on the map?
13. A dance band plays 6 tunes in three-quarters of an hour; how many does it play in 2 hr. at the same rate?
14. A clerk can address 140 envelopes in 40 min.; how many can he address in 1 hr.? How long will it take him to address 350 envelopes?
15. 5 equal bags of flour weigh 30 lb.; how many similar bags are there in a pile weighing 72 lb.?

EX. 21 (a). Direct and Inverse Variation

[Assume that the rates given in this exercise are uniform unless this assumption is contrary to common sense. If the unitary method cannot be used, give the reason but no other answer]

1. If 60 lb. of tea cost £9, find the cost of 200 lb. at the same rate.
2. It takes 6 hours to go a certain distance at 10 miles an hour; how long will it take at 24 miles an hour?
3. A boy can clean 5 pairs of boots in 20 min.; how many can he clean in 36 min.?
4. A boy grows 5 in. in 4 years; how much does he grow in the next 40 years?
5. If 14 lb. of jam cost 10s. 6d., find the cost of 21 lb. of jam of the same kind.
6. A journey takes 40 min. if the speed is 36 miles an hour; how long does it take if the speed is 60 miles an hour?
7. The annual tax on a 12-horse-power car is £9; what is it on a 16-horse-power car? The tax varies directly as the horsepower.
8. A batsman scored his runs at the rate of 25 runs in 35 min. How many runs did he make if his innings lasted 1 hr. 52 min.?
9. It takes me 20 min. from my house to the station if I walk at 4 miles an hour; how long will it take me at 5 miles an hour?
10. A man with bad teeth has 12 teeth extracted in 2 years; how many are extracted in 10 years?
APPENDIX

11. On a certain map 1 in. represents 4 mi.; what area on the map represents 80 sq. mi.? *12. A car travels 550 yd. in 25 sec.; find its speed in miles per hour.

13. 171 men can do a piece of work in 12 days; how many men are required to do it in 19 days?

14. If 3 men take 6 days to paint a fence, how long will 4 men take to paint a fence of the same height and design, but twice as long?

15. A case full of tea weighs 50 lb.; when empty it weighs 2 lb.; what does it weigh when half-full? when one-quarter full?

CHAPTER IV

FRACTIONS: ADDITION AND SUBTRACTION

EX. 23 (a). Lowest Terms

What fraction is:

1. $\frac{1}{8}$ of 1 lb.
2. $\frac{1}{16}$ of 1 lb.
3. $\frac{15}{16}$ of $\frac{1}{2}$ lb.
4. $\frac{3}{16}$ of 1 lb.
5. $\frac{4}{16}$ of $\frac{1}{2}$ lb.
6. $\frac{3}{16}$ of 1 lb.
7. $\frac{5}{16}$ of $\frac{1}{2}$ lb.
8. $\frac{1}{16}$ of 1 lb.
9. $\frac{11}{16}$ of $\frac{1}{2}$ lb.
10. $\frac{10}{16}$ of 1 lb.

Complete the following:

11. $\frac{3}{10} = \frac{3}{10}$
12. $\frac{3}{15} = \frac{1}{5}$
13. $\frac{3}{20} = \frac{3}{20}$
14. $\frac{8}{15} = \frac{8}{15}$

Express as simply as possible:

15. $\frac{9}{16}$
16. $\frac{16}{16}$
17. $\frac{8}{16}$
18. $\frac{8}{16}$
19. $\frac{8}{16}$
20. $\frac{8}{16}$
21. $\frac{21}{28}$
22. $\frac{22}{28}$
23. $\frac{23}{28}$
24. $\frac{24}{28}$
25. $\frac{25}{28}$
26. $\frac{26}{28}$
27. $\frac{27}{28}$
28. $\frac{28}{28}$
29. $\frac{29}{28}$

Write down the value of $x$ in the following equations:

30. $\frac{3}{9} = \frac{x}{18}$
31. $\frac{x}{3} = 1$
32. $\frac{x}{7} = 28$
33. $\frac{3}{x} = 21$
34. $\frac{9}{x} = 6$

APPENDIX

EX. 24 (a). Lowest Terms

Reduce to their lowest terms:

1. $\frac{4}{8}$
2. $\frac{7}{50}$
3. $\frac{9}{6}$
4. $\frac{7}{50}$
5. $\frac{5}{8}$
6. $\frac{4}{5}$
7. $\frac{3}{50}$
8. $\frac{4}{50}$
9. $\frac{9}{8}$
10. $\frac{9}{8}$
11. $\frac{9}{16}$
12. $\frac{11}{8}$
13. $\frac{11}{16}$
14. $\frac{9}{8}$
15. $\frac{11}{16}$
16. $\frac{9}{16}$
17. $\frac{11}{16}$
18. $\frac{11}{16}$
19. $\frac{11}{16}$
20. $\frac{9}{8}$

EX. 29 (a). Addition

1. $\frac{2}{3} + \frac{1}{3}$
2. $\frac{1}{6} + \frac{1}{6}$
3. $\frac{1}{6} + \frac{1}{6}$
4. $\frac{1}{6} + \frac{1}{6}$
5. $\frac{1}{6} + \frac{1}{6}$
6. $\frac{1}{6} + \frac{1}{6}$
7. $\frac{1}{6} + \frac{1}{6}$
8. $\frac{1}{6} + \frac{1}{6}$
9. $\frac{1}{6} + \frac{1}{6}$
10. $\frac{1}{6} + \frac{1}{6}$
11. $\frac{1}{6} + \frac{1}{6}$
12. $\frac{1}{6} + \frac{1}{6}$
13. $\frac{1}{6} + \frac{1}{6}$
14. $\frac{1}{6} + \frac{1}{6}$
15. $\frac{1}{6} + \frac{1}{6}$

EX. 32 (a). Addition and Subtraction

Simplify:

1. $\frac{2}{3} - \frac{1}{3}$
2. $\frac{1}{3} - \frac{2}{3}$
3. $\frac{8}{9} - \frac{5}{9}$
4. $\frac{3}{4} - \frac{1}{4}$
5. $\frac{2}{3} - \frac{1}{3}$
6. $\frac{2}{3} - \frac{1}{3}$
7. $\frac{1}{2} - \frac{1}{2}$
8. $\frac{1}{2} - \frac{1}{2}$
9. $\frac{1}{2} - \frac{1}{2}$
10. $\frac{1}{2} - \frac{1}{2}$
11. $\frac{1}{2} - \frac{1}{2}$
12. $\frac{1}{2} - \frac{1}{2}$
13. $\frac{1}{2} - \frac{1}{2}$
14. $\frac{1}{2} - \frac{1}{2}$
15. $\frac{1}{2} - \frac{1}{2}$
16. $\frac{1}{2} - \frac{1}{2}$
17. $\frac{1}{2} - \frac{1}{2}$
18. $\frac{1}{2} - \frac{1}{2}$

EX. 33 (a). Problems: Addition and Subtraction

1. The lengths of the sides of a triangle are $\frac{3}{4}$ in., $\frac{3}{4}$ in., $\frac{1}{4}$ in.; find its perimeter.
2. From a roll of cloth 10 yd. long, a man sells lengths of $\frac{5}{8}$ yd. and $\frac{3}{8}$ yd.; what length remains?
3. When I have done $\frac{1}{6}$ of a task, what fraction of it remains to be done? If this will take another 30 min., how long does the whole task take?
4. What must be added to the sum of \(4\frac{3}{7}\) and \(3\frac{1}{9}\) to make 10?

5. What must be subtracted from the sum of \(2\frac{1}{7}\) and \(2\frac{2}{9}\) to leave \(3\frac{1}{3}\)?

6. From a piece of string I cut off \(\frac{3}{4}\) of it; what fraction of it is left? If the remainder is 10 in. long, what was the length of the whole piece?

*7. Prove that \(\frac{5}{6}\) lies between \(\frac{3}{4}\) and \(\frac{1}{2}\).

8. A tile is \(4\frac{5}{8}\) in. long, \(3\frac{1}{2}\) in. wide; find its perimeter.

*9. A man spends \(\frac{5}{6}\) of his income at home, \(\frac{1}{3}\) of his income on holidays, and saves the rest. If he saves \$220 a year, what is his income?

*10. A can mow a field in 8 days, and B can mow it in 10 days. What fraction of the field can they mow in 1 day, if both work together? What fraction of the field remains to be done after both have been working for 4 days?

*11. Two taps together can fill a bath in 4 min., and one of the taps can fill it alone in 6 min. What fraction of the bath will the other tap fill by itself in 1 min. and how long will it take to fill it completely?

*12. One man can dig a trench in 18 days by himself, and with another man in 12 days. How long would the second man take to dig it by himself?

**CHAPTER V**

**FRACTIONS: MULTIPLICATION AND DIVISION**

**EX. 35 (a). Multiplication**

Simplify:

1. \(\frac{1}{4} \times \frac{1}{2}\).
2. \(\frac{2}{3} \times \frac{3}{4}\).
3. \(\frac{3}{5} \times \frac{1}{2}\).
4. \(\frac{2}{9} \times \frac{3}{4}\).
5. \(\frac{5}{7} \times \frac{3}{8}\).
6. \(\frac{9}{14} \times \frac{2}{3}\).
7. \(\frac{7}{12} \times \frac{3}{4}\).
8. \(\frac{3}{16} \times \frac{2}{5}\).
9. \(\frac{1}{8} \times \frac{3}{4}\).
10. \(\frac{2}{9} \times \frac{3}{5}\).
11. \(\frac{5}{12} \times \frac{3}{4}\).
12. \(\frac{4}{9} \times \frac{3}{4}\).
13. \(\frac{5}{9} \times \frac{2}{3}\).
14. \(\frac{3}{4} \times \frac{2}{5}\).
15. \(\frac{2}{5} \times \frac{3}{4}\).
16. \(\frac{9}{16} \times \frac{1}{2}\).
17. \(\frac{1}{8} \times \frac{3}{4}\).
18. \(\frac{3}{4} \times \frac{5}{12}\).
19. \(\frac{3}{5} \times \frac{2}{3}\).
20. \(\frac{4}{5} \times \frac{2}{5}\).
21. \(\frac{3}{4} \times \frac{2}{3}\).
22. \(\frac{1}{4} \times \frac{1}{2}\).

**EX. 37 (a). Division**

Simplify:

1. \(\frac{2}{3} \div \frac{1}{2}\).
2. \(\frac{1}{3} \div \frac{1}{2}\).
3. \(\frac{1}{2} \div \frac{1}{3}\).
4. \(2 \div \frac{3}{4}\).
5. \(\frac{2}{3} \div \frac{1}{2}\).
6. \(\frac{3}{5} \div \frac{1}{2}\).
7. \(\frac{3}{4} \div \frac{1}{3}\).
8. \(\frac{1}{2} \div \frac{1}{3}\).
9. \(1 \times \frac{1}{4}\).
10. \(2 \times \frac{1}{2}\).
11. \(\frac{1}{10} \times \frac{1}{10}\).
12. \(\frac{2}{3} \times \frac{1}{2}\).
13. \(\frac{3}{5} \times \frac{1}{2}\).
14. \(\frac{4}{5} \times \frac{1}{2}\).
15. \(\frac{1}{3} \times \frac{1}{2}\).
16. \(\frac{1}{4} \times \frac{1}{2}\).
17. \(\frac{1}{5} \times \frac{1}{2}\).
18. \(\frac{1}{6} \times \frac{1}{2}\).
19. \(\frac{1}{7} \times \frac{1}{2}\).
20. \(\frac{1}{8} \times \frac{1}{2}\).

**EX. 38 (a). Miscellaneous Fractions**

Simplify:

1. \(\frac{2}{3} - \frac{1}{2}\).
2. \(\frac{3}{4} - \frac{1}{2}\).
3. \(\frac{1}{2} - \frac{1}{3}\).
4. \(\frac{1}{4} \times \frac{1}{2}\).
5. \(\frac{5}{8} \times \frac{1}{4}\).
6. \(\frac{3}{4} \times \frac{1}{2}\).
7. \(\frac{1}{2} \times \frac{1}{4}\).
8. \(\frac{3}{5} \times \frac{1}{3}\).
9. \(\frac{2}{3} \times \frac{1}{2}\).
10. \(\frac{1}{2} \times \frac{1}{3}\).
11. \(\frac{1}{4} - \frac{1}{2}\).
12. \(\frac{1}{5} \times \frac{1}{2}\).
13. \(\frac{1}{6} \times \frac{1}{2}\).
14. \(\frac{1}{7} \times \frac{1}{2}\).
15. \(\frac{1}{8} \times \frac{1}{2}\).
16. \(\frac{1}{9} \times \frac{1}{2}\).
17. \(\frac{1}{10} \times \frac{1}{2}\).
18. \(\frac{1}{11} \times \frac{1}{2}\).
19. \(\frac{1}{12} \times \frac{1}{2}\).
20. \(\frac{1}{13} \times \frac{1}{2}\).

*21. \(\frac{1}{14} \times \frac{1}{2}\).
*22. \(\frac{1}{15} \times \frac{1}{2}\).
*23. \(\frac{1}{16} \times \frac{1}{2}\).
*24. \(\frac{1}{17} \times \frac{1}{2}\).
*25. \(\frac{1}{18} \times \frac{1}{2}\).
*26. \(\frac{1}{19} \times \frac{1}{2}\).
*27. \(\frac{1}{20} \times \frac{1}{2}\).
*28. \(\frac{1}{21} \times \frac{1}{2}\).
*29. \(\frac{1}{22} \times \frac{1}{2}\).
*30. \(\frac{1}{23} \times \frac{1}{2}\).

*21. \(\frac{1}{24} \times \frac{1}{2}\).
*22. \(\frac{1}{25} \times \frac{1}{2}\).
*23. \(\frac{1}{26} \times \frac{1}{2}\).
*24. \(\frac{1}{27} \times \frac{1}{2}\).
*25. \(\frac{1}{28} \times \frac{1}{2}\).
*26. \(\frac{1}{29} \times \frac{1}{2}\).
*27. \(\frac{1}{30} \times \frac{1}{2}\).
*28. \(\frac{1}{31} \times \frac{1}{2}\).
*29. \(\frac{1}{32} \times \frac{1}{2}\).
*30. \(\frac{1}{33} \times \frac{1}{2}\).

*21. \(\frac{1}{34} \times \frac{1}{2}\).
*22. \(\frac{1}{35} \times \frac{1}{2}\).
*23. \(\frac{1}{36} \times \frac{1}{2}\).
*24. \(\frac{1}{37} \times \frac{1}{2}\).
*25. \(\frac{1}{38} \times \frac{1}{2}\).
*26. \(\frac{1}{39} \times \frac{1}{2}\).
*27. \(\frac{1}{40} \times \frac{1}{2}\).
*28. \(\frac{1}{41} \times \frac{1}{2}\).
*29. \(\frac{1}{42} \times \frac{1}{2}\).
*30. \(\frac{1}{43} \times \frac{1}{2}\).
EX. 40 (a). Use of Fractions in Problems

1. What fraction is 1s. 6d. of half a crown?
2. Express $6\frac{2}{3}$ oz. as a fraction of 1 lb.
3. If 5 ml. = 8 km., express 3 ml. in kilometres, and express 5 km. in miles.
4. Divide $(2\frac{1}{3})^3$ by $(4\frac{1}{3})^2$.
5. Find the cost of $\frac{3}{4}$ yd. of material at 5s. 4d. per yard.
6. What must be added to $\frac{1}{2}$ to make $\frac{3}{4}$?
7. By what must 4$\frac{1}{2}$ be divided to give 2$\frac{1}{2}$?
8. The product of two numbers is $5\frac{1}{2}$; one of them is $7\frac{1}{3}$; find the other.
9. Find the volume of a rectangular block, 2$\frac{1}{2}$ in. long, 2$\frac{1}{4}$ in. wide, 2$\frac{3}{4}$ in. high.

*10. A pile of 8 equal note-books is 5 in. high; how high is the pile when 2 books have been removed?

11. How many pieces of tape, each 1$\frac{1}{2}$ in. long, can be cut from a piece 12$\frac{3}{4}$ in. long?
12. How many glasses, each holding $\frac{3}{4}$ pt., can be filled from a jug containing 10 pt.?
13. How many strips of cloth, each 2$\frac{1}{3}$ ft. long, can be cut from a roll 35 yd. long?

*14. A carpet is 12 ft. long, 10 ft. wide; what would be the increase in the area if the length and breadth were each 6 in. more?

15. If $\frac{3}{4}$ yd. of lace costs 7s. 6d., what is the cost of $\frac{2}{3}$ yd. of the lace?
16. $\frac{1}{4}$ pt. of milk are poured into an urn containing 2$\frac{1}{2}$ gal. of tea. What fraction of the mixture is milk?
17. If $\frac{2}{3}$ of a bath can be filled in 15 min., how long does it take to fill the whole bath?

*18. If $\frac{3}{4}$ of an acre is worth £38 10s., what is the value of $\frac{3}{8}$ of an acre at the same rate?

19. It takes 2 hr. to do $\frac{7}{9}$ of a journey; how long will the rest of it take at the same rate?
20. A man mows $\frac{4}{5}$ of a lawn in 32$\frac{2}{3}$ min.; how long will the rest of it take at the same rate?

CHAPTER VI

DECIMALS: NUMERATION, ADDITION AND SUBTRACTION

EX. 44 (a). Decimal Notation

Multiply each of the following numbers by 10, by 100, by 1000:—

1. 7.2; 70.5. 2. 0.8; 0.0816. 3. 40.7; 0.00605.

Divide each of the following numbers by 10, by 100, by 1000:—

4. 30; 30.5. 5. 406; 0.503. 6. 0.072; 0.009.

Write down the values of the following:—

7. 8.4 × 10. 8. 4.03 ÷ 10. 9. 0.704 × 100.
10. 0.72 ÷ 100. 11. 5.007 × 10. 12. 30.5 × 1000.
13. 3.0702 × 1000. 14. 400 ÷ 1000. 15. 0.0075 ÷ 1000.

Write down as decimals:

16. $\frac{7}{6}$. 17. $\frac{13}{5}$. 18. $\frac{3}{8}$. 19. $\frac{97}{100}$. 20. $\frac{8}{100}$.
21. $\frac{98}{100}$. 22. $\frac{0.8}{100}$. 23. $\frac{0.03}{100}$. 
Express as common fractions in their lowest terms:
24. 0·8. 25. 0·05. 26. 0·125. 27. 0·008. 28. 0·31.
29. 0·009. 30. 0·618. 31. 0·043.

Express in metres:
32. 0·075 km. 33. 30 cm. 34. 5 cm. 35. 25 mm.

Express in kilometres:
36. 420 m. 37. 75 m. 38. 180 cm. 39. 60 mm.

Complete the following:
40. 0·38 × ... = 38. 41. 5·6 ÷ ... = 0·0056. 42. 0·01 × ... = 10.
43. 100 ÷ ... = 0·1. 44. 0·045 × ... = 450. 45. 1 ÷ ... = 0·001.

**EX. 45 (a). Addition and Subtraction**

Add together, and express each answer in terms of the highest unit named:

<table>
<thead>
<tr>
<th>1. km. Hm. Dm.</th>
<th>2. m. dm. cm. mm.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 6 7</td>
<td>2 ... 7 5</td>
</tr>
<tr>
<td>2 9 8</td>
<td>4 3 8</td>
</tr>
<tr>
<td>2 3 5</td>
<td>1 6 2 5</td>
</tr>
</tbody>
</table>

Add together:

<table>
<thead>
<tr>
<th>3. 5·47</th>
<th>4. 3·56</th>
</tr>
</thead>
<tbody>
<tr>
<td>0·68</td>
<td>6·44</td>
</tr>
<tr>
<td>2·09</td>
<td>8·07</td>
</tr>
<tr>
<td>7·91</td>
<td>0·294</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>11. 8·725</th>
<th>12. 70·06</th>
</tr>
</thead>
<tbody>
<tr>
<td>0·096</td>
<td>9·9</td>
</tr>
<tr>
<td>7·08</td>
<td>15·48</td>
</tr>
<tr>
<td>9·129</td>
<td>20·06</td>
</tr>
</tbody>
</table>

| 15. 3·64, 0·568, 12·07, 5·109, 2·333. |

| 16. 54·01, 1·865, 12·505, 90·4, 6·28. |
| 17. 0·207, 0·0809, 0·0072, 0·105, 0·0048. |

Subtract the second quantity from the first, and express each answer in terms of the highest unit named:

<table>
<thead>
<tr>
<th>18. m. dm. cm.</th>
<th>19. km. Hm. Dm.</th>
<th>20. dm. cm. mm.</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 8</td>
<td>7 ... 5</td>
<td></td>
</tr>
<tr>
<td>2 3 5</td>
<td>2 9 8</td>
<td></td>
</tr>
</tbody>
</table>

Find the value of:
21. 3·6 - 0·8. 22. 4 - 0·4. 23. 3·1 - 0·06. 24. 2 - 0·05.
25. 0·7 - 0·03. 26. 0·01 - 0·001. 27. 2·34 - 0·8. 28. 0·837 - 0·79.
29. 0·736 - 0·06.

Subtract the second number from the first:
30. 6·48 - 0·762. 31. 5·43 - 0·501.
32. 2·18 - 0·48. 33. 2·76 - 0·164.
34. 0·74 - 0·072. 35. 1·56 - 0·357.
36. 9·69 - 0·0651. 37. 0·82.

Find the value of:
38. 0·615 - 0·475. 39. 17 - 8·634. 40. 2·001 - 1·738.
41. 10 - 0·655. 42. 100 - 9·95. 43. 1·01 - 0·101.
44. 1·5 + 7·3 - 3·8 - 1·2. 45. 4·6 - 2·9 + 0·1 - 1·8.
46. 8 - 7·6 + 0·5 - 0·59. 47. 0·06 + 0·04 - 0·05 - 0·005.

**EX. 46 (a). Problems: Addition and Subtraction**

1. A, B, C are 3 houses in this order along a straight road. The distance of A from B is 0·85 km, and the distance of B from C is 0·37 km.; find in metres the distance of A from C.
2. Find in metres the height of a pile of 10 books, if each book is 4·5 cm. thick.
3. If 1 m = 1·0936 yd., find in yards the difference between 100 m. and 110 yd.
4. A man walks 1·75 m. a second; find in kilometres the distance he walks in 100 sec.
5. A boy's temperature rises from 99·5 degrees to 101·2 degrees; find the increase in temperature.
6. A rectangle is 7·6 cm. long, 5·7 cm. wide; find in decimetres its perimeter.
7. If 1 mi. = 1·609 km., express 3·5 mi. in metres.
8. If 1 m. = 39·37 in., express 2 mm. in inches.
9. 10 turns of a screw-head make the point advance 2·34 cm.; how many millimetres does the point advance at each turn?
10. The rainfall for a year was 31.2 in.; 2.7 in. fell in January and 1.6 in. in February. How much fell in the last ten months?

11. Express the difference between 2 dm. and 155 mm. in metres.

12. A carpet 4.5 m. long, 3.8 m. wide is laid in a room, and this leaves a border 0.75 m. wide all the way round. Find the length and breadth of the room.

13. The internal dimensions of an open wooden box are 2.36 dm. by 1.54 dm. by 0.75 dm. high, and the wood is 12 mm. thick. Find in decimetres the external dimensions of the box.

14. A man walks 8.25 km. from his house to a town, and then returns home. When he has walked 5.7 km. back, how far has he walked altogether and how much farther has he to go?

CHAPTER VII

DECIMALS: MULTIPLICATION AND DIVISION

EX. 48 (a). Short Multiplication

Multiply:
1. 36.8 by 2, 4, 6.  2. 0.465 by 3, 8, 10.  3. 8.25 by 4, 7, 9.
4. 0.0625 by 5, 7, 8.  5. 0.0035 by 6, 10, 12.  6. 0.025 by 4, 8, 11.
7. 0.714 by 5, 9, 12.  8. 10.05 by 8, 11, 12.  9. 0.0096 by 5, 10, 12.
10. 0.674 by 300, 50.  11. 0.0085 by 6000.  12. 0.0575 by 800.
13. 0.0909 by 1200.  14. 3.0875 by 1100.  15. 0.4076 by 5000.

EX. 50 (a). Long Multiplication

[Nos. 1-26 are intended for oral work]

Write down the values:
1. 0.9 × 0.5.  2. 7 × 0.07.  3. 0.5 × 8.  4. 0.04 × 0.4.
5. 0.1 × 0.01.  6. 0.3 × 1.1.  7. 0.25 × 0.4.  8. 0.8 × 1.2.
9. (0.4)².  10. (0.01)².  11. (0.003)².  12. (0.11)².
13. (0.1)².  14. (0.2)².  15. 30 × 0.02.  16. 2.5 × 0.02.
17. 12 × 0.03.  18. 9 × 0.004.  19. 700 × 1.1.  20. 0.12 × 0.11.
21. 0.2 × 0.3 × 0.04.  22. 1.2 × 0.05 × 0.4.  23. 1.1 × 0.03 × 0.07.
24. (0.1)³ × 47.  25. (0.02)² × 0.5.  26. (0.3)³ × (0.1)².

APPENDIX

EX. 83 (a). Short Division

Express as decimals:
1. 0.05  \div 0.05  2. 0.05  \div 0.005  3. 0.005  \div 0.0005

Express as common fractions in their lowest terms:
4. 0.0875  5. 0.225  6. 0.0016  7. 0.775  8. 0.0704

Complete the following:
7. \frac{21}{0.07}  \div \ldots  8. \frac{0.45}{0.90}  \div \ldots  9. \frac{0.132}{0.011}  \div \ldots

Transform the following fractions so that the denominator becomes a whole number not greater than 12:
10. \frac{13}{0.0005}  11. \frac{4.8}{0.0012}  12. \frac{10.01}{11000}  13. \frac{1}{0.008}

Express as decimals (or whole numbers):
14. 2.5  \div 50  15. 0.84  \div 70  16. 0.07  \div 2  17. 12  \div 0.06.
18. 3  \div 0.4  19. 0.171  \div 30  20. 0.333  \div 0.09  21. 20.04  \div 1200.
22. 1.32  \div 11  23. 5.004  \div 800  24. 1.001  \div 110  25. 0.035  \div 70.
26. \frac{29}{40}  27. \frac{2.7}{0.05}  28. \frac{9}{650}  29. 57
EX. 56 (a). Division by Factors

Find the values of:

1. \(462 \div 22\).  2. \(10.5 \div 280\).  3. \(0.132 \div 2.4\).
4. \(36.4 \div 0.56\).  5. \(0.0816 \div 0.048\).  6. \(1242 \div 5400\).
7. \(7854 \div 1.68\).  8. \(9.45 \div 6300\).  9. \(0.231 \div 1.32\).

Evaluate correct to 2 places of decimals:

10. \(17 \div 27\).  11. \(0.382 \div 0.44\).  12. \(10.06 \div 7.2\).
13. \(0.01 \div 0.081\).  14. \(0.905 \div 0.108\).  15. \(100 \div 14.4\).

Express as decimals correct to 3 places of decimals:

16. \(\frac{17}{40}\).  17. \(\frac{2.67}{2.80}\).  18. \(\frac{8.3}{8.6}\).  19. \(\frac{2.5}{2.2}\).

EX. 57 (a). Long Division

Find the values of:

1. \(323 + 1.7; 3.23 \div 0.17; 3.23 \div 170\).
2. \(209 \div 3.8; 0.209 \div 3.8; 20.9 \div 0.38\).
3. \(98.9 \div 0.46; 9.89 \div 46; 0.989 \div 0.046\).
4. \(22.12 \div 7.9\).  5. \(933.1 \div 43\).  6. \(0.9267 \div 69\).
7. \(0.3431 \div 73\).  8. \(39.01 \div 83\).  9. \(0.4579 \div 241\).
10. \(0.1189 \div 0.58\).  11. \(8.05 \div 0.0023\).  12. \(337.26 \div 73\).
13. \(459.8 \div 0.076\).  14. \(0.3521 \div 7.38\).  15. \(0.124 \div 15.5\).
16. \(197.2 \div 0.136\).  17. \(1.5939 \div 49.5\).  18. \(1428 \div 0.255\).
19. \(1.2 \div 0.075\).  20. \(2.7633 \div 90.6\).  21. \(13.776 \div 0.123\).
22. \(672.88 \div 0.647\).  23. \(46332 \div 0.00162\).  24. \(0.033702 \div 24.6\).
25. \(0.0468 \div 0.002925\).  26. \(0.307326 \div 30.13\).
27. \(0.0011977 \div 0.1015\).  28. \(21.249 \div 0.030307\).
29. \(0.061103 \div 0.301\).  30. \(712.8576 \div 28560\).
31. \(1.3852242 \div 5130.46\).  32. \(0.29779974 \div 47.2698\).

Evaluate correct to 3 places of decimals:

33. \(0.087 \div 0.693\).  34. \(1 \div 2.87\).  35. \(0.683 \div 0.91\).
36. \(100 \div 836\).  37. \(0.0038 \div 0.053\).  38. \(1.01 \div 1.001\).

APPENDIX

EX. 59 (a). Decimalisation of Money

Express as a decimal of £1:

1. 11s. 6d.  2. 12s. 9d.  3. 7s. 3d.  4. 1s. 6d.
5. 13s. 7½d.  6. 19s. 10½d.  7. 8s. 8½d.  8. 5s. 9½d.
9. 3¾d.  10. 5½d.  11. £5 13s. 7½d.  12. £3 8s. 2¼d.

Express as a decimal of £1, correct to 3 places:

13. 9s. 8d.  14. 13s. 10d.  15. 7s. 9¾d.  16. 16s. 7¾d.
17. £4 9s. 2d.  18. £1 0s. 5d.  19. £9 7s. 1¾d.  20. £8 13s. 1¾d.

Express as a decimal of £1, correct to 4 places:

21. 16s. 5½d.  22. 1s. 8¾d.  23. £1 17s. 3½d.  24. £4 6s. 11¾d.

Find the value, correct to the nearest penny, of:

25. £0.47.  26. £0.325.  27. £0.886.  28. £0.627.
29. £0.905.  30. £0.513.  31. £3.0947.  32. £2.7048.
33. £2.6127.  34. £5.0886.  35. £1.9039.  36. £4.5108.

Express the first sum as a decimal of the second, correct to 3 places:

37. £1 14s. 10d.; £2.  38. £2 7s. 8¾d.; £5.  39. £7 19s. 6¾d.; £10.

Find the value, correct to the nearest penny, of:

40. 0·5186 of £4.  41. 0·6324 of £2 10s.  42. 0·3076 of £10 10s.
EX. 61 (a). Decimalisation of Compound Quantities

[Give the answers to Nos. 1-13 correct to 3 places of decimals]

Express as a decimal of 1 ton:
1. 13 cwt. 1 qr. 2. 8 cwt. 2 qr. 14 lb. 3. 17 cwt. 84 lb.
4. 5 cwt. 1 qr. 22 lb. 5. 1 cwt. 7 st. 11 lb. 6. 6 cwt. 55 lb. 6 oz.

Express as a decimal of 1 mile:
7. 3 fur. 7 ch. 8. 63 ch. 15 yd. 9. 7 fur. 15 p.

Express the first quantity as a decimal of the second:
10. 14 hr. 48 min.; 1 day. 11. 13 yd. 1 ft. 8 in.; 1 ch.
12. 3 gall. 2 qt. 1 pt.; 1 gall. 13. 5000 sec.; 1 hr.

Express in tons, cwt., qr., lb., to nearest lb.:
14. 5.5093 tons. 15. 1.005 tons. 16. 58.0709 cwt.

Express as compound quantities, correct to the nearest unit of the lowest given denomination:
17. 0.493 mi. (ch., yd., ft.). 18. 3.708 fur. (fur., yd., ft.).
19. 2.837 gall. (gall., qt., pt.). 20. 0.2728 cwt. (qr., lb., oz.).

Express the first quantity as a decimal of the second, correct to 3 places:
21. 9 cwt. 1 qr.; 4 tons. 22. 14 fur. 7 ch.; 5 mi. 23. 1 ft. 3½ in.; 2½ yd.

Find the value of:
24. 0.3074 of 4 tons, in tons, cwt., lb., to nearest lb.
25. 0.8127 of 5 mi., in mi., yd., to nearest yd.
27. 0.4185 of ¾ hr., in min., sec., to nearest sec.

EX. 63 (a). Miscellaneous Examples

1. Write down correct to 3 places of decimals:
   (i) 0.64357; (ii) 0.07018; (iii) 0.46973.
2. Subtract (0.4)² from (0.5)².
3. Express £3.7948 in £ s. d. to nearest farthing.
4. Simplify 0.0625 x 0.064.
5. Divide 0.042864 by 70.5.
6. Express 13s. 4½d. as a decimal of £1, correct to 4 places.
7. Express 0.67 ton in cwt., qr. to nearest qr.
8. Given that 1 km. = 0.6214 mi., find the number of yards in 1 km., to the nearest yard.
9. Simplify (3-3)² + (4-1)² - 2(4-2)².
10. Express £2.628125 in £ s. d.
11. Express as decimals: (i) ¾; (ii) 1/8; (iii) 8/32 x 1/54.
12. Express 6 hr. 15 min. 23 sec. as a decimal of 24 hr., correct to 3 places.
13. Find to the nearest penny the value of 0.318 of £2 5s.
14. Find in inches the difference between 0.135 yd. and 0.417 ft.
15. The perimeter of a rectangle is 17 cm.; its length is 5.65 cm., find its breadth.
16. Express as decimals to 5 places: (i) 3/8; (ii) 7/32; (iii) 5-1/100 - 2/100; and find which is nearest to 1/π = 0.31831.
17. The diameter of a penny is 1.2 in.; how many pennies are needed to form a line 1 mi. long, and what is their value?

Simplify the following:
18. 36.8 x 0.005 19. 5.5 x 0.175 20. 0.276 x 89
   0.125 0.28 x 0.275 1.6 x 7.5
21. 0.0065 x 2.25 22. (3.6)² x (1.6)² x 125
   0.125 x 0.39 0.28 x 0.275 x 80 x 0.25
23. A packet of paper 1.65 in. thick contains 450 sheets; find the thickness of each sheet correct to 1/1000 in.
24. Find x and y if x + y = 0.1 and x = 4y.
25. Equal pieces of string 20.7 cm. long are cut from a length of 50 m. How many pieces can be obtained and how much remains?
26. Construct the Siamese table of measure for lengths from the following approximate English equivalents:
   1 Niu = 0.83 inch; 1 Ru'p = 10 inches;
   1 Sen = 44.4 yards; 1 Roemeng = 2.525 miles.
CHAPTER VIII
METRIC SYSTEM

EX. 64 (a). Metric Units

1. Use a ruler graduated in cm. and a piece of squared paper ruled in tenths of an inch to express (i) 4 in. in cm.; (ii) 12 cm. in inches. Hence express approximately 1 in. in cm., and 1 cm. in inches.

2. 1 in. = 2.54 cm.; express in cm. (i) 10 in., (ii) 1 ft.; and express 1 yd. in metres to 2 places of decimals.

3. 8 km. = 5 mi. About how many miles and how many kilometres can you bicycle in 1 hr.?

4. How many sq. cm. are there in 1 sq. m.? Express 500 sq. cm. in sq. m.

5. Express in litres, 2 cu. dm., 5000 c.c., 250 c.c.

6. Express in c.c., 3 litres, 0.5 litre, 0.04 litre.

7. What is the weight of (i) 50 c.c. of water; (ii) 3 cu. dm. of water; (iii) 5 litres of water?

8. How many c.c. of water weigh 5 kg., 0.08 kg.?

9. 1 kg. = 2.2 lb.; find approximately the weight in lb. of 3 litres of water.

10. Express (i) 4 fr. 75 c. in fr.; (ii) 20 dollars 80 cents in dollars.

11. Cork is about one-quarter of the weight of water. Find in gm. the weight of 2 cu. dm. of cork, and the volume in c.c. of a piece of cork which weighs 100 gm.

12. A gallon of water weighs 10 lb. and a litre of water weighs 2.2 lb. approximately. Express a litre as a decimal of a gallon.

EX. 65 (a). Addition and Subtraction

1. Express in grams:
   (i) 5 Dg. 6 dg.; (ii) 8 cg.; (iii) 3 dg. 25 mg.

Add and give the answers in Kg.:

2. 2 Kg. 5 Hg.; 6 Hg. 7 gm.; 5 Dg. 3 gm.

3. 85 gm.; 4 Dg. 5 dg.; 45 dg.

4. Subtract the second quantity from the first, and give the answers in Kg.:

5. 3 Kg. 4 Hg.; 650 gm.

6. 5 Hg. 5 gm.; 23 Dg.

7. Add 3 Kg. 17 gm.; 25 Hg.; 47 Dg.; answer in Kg.

8. Express in dollars the sum of $27 80c., $15 50c., $16 75c.; and subtract the sum from $75.

Find the values of:

9. 3 Kg. 8 Hg. 7 gm. + 9 Hg. 4 Dg. 6 gm. + 2 Kg. 5 Dg. 3 gm.

10. 15 l. 2 dl. 1 cl. – 8 l. 4 dl. 3 cl. (in l.).

11. 6 Dm. 4 cm. + 2 Hm. 5 m. 1 dm. – 9 m. 6 dm. 8 cm. (in m.).

12. The capacities of three bottles are 350 c.c., 425 c.c., 675 c.c., respectively. Find their total capacity in litres.

13. A tank of capacity 1 cu. m. contains 80 litres of water; how many more litres will it hold?

14. The external diameter of a hollow pipe is 9.2 cm., and the metal is 7 mm. thick; find the internal diameter in cm.

15. A flask weighs 51.64 gm. when empty and 143.52 gm. when full of water. Find (i) the weight of the water; (ii) the capacity of the flask in c.c.

16. From a cubical wooden block of edge 1 dm., a cuboid 5 cm. by 6 cm. by 9 cm. is removed. Find in cu. dm. the volume of what remains.

17. What change remains from 400 francs after paying bills of amounts 24 fr. 50 c., 182 fr. 75 c., 94 fr. 75 c.?

EX. 66 (a). Multiplication and Division

1. If £1 = 82 fr. 50 c., express in francs (i) £4; (ii) £2 10s.

2. 25 equal marbles weigh 1.5 Kg.; find the weight of 1 marble in grams.

3. Express 50 gall. in litres, to the nearest l. [1 gall. = 4.546 l.]

4. Express in sq. cm. the area of a rectangle 0.8 m. long, 6.5 mm. wide.
5. Find the cost of 6.5 m. of ribbon at 4 fr. 50 c. per metre.
6. 2\(\frac{1}{2}\) litres of petrol weigh 1.7 Kg.; find the weight of petrol in grams per c.c.
7. If 17.5 metres of tape cost 14 fr., find the cost per metre.
8. Find in Kg., the weight of a cuboid of lead, 8 cm. by 6 cm. by 1.5 cm., if 1 c.c. of lead weighs 11.5 gm.

9. The area of a rectangular mat is 1.5 sq. m.; its breadth is 75 cm., find its length in dm.

10. How many glasses each holding 450 c.c. can be filled from a jug containing 4 litres of beer, and how much remains?

11. The bridge at Brooklyn is 5990 ft. long; express this in kilometres to 2 places of decimals. [1 ft. = 0.3048 m.]

12. Express 2 gall. 3 qt. in litres to nearest cl. [1 gall. = 4.546 l.]

13. Taking £1 = 82.5 fr., express in francs, to the nearest franc, (i) £3 12s.; (ii) 16s. 10d.

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Find the value of:

13. 865 + 704 + 356.
14. 538 + 98 + 607.
15. 7855 + 19025 + 8017 + 10779 + 8693.
16. 3560 + 6595 + 20706 + 389 + 4093.

Subtract the second number from the first:

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D 2. MULTIPLICATION

Multiply:
1. 19, 37, 56, 48 by 7. 2. 25, 74, 86, 39 by 9.
3. 40, 28, 93, 57 by 11. 4. 73, 66, 29, 85 by 12.
5. 6087 by 5, 6, 8, 40. 6. 5974 by 20, 300, 5, 9.
7. 73084 by 39, 85. 8. 60809 by 74, 630.
16. 1090 × 999. 17. 20748 × 5695. 18. 8097 × 6085.
19. 7090 × 8206. 20. 92809 × 60007. 21. 75135 × 46005.
25. 603 × 433 × 976.

D 3. DIVISION

Find the quotient and remainder:
1. 32468 ÷ 6. 2. 80719 ÷ 7. 3. 41967 ÷ 8.
4. 67094 ÷ 9. 5. 48070 ÷ 11. 6. 69358 ÷ 12.
7. 98708 ÷ 40. 8. 77165 ÷ 300. 9. 37050 ÷ 200.
10. 68635 ÷ 500. 11. 98989 ÷ 110. 12. 74940 ÷ 90.
16. 364917 ÷ 409. 17. 710230 ÷ 717. 18. 829645 ÷ 529.
22. 3876125 ÷ 39. 23. 8050563 ÷ 84. 24. 4721835 ÷ 75.

D 4. REDUCTION

Reduce:
1. £3 8s. 2d.; £5 11s. 8d.; £8 17s. 10d.; to pence.
2. £3 17s.; £12 13s. 6d.; £19 1s. 6d.; to sixpences.
3. £10 11s.; £14 7s. 6d.; £20 14s. 9d.; to threepences.
4. 8693d.; 10,715d.; 91,447d.; to £ s. d.
5. 307,042 farthings; 89,761 halfpence; 5708s. shillings; to £ s. d.
6. 7 tons 4 cwt. 3 qr. to qr. 7. 13 cwt. 2 qr. 20 lb. to lb.
8. 3 tons 11 cwt. 75 lb. to lb. 9. 5 cwt. 1 qr. 12 lb. to oz.
10. 4 mi. 5 fur. 6 ch. to ch. 11. 1 mi. 3 fur. 2 ch. to yd.
12. 3 fur. 100 yd. to ft. 13. 5 mi. 1000 yd. to in.
14. 3 ac. 6 sq. ch. to sq. yd. 15. 8 sq. mi. 250 ac. to ac.

D 5. COMPOUND ADDITION AND SUBTRACTION

Add together:
1. 1 qr. 12 lb. 8 oz., 3 qr. 19 lb. 14 oz., 2 qr. 23 lb. 11 oz., 3 qr. 15 lb.
2. 13 cwt. 57 lb., 12 cwt. 43 lb., 9 cwt. 100 lb., 8 cwt. 3 qr. 14 lb.
3. 2 tons 7 cwt. 35 lb., 14 tons 16 cwt. 84 lb., 19 tons 18 cwt. 105 lb.
4. 13 cwt. 1 qr. 12 oz., 17 cwt. 80 lb. 9 oz., 11 cwt. 2 qr. 17 lb.
5. 2 tons 16 cwt. 3 qr., 17 cwt. 2 qr. 20 lb. 12 oz., 2 qr. 16 lb. 4 oz.
6. 5 ch. 17 yd. 2 ft., 4 ch. 9 yd. 2 ft., 3 ch. 8 yd. 1 ft., 100 yd.
7. 2 mi. 5 fur. 4 ch., 13 mi. 6 fur. 3 ch., 1 mi. 3 fur. 5 ch., 220 yd.
8. 3 ch. 12 yd. 1 ft. 9 in., 4 ch. 16 yd. 2 ft. 4 in., 5 ch. 9 yd. 1 ft. 8 in.
9. 2 mi. 5 fur. 20 p., 1 mi. 6 fur. 32 p., 3 mi. 7 fur. 25 p., 880 yd.
10. 5 ac. 6 sq. ch. 300 sq. yd., 2 ac. 5 sq. ch. 250 sq. yd., 25 sq. ch. 100 sq. yd.
11. 3 pk. 1 gall. 2 qt., 2 pk. 3 qt. 1 pt., 1 gall. 2 qt. 1 pt., 10 pts.
12. 6 bush. 2 pk. 5 qt., 4 bush. 3 pk. 6 qt., 3 bush. 1 pk. 3 qt., 20 pts.

Subtract:
13. 2 qr. 17 lb. from 1 cwt. 14. 35 lb. 10 oz. from 3 qr.
15. 8 cwt. 50 lb. from 1 ton. 16. 13 cwt. 1 qr. 17 lb. from 1 ton.
17. 16 cwt. 2 qr. 16 lb. 12 oz. from 3 tons 5 cwt. 1 qr. 7 lb. 8 oz.
18. 14 cwt. 3 qr. 22 lb. 8 oz. from 5 tons 1 cwt. 2 qr. 13 lb. 4 oz.
19. 3 fur. 4 ch. from 1 mi. 20. 6 fur. 25 p. from 2 mi.
21. 6 ch. 15 yd. from 2 fur. 22. 7 yd. 1 ft. 8 in. from 1 ch.
23. 5 fur. 6 ch. 17 yd. 2 ft. 3 in. from 1 mi. 3 fur. 4 ch. 20 yd.
24. 6 fur. 175 yd. 8 in. from 3 mi. 2 fur. 48 yd. 3 in.
25. 7 ac. 6 sq. ch. 150 sq. yd. 6 sq. ft. 100 sq. in. from 10 ac. 3 sq. ch.
26. 3 bush. 3 pk. 1 gall. 3 qt. 1 pt. from 1 qr. 1 bush. 2 qt.
27. 6 gall. 3 qt. 1 pt. from 2 qr. 1 pk. 1 qt.
D 6. COMPOUND MULTIPLICATION AND DIVISION

Multiply:

1. 7s. 3d. by 4.
2. 2s. 11d. by 9.
3. 4s. 10d. by 5.
4. 17s. 6d. by 8.
5. 3s. 8d. by 12.
6. 9s. 11d. by 6.
7. 8s. 10d. by 11.
8. 14s. 8d. by 10.
9. 12s. 9d. by 7.
10. £2 7s. 4d. by 17 and 23.
11. £3 12s. 5d. by 45 and 86.
12. £16 13s. 7d. by 94 and 50.
13. £123 9s. 5d. by 142.
14. £82 16s. 7d. by 315.
15. £107 11s. 6d. by 182.
16. 7 cwt. 3 qr. 12 lb. by 24.
17. 21 cwt. 80 lb. 12 oz. by 56.
18. 3 tons 7 cwt. 1 qr. 15 lb. by 37.
19. 8 tons 14 cwt. 45 lb. by 130.
20. 20 yd. 2 ft. 10 in. by 207.
21. 8 ch. 13 yd. 1 ft. 9 in. by 143.
22. 6 bush. 3 pk. 1 gall. 2 qt. by 67.

Divide:

30. £127 12s. 3d. by 9.
31. £194 16s. 8d. by 14.
32. £103 14s. 9d. by 6.
33. £113 3s. 7d. by 5.
34. £402 15s. 6d. by 17.
35. £2670 19s. 6d. by 29.
36. £239 13s. 1d. by 113.
37. £660 14s. 10d. by 47.
38. £485 7s. 3d. by 258.
39. £922 16s. 8d. by 791.
40. 52 yd. 2 ft. 3 in. by 9.
41. 1 fur. 51 yd. 2 ft. 6 in. by 21.
42. 1 mi. 4 fur. 106 yd. 16 in. by 17.
43. 272 tons 3 qr. 4 lb. by 38.
44. 7 tons 4 cwt. 3 qr. 6 lb. by 102.
45. 5701 tons 4 cwt. 1 qr. 20 lb. by 1083.

Divide and give the remainder in the lowest unit named:

46. £1705 16s. 3d. by 382.
47. £10,724 13s. 5d. by 307.
48. 427 mi. 800 yd. 2 ft. by 461.
49. 305 mi. 7 fur. 8 ch. 15 yd. by 95.
50. 604 tons 13 cwt. 100 lb. by 429.
51. 17 cwt. 3 qr. 21 lb. 12 oz. by 68.
52. 13 ac. 7 sq. ch. 250 sq. yd. by 47.
53. 23 bush. 5 gall. 6 pt. by 59.

D 7. FRACTIONS, ADDITION AND SUBTRACTION

1. \( \frac{1}{2} + 3\frac{1}{2} \)
2. \( 2\frac{1}{2} + \frac{3}{4} + \frac{1}{2} \)
3. \( \frac{1}{2} + \frac{1}{3} + \frac{2}{3} \)
4. \( \frac{1}{2} + \frac{3}{4} + \frac{1}{4} \)
5. \( \frac{1}{2} + \frac{1}{2} + \frac{1}{2} \)
6. \( \frac{2}{3} + \frac{1}{4} + \frac{2}{3} \)
7. \( \frac{2}{3} - \frac{1}{2} + \frac{1}{4} \)
8. \( \frac{1}{2} - \frac{1}{3} + \frac{1}{4} \)
9. \( \frac{1}{3} - \frac{1}{4} + \frac{1}{5} \)
10. \( \frac{1}{2} + \frac{1}{3} + \frac{1}{4} \)
11. \( \frac{1}{3} + \frac{1}{4} + \frac{1}{5} \)
12. \( \frac{1}{3} - \frac{1}{4} + \frac{1}{5} \)
13. \( \frac{1}{4} + \frac{1}{5} + \frac{1}{6} \)
14. \( \frac{1}{5} - \frac{1}{6} + \frac{1}{7} \)
15. \( \frac{1}{6} + \frac{1}{7} + \frac{1}{8} \)
16. \( \frac{1}{4} + \frac{1}{5} + \frac{1}{6} \)
17. \( \frac{1}{5} + \frac{1}{6} + \frac{1}{7} \)
18. \( \frac{1}{3} + \frac{1}{4} + \frac{1}{5} \)
19. \( \frac{1}{2} + \frac{1}{3} + \frac{1}{4} \)
20. \( \frac{1}{3} + \frac{1}{4} + \frac{1}{5} \)
21. \( \frac{1}{3} + \frac{1}{4} + \frac{1}{5} \)
22. \( \frac{1}{2} + \frac{1}{3} + \frac{1}{4} \)
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25. \( \frac{1}{5} + \frac{1}{6} + \frac{1}{7} \)
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30. \( \frac{1}{10} + \frac{1}{11} + \frac{1}{12} \)
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36. \( \frac{1}{16} + \frac{1}{17} + \frac{1}{18} \)
37. \( \frac{1}{17} + \frac{1}{18} + \frac{1}{19} \)
38. \( \frac{1}{18} + \frac{1}{19} + \frac{1}{20} \)
39. \( \frac{1}{19} + \frac{1}{20} + \frac{1}{21} \)
40. \( \frac{1}{20} + \frac{1}{21} + \frac{1}{22} \)

D 8. FRACTIONS, MULTIPLICATION AND DIVISION

1. \( \frac{3}{4} \times \frac{2}{3} \)
2. \( \frac{3}{4} \times \frac{2}{3} \)
3. \( \frac{1}{3} \times \frac{2}{3} \)
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8. \( \frac{2}{3} \times \frac{3}{4} \)
9. \( \frac{3}{4} \times \frac{2}{3} \)
10. \( \frac{2}{3} \times \frac{3}{4} \)
13. $\frac{8}{9}$ of $12 = \left(\frac{2}{3}\right)$ of $4\frac{1}{2}$.
14. $4\frac{1}{2} \times 3\frac{3}{8} \times \frac{3}{5} + 7\frac{1}{3}$.
15. $\frac{1}{7}$ of $2\frac{3}{4} - 2\frac{3}{4}$.
16. $\frac{1}{5} \times 2\frac{3}{4} - 2\frac{1}{2} + 20$.
17. $\frac{1}{9} \times \frac{1}{8} \times 1\frac{1}{4}$.
18. $2\frac{1}{3} \times 3\frac{3}{4} \times \frac{1}{9}$.
19. $\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} + 1\frac{1}{2}$.
20. $\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$.
21. $\frac{1}{2} = \frac{1}{2}$.
22. $\left(\frac{3}{1} + \frac{3}{3}\right) \times \left(\frac{3}{1} + \frac{3}{3}\right)$.
23. $\left(\frac{15}{5} + \frac{7}{9}\right) \times \left(\frac{6}{1} + \frac{1}{1}\right)$.
24. $(\frac{2}{3}) \times \left(\frac{2}{3} + 1\frac{1}{2}\right)$.

**D 9. MISCELLANEOUS FRACTIONS**

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<tr>
<th>Fraction</th>
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<tbody>
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<td>2. $\frac{8}{7} + \frac{2}{3}$</td>
<td>$\frac{2}{3} - 4\frac{1}{7}$</td>
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<td>5. $\frac{1}{3} - \frac{1}{4}$</td>
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<td>13. $\frac{1}{3} - \frac{1}{4}$</td>
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<td>14. $\frac{1}{3} - \frac{1}{4}$</td>
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<td>15. $\frac{1}{3} - \frac{1}{4}$</td>
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<td>16. $\frac{1}{3} - \frac{1}{4}$</td>
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<td>17. $\frac{1}{3} - \frac{1}{4}$</td>
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<td>18. $\frac{1}{3} - \frac{1}{4}$</td>
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<td>19. $\frac{1}{3} - \frac{1}{4}$</td>
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<td>20. $\frac{1}{3} - \frac{1}{4}$</td>
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<td>21. $\frac{1}{3} - \frac{1}{4}$</td>
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<td>22. $\frac{1}{3} - \frac{1}{4}$</td>
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<td>23. $\frac{1}{3} - \frac{1}{4}$</td>
<td>$\frac{1}{3} - \frac{1}{4}$</td>
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<tr>
<td>24. $\frac{1}{3} - \frac{1}{4}$</td>
<td>$\frac{1}{3} - \frac{1}{4}$</td>
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</tbody>
</table>

**D 10. DECIMALS, ADDITION AND SUBTRACTION**

Find the value of:

1. $1.04 + 2.76$.
2. $0.02 + 0.98$.
3. $1.1 + 0.11$.
4. $0.2 - 0.04$.
5. $1.09 - 0.7$.
6. $12 - 0.04$.
7. $3.06 + 0.6$.
8. $2.2 - 0.22$.
9. $0.37 - 0.007$.
11. $0.962 - 0.2807$.
12. $0.03 - 0.0048$.

Add the following (i) in columns, (ii) in rows:

<table>
<thead>
<tr>
<th>i</th>
<th>13.</th>
<th>14.</th>
<th>15.</th>
<th>16.</th>
</tr>
</thead>
<tbody>
<tr>
<td>17.</td>
<td>27.49</td>
<td>9.62</td>
<td>10.08</td>
<td>0.75</td>
</tr>
<tr>
<td>18.</td>
<td>3.01</td>
<td>12.8</td>
<td>0.99</td>
<td>31.26</td>
</tr>
<tr>
<td>19.</td>
<td>5.8</td>
<td>0.07</td>
<td>26.54</td>
<td>17.9</td>
</tr>
<tr>
<td>20.</td>
<td>30.75</td>
<td>48.3</td>
<td>7.06</td>
<td>0.54</td>
</tr>
</tbody>
</table>

Find the value of:

21. $16.49 - 7.018 + 0.623$.
22. $0.907 + 11.42 - 1.86$.
23. $4.7016 - 0.903 - 1.827$.
24. $5.0103 - 0.3072 + 1.008$.
25. $907.2 + 86.07 - 100.5$.
26. $12.42 - 0.706 - 0.639$.
27. $61.37 + 104.8 + 9.623 + 20.075 + 9.812 + 713.06$.
28. $36.01 - 28.037 + 4.729 - 0.0082 + 0.917 - 10.805$.
29. $125.2 - 69.663 - 5.9782 + 3.0704 + 47.308 - 78.07$.
30. Subtract the sum of $27.023, 0.99, 18.507$ from the sum of $1834.79, 4.358$.

**D 11. MULTIPLICATION OF DECIMALS**

Multiply:

<table>
<thead>
<tr>
<th>1.</th>
<th>2.</th>
<th>3.</th>
<th>4.</th>
<th>5.</th>
<th>6.</th>
<th>7.</th>
<th>8.</th>
<th>9.</th>
<th>10.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2.17 \times 3, 0.3, 0.4, 0.4$</td>
<td>$10.85$ by $6.06, 70, 0.07$</td>
<td>$0.65$ by $8, 0.008, 900, 0.9$</td>
<td>$0.094$ by $5, 0.05, 11, 12$</td>
<td>$12.67$ by $5, 8, 10, 12$</td>
<td>$25.08$ by $4, 5, 7, 11$</td>
<td>$0.058$ by $17, 23, 45$</td>
<td>$0.0097$ by $19, 68, 370$</td>
<td>$2.63 \times 1.08$</td>
<td>$37.5 \times 0.128$</td>
</tr>
<tr>
<td>$0.407 \times 6.14$</td>
<td>$70.05 \times 0.408$</td>
<td>$0.064 \times 0.125$</td>
<td>$0.011 \times 0.101$</td>
<td>$86600 \times 5.005$</td>
<td>$0.7105 \times 640$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**COMP.**
APPENDIX

MULTIPLY:
18. 162.075 x 0.48 19. 70.083 x 6.09 20. 0.08104 x 675.
21. 909.47 x 86.4 22. 0.00725 x 0.236 23. 0.60308 x 0.0205.
24. 2.0 x 0.02 x 0.004 25. 0.01 x 0.101 x 0.001.
26. (0.03) x (1.2)² 27. (0.1)³ x 1001 x (0.02)².
28. 0.365 x 7.12 x 5.04 29. 2.001 x 0.025 x 128.

Express as compound quantities to the nearest whole number of the unit named in brackets:
30. £48732 (d.) 31. 80.0865 (f.) 32. £2.5873 (£d.)
33. £5.4096 (£f.) 34. £3.7109 (£fd.) 35. £7.3945 (£d.).
36. 0.37258 ton (lb.) 37. 762445 cwt. (oz.).
38. 1.807236 mi. (yd.) 39. 4560741 ac. (sq. yd.).
40. 0.7184 days (sec.) 41. 6301616 km. (nm.).
42. 1 km. = 0.6214 mi., find 1 km. in yd., to nearest yd.
43. 1 yd. = 0.9144 metres, find 1 mile in metres, to nearest m.
44. 1 lb. = 0.4536 Kg., find 1 ton in Kg., to nearest Kg.
45. 1 cu. in. = 16.39 cu. cm., find 1 cu. ft. in cu. dm. to 3 figures.

D 12. DIVISION OF DECIMALS

Divide:
1. 0.96 by 4, 0.3, 60, 0.08. 2. 1.44 by 6, 0.04, 900, 0.5.
3. 0.072 by 80, 1.2, 0.006, 900. 4. 10.8 by 300, 0.005, 0.12, 0.009.
5. 17.28 by 0.012, 4000, 0.27, 640.
6. 0.0192 by 8, 0.03, 2400, 0.16.
7. 6 + 0.008. 8. 3.1 + 0.025. 9. 2.86 + 0.013.
10. 4.8 + 0.0016. 11. 1.735.6 + 6.5. 12. 0.07344 + 0.34.
13. 10.58 + 8.4. 14. 0.32064 + 0.0247. 15. 0.001725 + 0.46.
16. 100.1 + 0.47. 17. 1.840284 + 61.2. 18. 105.706 + 0.0502.
19. 36.477 + 0.021. 20. 15.656 + 1.52. 21. 0.02266 + 0.515.
22. 1.4014 + 0.035. 23. 31.1 + 0.0305. 24. 3.3077 + 0.011.
25. 0.07504 + 23.45. 26. 0.004408 + 0.0112.
27. 2515.611 + 0.0785. 28. 67.288 + 647.
29. 1708.4592 + 0.072.
30. 10.724 + 0.003125.

APPENDIX

In Nos. 31-47, give quotients correct to the number of places of decimals stated in brackets:
31. 3 ÷ 7 (3). 32. 23 ÷ 109 (3). 33. 1 ÷ 19 (4).
34. 1 ÷ 109 (5). 35. 1 ÷ 714 (2). 36. 0.01 ÷ 708 (7).
37. 40 ÷ 13 (2). 38. 99 ÷ 999 (4). 39. 60 ÷ 75 (2).
40. 107.05 ÷ 0.608 (1). 41. 3.008 ÷ 83.26 (4). 42. 80.7 ÷ 3.1416 (1).
43. 0.0708 ÷ 0.0101 (2). 44. 0.8369 ÷ 0.95 (5).
45. 10 ÷ 11 (1). 46. 1 ÷ 2.7183 (3). 47. 1 ÷ 0.33 (3).
48. Express as a decimal of £1, correct to 4 figures, 17a. 8d.; 13s. 10d.; 5s. 8d.; 9s. 4d.; 11s. 9d.
49. Express 1 fur. 33 yd. as a decimal of 1 mile.
50. Express £1 19s. 4½d. as a decimal of 10 guineas.

REVISION PAPERS

PAPERS A 1-8 (Elementary work)

Paper A 1
1. What must be added to the difference between 18 and 81 to make 100?
2. (i) How many times is 59 contained in 253?
   (ii) When a certain number is divided by 908, the quotient is 37 and the remainder is 235; find the number.
3. A man was 57 years old on the 1st of January 1934; how old will he be in 1948? How old was he in 1905? When was he born?
4. What must be added to the sum of £1 15s. 6d.; 19s. 4½d.; £1 12s. 7½d. to make £5?
5. (i) Divide £83 7s. 6d. by 30.
   (ii) Multiply £2 0½s. 5d. by 21.
6. A car is driven from A to B at 47 mi. an hour for 2 hr. and at 39 mi. an hour for 3 hr. How far is A from B?

Paper A 2
1. Add: 86145; 90728; 70608; 564; 13724; 9006.
2. Find the cost of 72 pianos at £37 10s. each.
3. A has 51 oranges, B has 17 oranges; how many must A give B so that they have equal amounts?
4. (i) I buy 3 articles for 2s. 7½d., 5s. 11½d., 8s. 9½d. respectively. What change do I receive from £1? (ii) How many golf balls at 1s. 6d. each can be bought for £1, and how much remains over?

5. How many charabancs, each holding 42 passengers, are required for 630 people?

6. If I walk 120 yd. a minute, I take 14 min. to go from my house to the station. How fast do I walk if I take 15 min.?

Paper A 3

1. I leave home at 8.40 a.m. and arrive back at 5.25 p.m.; how long, in hours and minutes, am I away?

2. What number exceeds 7284 by the same amount that 7284 exceeds 4917?

3. (i) Divide £75 12s. 6d. by 55.
   (ii) Multiply £4 7s. 9d. by 55.

4. Eggs are sold at 7 for 10 pence; how many can I buy for 16s. 8d.?

5. A man buys 6 dozen articles at 4 for 3d. and 6 dozen at 3 for 2d.; he sells them all at 6 for 5¼d. Find his profit.

6. Two motorists start at the same time from two towns, 130 mi. apart, and drive towards each other, one at 27 mi. an hour and the other at 32 mi. an hour. How far apart are they after 2 hr.?

Paper A 4

1. Add: 704127; 630918; 41723; 518319; 856194.

2. Light travels 186,000 mi. in 1 sec. How far does it travel in 4 min.?

3. (i) Divide £66 4s. 7d. by 17.
   (ii) Multiply £2 13s. 8d. by 31.

4. How many articles at 6d. each can be bought for four and a half guineas?

5. Telegraph poles are set up at intervals of 55 yd. along a road; how many poles are there every 7 miles? (1 mile = 1760 yards.)

6. A man pays each year £75 in rent, £28 in rates, £96 in wages, £525 on other household expenses, and £42 on holidays. He earns £950 a year. How much money has he left?

Paper A 5

1. Letters carried by air mail increased from 2911345 in 1928 to 5104365 in 1930. Find the increase for this period. The increase in the next year was 1245355; find the number carried in 1931.

2. An estate of value £40182 is divided between A, B, C. How much will C get if (i) the shares are equal, (ii) A gets £19425 and B gets £13788?

3. Find the total cost of 640 stamps at 1¾d. each and 640 envelopes at one farthing each.

4. The circumference of a wheel is 18 ft. How many revolutions does the wheel make in travelling 3 mi.

5. A dealer buys a car for £178, spends £35 on repairs, and sells it for £250. What profit does he make?

6. How many articles at 5½d. each can be bought for 30 shillings, and how much money is over?

Paper A 6

1. A collection contained 47 pennies, 31 sixpences, 26 shillings, and 7 half-crowns; find the value in £ s. d.

2. I start from a place 2178 ft. above sea-level, then ascend 944 ft., then descend 639 ft. How much must I now ascend to reach a peak 3024 ft. above sea-level?

3. (i) How many sacks are needed for 6048 lb. of potatoes if each sack holds 56 lb.?
   (ii) How many chairs at 13s. 6d. each can be bought for £50, and how much money remains over?

4. A stack of bricks is 18 bricks thick, 32 bricks long, and contains 11 layers. How many bricks are there in the stack? How many more would there be if the stack was 1 brick thicker, 1 brick longer, and contained 1 more layer?

5. An excursion party of 724 people is charged at the rate of 7½d. each for tea. What is the total charge?

6. A motorist leaves home at 8 a.m. and has to reach a place 135 mi. away by 11.30 a.m. He travels for 3 hr. at 38 mi. an hour; at what rate must he go for the rest of the way to arrive in time?
APPENDIX

Paper A 7

1. A boy is told to work out the examples numbered 1, 5, 9, 13, etc., in an exercise containing 50 examples. How many has he to do?

2. How many halfpennies weigh 1 ton, if 5 halfpennies weigh 1 oz.?

3. What must be added to the sum of £ 17 9s. 7d., £ 34 18s. 10d., £ 9 7s. 9d., £ 28 13s. 5d. to make £ 100?

4. A cage is lowered down a mine-shaft by a chain which passes over a wheel whose circumference is 28 ft. How many turns of the wheel are required to lower the cage 1036 ft.?

5. 754 tickets at 9d. each are sold for a raffle. There are two prizes of values 12 guineas and 5 guineas respectively. How much profit is made?

6. A pail full of water weighs 31 lb. and half-full weighs 17 lb. What is the weight of the pail when empty?

Paper A 8

1. Poor relief for England and Wales rose from £ 36,841,768 in 1925 to £ 40,630,903 in 1930. Find the increase. The amount for 1925 was less by £ 12,933,148 than that for 1927. Find the poor relief in 1927.

2. Multiply £ 87 7s. 5d. by 73.

3. A sheet of paper 300 millimetres high is ruled with lines 9 millimetres apart, beginning 7 millimetres from the top. How many lines can be ruled, and what is the distance of the last line from the foot of the sheet?

4. A bucket is raised by a rope passing round a wheel whose circumference is 57 in. How many feet will the bucket rise for 16 turns of the wheel?

5. A hotel uses during the year 40 tons of coal at 39s. 6d. a ton, 45 tons at 48s. a ton, and 65 tons at 44s. a ton. Find the total cost of the coal used. If all this coal had been bought at the same price per ton, and if the total cost was the same as before, what would have been the price per ton?

6. A man attains the age of 63 on March 10, 1921, and his son is 28 after another 6 months. How old is the man when his son is just 40? How old was the son when his father was just 48?

Paper A 9

1. What is the least number by which 252 must be multiplied to give a perfect square?

2. (i) A hurdle is 6 ft. 9 in. long; find in yards the length of a fence containing 80 hurdles.

   (ii) Express 8370 lb. in tons, cwt., lb.

3. How many times is 3 ft. 8 in. contained in 8 ch.?

4. If 3 tons of coal cost £ 10 16s., find the cost of 5 tons.

5. The diagram represents the cross-section of a solid 3 ft. long. The corners are right-angled and the dimensions are shown in inches. Find (i) the area of the cross-section in sq. in., (ii) the volume of the solid in cu. ft., cu. in.

6. A box without a lid is made of wood 1 in. thick. The box is 1 ft. 3 in. long, 1 ft. wide, 7 in. high, measured externally. Find in cu. in. the volume of the wood used for the box.

Paper A 10

1. (i) Add: 7 cwt. 3 qr. 16 lb.; 5 cwt. 2 qr. 21 lb.; 9 cwt. 3 qr. 13 lb.; 4 cwt. 1 qr. 9 lb.

   (ii) Multiply 3 ch. 15 yd. 2 ft. by 29. (Answer in mi., ch., etc.)

2. (i) How many layers of brickwork 5 in. high are there in a wall 6 ft. 3 in. high?

   (ii) A clerk is paid £230 a year (365 days). How much is this per day? Disregard fractions of a penny.

3. The price of tea is raised by 1d. per lb.; find the increase in the price per ton.

4. A boy takes 4 hours to bicycle a certain distance at 10 miles an hour. How many minutes would he take in a car travelling 32 miles an hour?

5. A photograph 6 in. long, 5 in. wide is mounted on a rectangular card so that there is a margin 1/4 in. wide all round. What area of the cardboard is visible.
6. A tank measuring 10 ft. by 6 ft. by 4 ft. 6 in. is full of water. If the water is run into a larger empty tank, 9 ft. long, 7 ft. 6 in. wide, find the depth of water in this tank.

**Paper A 11**

1. (i) Add: 5000 in., 837 ft., 137 yd. (Answer in ch., yd., etc.)
   (ii) Divide 18 tons 9 cwt. 1 qr. by 7.
2. (i) How many times is 3 qt. 1 pt. contained in 15 gall. 3 qt.? (ii) How many articles at 1s. 9d. each can be bought for £10, and how much money is over?
3. 60 lb. of coffee cost 4 guineas. Find the cost of 200 lb.
4. A steel rail is 32 ft. long and weighs 300 lb. Find the weight per mile of the rails in tons, cwt., lb.
5. A cube of lead, edge 6 in., is melted and recast into a rectangular block 9 in. long, 8 in. wide. Find its height.
6. With the data of No. 5, find the difference between the areas of the total surfaces of the original cube and the new block.

**Paper A 12**

1. Express 7546 in prime factors. What is the least number by which it must be multiplied to give (i) a perfect square, (ii) a perfect cube?
2. (i) Subtract 7 cwt. 3 qr. 19 lb. from 1 ton 2 qr.
   (ii) Divide 2608 mi. 5 fur. 71 yd. by 203.
3. (i) A man's stride is 2 ft. 9 in. long; how many steps does he take in walking half a mile?
   (ii) A barrel of pork contains 224 lb.; how many tons of pork are there in 750 barrels?
4. A hotel charges £8 5s. for 12 days. What is the charge for 14 days at the same rate?
5. The floor of a room is 14 ft. long, 9 ft. wide, and is covered with tiles 4 in. square. How many tiles are used?
6. A box without a lid is 6 ft. long, 4 ft. wide, 3 ft. deep, internal measurements. What area of tin sheeting is required to line inside the base and sides? If the box contains 30 cu. ft. of water, find in inches the depth of the water.

**Paper A 13**

1. Express in prime factors $48^2 \times 18^8$. Divide it by $36^2$.
2. (i) Reduce 12450 in. to ch., yd., etc.
   (ii) A van can carry a load of 1 ton 7 cwt. 80 lb.; what load can be carried by 13 of these vans?
3. A man travelling by air is allowed 90 lb. of luggage free and is charged at the rate of 9d. for every 4 lb. above that amount. What is the charge for 158 lb. of luggage?
4. In a sale, a pair of stockings is reduced from 12s. 6d. to 7s. 6d.; find the reduction in the price of a scarf originally marked £1 12s. 6d., if reduced at the same rate.
5. Find the total area of the walls of a room, 16 ft. long, 14 ft. wide, 9 ft. high. What is the remaining area, if allowance is made for two windows, each 3 ft. by 5 ft., a door 7 ft. by 4 ft., and a fire-place 6 ft. by 3 ft.?
6. A beam of timber is 12 ft. long, 2 ft. wide, 1 ft. deep; the timber weighs 42 lb. per cu. ft. Find the weight of the beam in cwt.

**Paper A 14**

1. Find in prime factors the L.C.M. of 24, 42, 63, 105, 135.
2. (i) Reduce 12560 yd. to mi., ch., yd.
   (ii) A bag of flour contains 140 lb.; how many bags contain 12 tons 15 cwt. of flour.
3. By selling tea at 1s. 10d. per lb., a grocer makes a profit of £48 16s. 8d. per ton. What does he pay for 1 ton of tea?
4. A car which runs 28 mi. to the gallon of petrol uses 12 gall. for a journey; how much will a car which runs 21 mi. to the gallon use for the same journey, and what will the petrol cost for each car for this journey at 1s. 5d. per gall.?
5. A rectangular piece of land is 1 mi. 4 fur. long and 6 fur. wide. Find its area in acres. If it is rented at 1s. 6d. per acre find the rent paid.
6. How many cu. ft. of water must be pumped into a swimming-bath 75 ft. long, 18 yd. wide to raise the water-level 2 ft.
Paper A 15

1. (i) Find the fourth root of 20736.
   (ii) Find the least number for which the remainder is 11 when it is divided either by 18 or by 30 or by 45.

2. Multiply 3 fur. 5 ch. 14 yd. by 29.

3. A grocer buys 1 cwt. of tea at 2s. 1d. per lb., 100 lb. of tea at 1s. 10½d. per lb., and 48 lb. of tea at 1s. 9¼d. per lb. He mixes it all together and sells it at 2s. 3d. per lb.; what profit does he make?

4. A tin sheet, 2 ft. 6 in. by 3 ft. 4 in. weighs 8 oz. Find the weight of a similar tin sheet, 6 ft. 3 in. by 3 ft. 6 in.

5. A window 3 ft. high, 2 ft. wide, contains 9 panes of glass, each 11 in. high, 7 in. wide. Find in sq. in. the area not occupied by the glass.

6. The volume of a rectangular tank is 1008 cu. ft. (i) Find its height if the base is 4 yd. square. (ii) Find its breadth if its length is 21 ft. and height is 6 ft. (iii) Find its length if the area of the end face is 8 sq. yd.

Paper A 16

1. What is the least sum of money which is an exact number of guineas, half-crowns, and florins?

2. (i) Add: 6000 oz., 325 lb., 18 stone, 14 qr.
   (ii) Telegraph wire weighs 460 lb. per mile. Find in tons, cwt., the total weight of wire required for a line 28 miles long.

3. 195 men can do a piece of work in 17 days. How many men are required to do it in 2 days less time, if all work at the same rate?

4. A man spends £605 in buying francs when 84 francs are worth £1. When he sells all the francs he has bought, he receives £1 for every 88 francs. Find his gain or loss.

5. The area of a rectangular field, 10 chains wide, is 15 acres. Find in yards the length of the fence which encloses it.

6. A rectangular piece of wood 14 in. long, 11 in. wide, has two rectangular holes, each 5 in. by 4 in., pierced in it; the wood is 2 in. thick. Find its weight in lb., oz. if 3 cu. in. of wood weigh 1 oz.

Paper A 17

1. (i) Simplify $\frac{1}{2} - \frac{3}{4} + \frac{1}{8} - \frac{1}{2}$.
   (ii) Express 7 cwt. 2 qr. as a fraction of 1 ton.

2. (i) Express 0.0075 as a vulgar fraction.
   (ii) Evaluate $2.7633 \div 90.6$.

3. (i) Express £9-73125 in £ s. d.
   (ii) Taking 35 yd. as equivalent to 32 m., express 7 ft. 7 in. in cm., to the nearest cm.

4. If 1½ tons of coal cost £3 15s., find the cost of 2 tons 16 cwt.

5. Lead is 11½ times as heavy as water, and 1 cu. ft. of water weighs $62\frac{1}{2}$ lb.; find in lb., to 1 place of decimals, the weight of 1 cu. ft. of lead.

6. A rectangular block of metal is 2½ in. long, 2½ in. wide, 2 in. high. It is filed down to the form of a cube whose edge is 2 in.; what fraction of the original block has been removed?

Paper A 18

1. In March 1931, there were 5,514,401 pupils in the elementary schools. Of these 1,703,330 were between 5 and 8 years old; 2,009,656 were between 8 and 11; 1,642,988 were over 11. How many were under 5 years old?

2. (i) Simplify $\frac{3}{4} - (\frac{3}{4} - \frac{1}{4})$.
   (ii) A boy has 13s. 4d.; what fraction of this is left after he has spent 9s.

3. (i) Express $\frac{7}{10}$ as a decimal.
   (ii) Evaluate $0.1608 \times 0.7045$.

4. Find the duty on $\frac{1}{2}$ lb. of tobacco at 8s. 4d. per lb.
   - £48 16s. 6d. is divided between 3 men and 9 women so that each man receives £3 5s. more than each woman. How much does each man get?

5. A boy is promised half a crown for every ½ of a second that his time for a 100 yd. race is less than 12 sec. He runs at the rate of 26 ft. per second. How much money does he win?
APPENDIX

Paper A 19

1. (i) Simplify $3\frac{2}{3} \times \frac{2}{3} + \left( \frac{1}{3} + 1\frac{1}{3} \right)$.
   (ii) Express £1 6s. 3d. as a fraction of £2 5s. 6d.
2. (i) Express in km. the sum of: 4 Hm. 8 Dm. 9 m.; 3 Hm. 2 Dm. 8 m.; 75 Dm.; 650 m.; 704 m.
   (ii) Evaluate $0.27633 \div 90.6$.
3. A boy bicycles $31\frac{1}{2}$ mi. in $2\frac{1}{2}$ hr.; how long will he take to go $11\frac{3}{4}$ mi. at the same rate?
4. Aeroplane fabric is sold at 35 shillings per dozen yards, and odd lengths less than this at 3s. 1d. per yard. Find the cost of 40 yd.
5. Two rods each 1 ft. long are divided, one into 9 equal parts and the other into 10 equal parts. They are placed side by side so that the seventh points of division in each coincide. Find in inches how much the end of one is short of the end of the other.
6. The upright portion of a metal cross is 2 ft. 6 in. long, 5 in. wide, and the horizontal portion is 1 ft. 8 in. long, 4 in. wide. The cross is half an inch thick. Find (i) the volume in cu. in., (ii) the area of the total surface.

Paper A 20

1. (i) Simplify $\left( 2\frac{2}{3} + 1\frac{1}{6} + 1\frac{1}{3} \right) + \left( 2\frac{2}{3} - 1\frac{1}{3} \right)$.
   (ii) What fraction is $\frac{8}{3}$d. of half a crown?
2. (i) Evaluate $0.7364 + 0.805 - 0.0909 - 0.8124$.
   (ii) Multiply $0.03928$ by $0.0$.
3. (i) Express 0.01875 ton in lb.
   (ii) The volume of a stack of 4500 bricks is 135 cu. ft.; find the size of a brick in cu. in.
4. If 2\frac{1}{2} lb. of tea cost 4s. 2d., find the cost of 1\frac{1}{2} lb.
5. An aeroplane flying at 80 mi. an hour passes over a destroyer steaming in the same direction at 35 knots. How many feet short of 2 mi. is the aeroplane ahead of the destroyer after 3 min.?
6. A testimonial is raised by subscriptions of 5s. each, and the expenses amount to £1 3d. for every shilling received. What must be the least number of subscribers to secure that the money available should not be less than £50?

APPENDIX

Paper A 21

1. (i) Find by factors the value of $53^2 - 47^2$.
   (ii) Prove that $12066^2 + 887^3 = 12073^3 + 786^2$.
2. (i) Simplify $5\frac{3}{4} + (2 - \frac{1}{2}) + \frac{2}{3} + 1\frac{3}{4}$.
   (ii) Express 5 fur. 100 yd. as a fraction of 1 mi. 4 fur.
3. (i) Express 0.3047 cwt. in lb., oz., to the nearest oz.
   (ii) Evaluate $0.022344 \div 73.5$.
4. Taking 1 kg. = 2.205 lb., express 50 lb. in kg., correct to one place of decimals.
5. A train travelling 37\frac{1}{2} mi. per hour takes 7 sec. to pass a telegraph pole. Find the length of the train in feet.
6. Newspaper wrappers with 4d. stamps on them can be bought in packets of 22 for 1s., and with 1d. stamps in packets of 23 for 2s. Find how much a firm which uses 60 packets of the former and 40 packets of the latter saves by using these instead of buying unstamped wrappers at 20 for 1d. and stamping them.

Paper A 22

1. (i) Simplify $4\frac{2}{3} - \frac{1}{6} + 2\frac{1}{3} - 1\frac{3}{4} \times \frac{1}{3}$.
   (ii) Simplify $6^3 \times 8^2 \div (12^3 \times 4^3)$.
2. (i) Express £4.583 in f. s. d., to the nearest penny.
   (ii) Evaluate $(0.025)^2 \times (0.12)^3$.
3. (i) Express 4 tons 12 cwt. 3 qr. as a decimal of 5 tons.
   (ii) Taking 1 kg. = 2.205 lb., express 14 kg. in lb., oz., to the nearest oz.
4. Bleriot's first Channel flight in 1909 took 37 min. and the distance was 26 mi. Find the speed in miles per hour, to the nearest mile.
5. How many gross of exercise books containing 22 leaves each 6 in. by 8 in. can be made from a strip of paper 2 ft. wide, 1 mi. long?
6. I can obtain coal from the pit-head at 33s. per ton for the first 10 tons and 32s. 6d. for every additional ton. The transport costs me £8 per truck-load of 14 tons. How much do I save by obtaining a truck-load direct from the colliery instead of buying the same coal from a local dealer who delivers it for 48s. per ton?
Paper A 23

1. (i) Simplify \((\frac{3}{4} - \frac{1}{4}) + (\sqrt{2} - \sqrt{3} + 1)\).
   (ii) The area of the floor of a room 18 ft. long is 275 sq. ft.; find the breadth of the room.

2. (i) Divide 3:0537 by 0:648.
   (ii) Express 8s. 5/8d. as a decimal of £1, correct to 4 places.

3. 3 m. 35 mm. of platinum wire cost 310 fr. 25 c.; find the price per cm., to the nearest centime.

4. A man smokes 3 oz. of tobacco a week at 9d. per oz. and 5 cigarettes a day at 5s. per hundred. What does he spend on smoking in 52 weeks?

5. How long do I take to walk 6 mi. if I take 90 steps of 2 ft. 9 in. every minute?

6. Beech trees planted for timber are allowed 36 sq. yd. per tree. The average amount of timber per tree is 11 cu. ft., and the timber is sold at £1 10s. per load of 25 cu. ft. Find the value of the timber on an acre.

Paper A 24

1. The number of bunches of bananas imported into the United Kingdom in 1931 was as follows: British West Indies 6,970,810; Honduras 2,700,091; Colombia 1,675,275; Costa Rica 1,620,734; other countries 3,194,990. Find the total number imported. If the average weight of a bunch is 34 lb., find the total weight, to the nearest thousand tons.

2. (i) Simplify \(\frac{1}{2}(\frac{3}{4} + \frac{1}{6}) - 1 + \frac{1}{2}(1\frac{1}{2} + 2\frac{1}{2})\).
   (ii) When \(\frac{3}{4}\) of a tank has been filled, there is still room for 45 gall. How much can the tank hold?

3. (i) Evaluate \((0.2)^3 \times 0.101 \times (0.05)^9 \times 100).
   (ii) Express 13 lb. 12 oz. as a decimal of 1 qr., to 3 places.

4. In a long division sum, the dividend is 2835891 and the successive remainders are 445; 156; 135; 395. Find the divisor and quotient.

5. A housekeeper makes 20 lb. of marmalade with 21 oranges and 15 lb. of sugar. The oranges cost 14 a shilling and the sugar costs 3\(\frac{3}{4}\)d. per lb.; the boiling costs 4\(\frac{3}{4}\)d. How much does she save by making it instead of buying it at 8d. per lb.
APPENDIX TO PART II

CHAPTER X

FRACTIONAL PARTS AND PRACTICE

EX. 73 (a). Fractional Parts

Find the value of:

1. \(\frac{3}{4}\) of £1. 2. \(\frac{7}{10}\) of £1. 3. \(\frac{5}{8}\) of £1. 4. \(\frac{7}{9}\) of £1.
5. \(\frac{5}{6}\) of 10s. 6. \(\frac{3}{4}\) of 7s. 6d. 7. \(\frac{7}{8}\) of £3. 8. \(\frac{7}{12}\) of £1.
9. \(\frac{5}{8}\) of 1 ft. 10. \(\frac{7}{10}\) of 1 mi. 11. \(\frac{4}{9}\) of 1 cwt. 12. \(\frac{3}{5}\) of 1 ton.
13. \(\frac{2}{5}\) of £2 9s. 2d. 14. \(\frac{7}{10}\) of £5 4s. 6d. 15. \(\frac{3}{7}\) of £2 13s. 8d.
16. \(\frac{2}{3}\) of £3 14s. 9d. 17. \(\frac{5}{6}\) of 10 yd. 1 ft. 6 in.
*18. \(\frac{3}{4}\) of 3 tons 7 cwt. 2 qr. *19. Express \(\frac{2}{3}\) of 18\(\frac{2}{3}\) mi. in yards.
*20. Find the value of \(\frac{1}{4}\) of £10 2s. 6d.; \(\frac{2}{3}\) of £2 17s. 1d.

EX. 76 (a). Simple Practice

Find by Practice the cost of the following articles:

1. 37 at £1 6s. 6d. each. 2. 52 at £3 7s. 3d. each.
3. 88 at £2 3s. 5d. each. 4. 69 at £4 18s. 6d. each.
5. 112 at £3 11s. 8d. each. 6. 156 at £3 2\(\frac{1}{4}\)d. each.
7. 274 at £2 12s. 8\(\frac{1}{4}\)d. each. 8. 578 at £3 7s. 9\(\frac{1}{2}\)d. each.

Find by Practice the total weight of the following:

9. 116 loads, each 8 cwt. 3 qr. 21 lb.
10. 265 loads, each 1 ton 14 cwt. 78 lb.

Find by Practice the total length of the following:

*11. 582 fence-sections, each 3 yd. 1 ft. 9 in. long; answer in ch., yd., etc.

*12. 465 rolls of wire, each 26 yd. 2 ft. 10 in. long; answer in ch., yd., etc.
APPENDIX

Find by Practice methods:
13. The cost of 75 knives at 1s. 4½d. each.
14. The amount of 365 days' pay at 16s. 4d. a day.

Use Subtraction methods to find:
15. The cost of 92 yd. at 11½d. a yard.
*16. The cost of 38½ yd. at 3s. 5½d. a yard.
*17. The cost of 127 tons at £2 18s. 4d. per ton.
*18. 4 yd. 2 ft. 10½ in. x 240.

EX. 77 (a). Compound Practice

Find by Practice the cost of:
1. 4 cwt. 3 qrs. at £1 6s. per cwt.
2. 1 ton 11 cwt. 1 qr. at £5 10s. per ton.
3. 7 tons 16 cwt. 1 qr. at £3 13s. 4d. per ton.
4. 11 cwt. 3 qr. 5 lb. at £9 6s. 8d. per cwt.
5. 12 yd. 2 ft. 6 in. at £1 14s. 9d. per yard.
*6. 5 ac. 2 r. 28 p. at £3 16s. 8d. per acre.

Find, correct to the nearest penny, the cost of:
7. 12 cwt. 2 qr. 8 lb. at £5 7s. 6d. per ton.
8. 3 tons 14 cwt. 3 qr. at £7 14s. 8d. per ton.
9. 3 ch. 17 yd. 1 ft. at £3 18s. 9d. per chain.
*10. 5 ac. 2 r. 16 p. at 35 guineas per acre.
*11. 3 tons 5 cwt. 3 qr. 2 lb. at £3 1s. 10d. per ton.
*12. 1 ton 6 cwt. 2 qr. 8 lb. at £5 8s. per cwt.

Find by Practice the dividend on:
13. £135 10s. at 3s. 10d. in the £.
14. £516 12s. 6d. at 5s. 4d. in the £.

Find by Practice the tax payable on:
15. £206 at 17s. 5½d. in the £.
16. £2492 at 9s. 2d. in the £.

Find, correct to the nearest penny, the cost of:
*17. 215 tons 7 cwt. 3 qr. at £1 7s. 10d. per ton.
*18. 835 tons 13 cwt. 1 qr. at £4 18s. 5d. per ton.

APPENDIX

EX. 78 (a). Miscellaneous Examples

1. Find the cost of 210 towels at 1s. 11½d. each.
2. Find the cost of 2 lb. 12 oz. of tobacco at 14s. 4d. per lb.
3. Multiply 2 yd. 1 ft. 6 in. by 2½ by a practice method.
4. Find the rates payable on an assessment of £85 at 7s. 9d. in the £.
5. Taking 1 lb. = 0.4536 kg., express 13½ oz. in grams, to the nearest gram.
6. Find the duty on 12 cwt. 3 qr. at £2 8s. per cwt.
7. A swimming-bath holds 37,500 gall.; find the cost of filling it at 9d. per 1000 gall.
8. Use a practice method to evaluate 0.275 of £5 16s. 8d.
9. Find to the nearest half penny the cost of 3 lb. 11 oz. at 6½d. per lb.
10. A shopkeeper makes an average profit of 6s. 10d. on each £1 he takes. What is his profit for a year in which he takes £3242?
11. Use the fact that 1000 articles at 6d. each cost £25 to find the cost of (i) 1000 articles at 5½d. each; (ii) 555 articles at 5½d. each.
12. A man who insures his life at the age of 25 pays annually £1 6s. 3d. for each £100 insurance. What is his annual payment on an insurance for £640?
13. A train travels 47 mi. an hour; how far, to the nearest ½ mi., does it go in 2 hr. 38 min.?
14. Find the cost of 3 cwt. 2 qr. 8 lb. at 4d. per lb.
15. Find to the nearest penny the dividend on £867 8s. 4d. at 7s. 9d. in the £.
16. A bankrupt owes £3714 15s., but is only able to pay at the rate of 11s. 4d. in the £. Find his assets.

CHAPTER XI

RATE, RATIO AND PROPORTION

EX. 80 (a). Ratio and Rates

1. Find the ratio of the distances from London of Portsmouth (70 miles) and of Leicester (98 miles).
2. A man dies at the age of 49. For what fraction of his life was he over 21? Find the ratio of the length of his life over 21 to that under 21.
3. In a box of 12 dozen eggs, 18 eggs are bad. Find the ratio of the number of bad eggs to good eggs.

4. A house rented at £80 a year is assessed for rates at £64 a year. Find the ratio of the assessment to the rent.

5. The edges of two cubes are 9 in., 6 in. long. Find the ratio of (i) their volumes; (ii) the areas of their surfaces.

6. Anthracite nuts cost £3 13s. 6d. per ton and cobbles cost £2 12s. 6d. per ton. Find the ratio of their prices.

7. A man buys a plot of land for £420 and spends £1400 in building a house on it. Find the ratio of the cost of the house to the total expenditure.

8. Find the ratio of a speed of 400 yd. a minute to a speed of 30 ft. per second.

9. A sponge costing 5s. lasts me 32 weeks and one costing 7s. 6d. lasts me 60 weeks. Find the ratio of the worth to me of the first sponge to that of the second.

10. 15 men can be fed for a week at the same cost as 24 children. Find the ratio of the daily cost of a man's food to that of a child's food.

11. The rateable value of a house is \( \frac{5}{6} \) of the annual rent. If the rent is £84 a year, and if a rate of 11s. 3d. in the £ is demanded, how much must be paid?

12. In 1932 the rateable value of Westminster was £10,850,000 and rates were levied at 9s. 8d. in the £. How much money was obtained from the rate?

*13. The rateable value of a town is £342,650. If it is necessary to raise £244,500 to meet the cost of administration, find what rate in the £, correct to the nearest penny, must be demanded.

*14. Find the tax on an income of £300, if no tax is paid on the first £150, if tax is paid at the rate of 2s. 3d. in the £ on the next £175, and at 4s. 6d. in the £ on the rest.

*15. After tax at the rate of 4s. in the £ has been deducted, there remains £5 12s.; what was the original amount?

16. A bankrupt paid 8s. 6d. in the £. His assets were £1360, what were his liabilities?

17. A bankrupt who owes a man £172 is only able to pay at 13s. 9d. in the £; how much does the man lose?

*18. A bankrupt's assets were declared to be £1847; the receiver ruled that £432 ranked as claims which must be paid in full. The unsecured liabilities amounted to £3420. At what rate in the £, to the nearest penny, were the unsecured creditors paid?

EX. 81 (a). Ratio and Scales

Express the following ratios in the form \( m : 1 \):

1. 13 : 10.
2. 1 hr. to 25 min.
3. \( \frac{1}{4} \) mi. to 1000 yd.

Find \( m \) for the following pairs of equal ratios:

4. 12 oz. : 1 lb. = \( m : 1 \).
5. 3 pt. : 1 gall. = \( m : 100 \).

Express the following ratios in the form \( m : 1 \), giving \( m \) correct to 2 places of decimals:

6. 1 ft. 5 in. to 1 yd.
7. £4 15s. to £3 5s.

*8. 13s. 9d. to £1 7s. 3d.
*9. 3 cwt. 1 qr. to 1 ton 1 \( \frac{1}{2} \) tons.
10. The scale of a map is 6 in. to the mile. Find its R.F.

11. On a map of scale 1 : 2,500,000, the distance between Calais and Marseilles is 35-3 cm. Find the actual distance in km.

12. The scale of a map is 1 in. to 4 mi. How far apart on the map are two churches distant 9\( \frac{1}{2} \) mi. from one another?

13. The scale of a map is 1 in. to the mile. Find in yards, to the nearest 100 yd., the length of a road which is 3-62 in. long on the map.

*14. On a map, scale 1 in. to 2 mi., a wood has an area of 0-43 sq. in. Find its actual area in acres, to the nearest 10 acres.

*15. What area, correct to 3\( \frac{1}{2} \) sq. in., is occupied by a farm of 100 acres on a map of scale 25 in. to the mile?

16. A railway mounts from sea-level to a height of 745 ft. above sea-level. For the first 9\( \frac{1}{2} \) mi. it is on a gradient of 1 in 110, and for the rest on a gradient of 1 in 60. Find the length of the railway.
**EX. 82 (a). Multiplying Factors**

1. Decrease 12s. in the ratio 7 : 16.
2. Increase 2 ft. 8 in. in the ratio 9 : 4.
3. Increase £1 15s. in the ratio 5 : 3.
4. Decrease 1 ton 13 cwt. in the ratio 3 : 4.
5. In making a model, the height 8 ft. of a statue is reduced in the ratio 5 : 48; find the height of the model.
6. The ratio of the summer to winter charges of a hotel is 7 : 4.
   If the winter charge is £3 12s. a week, find the summer charge.
7. In what ratio must £1 be decreased to become 12s.?
8. In what ratio must 12$\frac{1}{2}$ oz. be increased to become 1$\frac{1}{2}$ lb.?
9. In what ratio must half a yard be increased to become 2 ft.?
10. What multiplying factor increases £1 to £1 5s.?
11. What multiplying factor alters a speed of 30 mi. an hour into a speed of 16 yd. a second?
12. What multiplying factor increases 2 lb. by 12 oz.?
13. What multiplying factor reduces a charge of eight guineas to £7?
14. Two sums of money are in the ratio 10 : 7; the larger is 15s.; what is the smaller?
15. Two distances are in the ratio 8 : 15; the smaller is 200 yd.; what is the larger?

*16. A number is diminished by $\frac{1}{3}$ of itself. In what ratio has it been decreased?*

*17. A number is increased by $\frac{3}{4}$ of itself. In what ratio has it been increased?*

*18. What multiplying factor increases a number by $\frac{3}{4}$ of itself?*

19. A photograph measuring 6 in. by 3 in. is enlarged so that the shorter side becomes 10$\frac{1}{2}$ in. What does the larger side become? In what ratio is the area increased?

*20. When the price of coal rises from £2 a ton to £2 5s. a ton, I reduce my annual consumption from 12 tons to 10 tons. In what ratio is my coal bill altered?*

21. The price £10 10s. of a table is reduced in a sale by £3 10s. What is the sale price of a sofa, usual price £7 10s., if reduced in the same ratio?

**EX. 83 (a). Direct and Inverse Variation**

[Assume that the rates given in this exercise are uniform]

1. 28 lb. of preserving sugar cost 6s.; find the cost of 21 lb.
2. If 100 tablets of soap cost £2, find the cost of 120 tablets.
3. 18 men can repair a road in 14 days; how long will 21 men take to do so?
4. A garrison of 48 men have enough food for 15 days; how long would this food last 40 men?
5. 8 lb. of prunes cost 11s.; find the cost of 14 lb.
6. A car travelling at 36 mi. per hour takes 2$\frac{1}{2}$ hr. for a journey; how long will it take at a speed of 40 mi. an hour?
7. 48 gall. of oil cost £2 16s.; find the cost of 13 gall. 2 qt.
8. 6 yd. of stair-carpet cost £1 14s. 6d.; find the cost of 10 yd.
9. 5 men can load a van in half an hour; how long will it take 6 men to do so?

*10. A car runs 72 ml. on 3 gall. of petrol, costing 1s. 3d. per gallon; how far will it run on £1 worth of petrol?*

11. If a money prize is shared equally between 16 boys, each receives 3s. 4d.; what would each get if it was shared equally between 20 boys?

12. Matting for a passage 8 yd. long, 5 ft. wide, costs 36s.; find the cost for a passage 10 yd. long, 6 ft. wide.

13. If I spend 17s. 6d. a day I have enough money for 18 days. How long will my money last if I spend 22s. 6d. a day?

*14. On a map whose scale is 4 mi. to the inch, the distance between two churches is 1$\frac{1}{2}$ in.; what would be the distance on a map whose scale is 5 mi. to the inch?*

*15. A man pays 2s. 3d. in the £ income tax on the part of his income above £120. His tax is £31 10s., find his income.*

**EX. 84 (a). Compound Variation**

[Assume that the rates given in this exercise are uniform]

1. 25 horses eat 5 bushels of corn in 12 days; how much will 10 horses eat in 18 days?
2. If 10 men earn £20 in 4 days, in what time will 12 men earn £48?
9. The ratios of the sides of a quadrilateral are 3 : 4 : 6 : 7, and its perimeter is 1 ft. Find the length of each side.

10. A, B, C provide £400, £350, £250 respectively for a business, and share the profits £120 in the ratios of the capital each provides. What does each receive?

*11. Divide three guineas into an equal number of florins, shillings, and sixpences.

12. Divide 8s. 2d. into two shares such that one is 2\(\frac{3}{4}\) times the other.

Find the three smallest whole numbers proportional to \(x, y, z\) in Nos. 13–16:

13. \(\frac{x}{y} = \frac{3}{10}; \frac{y}{z} = \frac{15}{7}\)  
14. \(\frac{x}{y} = \frac{4}{9}; \frac{x}{z} = \frac{6}{5}\)  

*15. \(2x = 3y = 5z\)  
*16. \(5x = 7z; x = y + z\)

17. Divide four guineas between A, B, C so that A has half as much again as B, and B has twice as much as C.

*18. Divide a load of 120 lb. between A, B, C so that A carries as much as B and C together, and B carries \(\frac{3}{4}\) of what C carries.

19. A chair costs 3 times as much as a rug, and a table costs twice as much as the chair. The three together cost £8. Find the cost of each.

*20. A certain piece of work can be done by A in 5 days, and by B in 7 days, working separately. If £4 is paid for the job, what should each receive if they work together?

*21. A man borrows £500 for 5 months from B, £400 for 4 months from C, £300 for 3 months from D. He pays £25 interest altogether; how is this shared between B, C, D, if the rates of interest are equal?

**EX. 85 (a). Proportional Parts**

1. Divide 18s. in the ratio (i) 2 : 1; (ii) 5 : 4; (iii) 1 : 3.
2. Divide 5 yd. in the ratio (i) 5 : 4; (ii) 7 : 3; (iii) 8 : 1.
3. Divide £40 into three shares in the ratios 3 : 2 : 5.
4. Divide £3 10s. into three shares in the ratios 1 : 2 : 3.
5. Divide 16s. into four shares proportional to 6, 1, 7, 10.

Find as simply as possible three whole numbers proportional to:

6. £2 5s.; £1 10s.; £7 10s.
7. 25 cm.; 1 m.; 4 dm.

8. The lengths of the sides of a triangle are proportional to 2, 5, 6, and its perimeter is 7\(\frac{8}{10}\) cm. Find the length of each side.

**EX. 86 (a). Problems on Ratios**

1. If \(\frac{3}{5}\) of a number is 66, find the number.
2. After driving \(\frac{3}{5}\) of the distance from A to B, a man has still 99 mi. to go; how far is A from B?
3. If 3d. in the shilling is deducted from a bill, 8s. 3d. is left. How much was the bill?
4. When $\frac{4}{3}$ of the coal in a cellar has been used, $2\frac{3}{4}$ tons remain. How much coal was there at first?

5. After paying tax at the rate of 2s. in the £, a man has £504 left. How much had he at first?

6. 3d. in the shilling is added to the price of a front seat for entertainment tax. If the seat costs 8s. 9d., how much has been added for tax?

7. A dealer makes a profit of 7s. in the £ on the cost price of his goods. What does he pay for an article which he sells for £6 15s.?

8. A shopkeeper makes a profit of 5s. in the £ on the cost price of his goods. What is his profit on an article he sells for 35 shillings?

9. By selling my house for £900 I lose at the rate of 1s. 3d. in the £ on the price I paid for it. How much do I lose?

*10. After paying tax at an average rate of 4s. 6d. in the £, a man's income was £621 11s. Find his gross income, that is his income before tax was paid.

11. A and B invest £169 18s. and £243 10s. at the same date in Savings Certificates. What is the value to the nearest £ of B's holding when A's holding is worth £210?

*12. 100 lb. of wheat yields 70 lb. of flour, and there is 100 lb. of flour in 130 lb. of bread. How much wheat, to the nearest lb., is used in making 100 lb. of bread?

*13. A clock loses 8-5 sec. an hour when the fire is alight and gains 5-1 sec. an hour when the fire is out. If on the whole it neither gains nor loses, how long is the fire burning in every 24 hr.?

*14. A and B provide respectively £3500 and £5000 for a business, in the working of which B takes no part. From the profits, A is paid a fixed salary as manager and the remainder is divided in the ratio of the capitals invested. If at the end of 1 year, A receives £690 and B receives £450, find the amount of A's salary.

*15. A small flask weighs 27 gm. when empty and 98-5 gm. when full of water. If some mercury is put into the empty flask it weighs 182 gm., and if this is now filled up with water, the total weight is 242 gm. Find the ratio of the weights of equal volumes of mercury and water in the form $n:1$, giving $n$ to the nearest whole number.

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CHAPTER XII

PERCENTAGE

EX. 88 (a). Simple Percentage

Express the following percentages as fractions —

1. $20\%$, $40\%$, $160\%$.
2. $6\frac{1}{2}\%$, $108\frac{3}{4}\%$, $16\frac{2}{3}\%$.

Express the following as percentages —

3. $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$.
4. $\frac{2}{3} + \frac{3}{7} + 0.075 + 1.25$.

Write down as decimals the following percentages —

5. $30\%$, $5\%$, $208\%$.
6. $116\frac{1}{4}\%$, $2\frac{2}{3}\%$, $\frac{1}{2}\%$.

Find the values of the following —

7. $40\%$ of £1.
8. $25\%$ of 10s.
9. $210\%$ of 2s. 6d.
10. $66\frac{2}{3}\%$ of 1s.
11. $112\frac{3}{4}\%$ of 2 yd.
12. $48\%$ of 2½ tons.

Express the first quantity in each pair as a percentage of the second quantity:

13. 4s.; £1.
14. 15s.; £1 10s.
15. 2s. 6d.; 1s.
16. 23 cwt.; 1 ton.
17. 150 m.; 1 km.
18. 8 in.; 4 ft.
19. 15s. 9d.; £8 15s.
20. £13 15s.; £17 12s.
21. 25s. 3d.; £50 10s.

22. 15 per cent. of the pages in a magazine are advertisements. There are 60 pages in all; what is the number of pages of advertisements?

23. There are 424 pupils in a school. If $12\frac{1}{4}\%$ are absent ill, how many are present?

24. $62\frac{1}{2}\%$ of the cost of manufacture of a book-case is due to labour. If the cost is £20, how much is spent on labour?

25. A man earns £320 a year and spends £304 a year. What percentage of his income does he (i) spend, (ii) save?

26. A school contains 187 girls and 153 boys. What percentage of the school are (i) girls, (ii) boys?
APPENDIX

27. Bronze contains by weight 95\% copper, 4\% tin, 1\% zinc. How much of each is there in 5 tons of bronze?

28. A boy's marks for 3 test papers are 22 out of 24, 29 out of 40, 12 out of 20, respectively. What percentage does he obtain on the whole test?

Find, correct to the nearest penny, the value of:

29. 3\% of £8 14s. 9d. 30. 8\% of £7 6s. 4d.

31. 5\frac{1}{2}\% of £36 17s. 10d. 32. 4\frac{3}{4}\% of £87 13s. 2d.

33. 7\frac{5}{8}\% of £52 9s. 7d. 34. 2\frac{3}{4}\% of £63 11s. 5d.

35. A block of wood is 2 in. by 2\frac{1}{2} in. by 2\frac{1}{4} in. It is cut down into the shape of a cube of edge 2 in.; what percentage of the original block has been cut away?

36. A man insures his life for £6225 by paying annually a premium of 2\frac{3}{8}\% per cent. of the amount. How much is the premium he pays?

37. The contents of a private house, valued at £1720, are insured against fire at 1s. 6d. per cent. and against burglary at 1s. 9d. per cent. Find the premium, to the nearest penny, that must be paid to cover each of these risks.

38. A jeweller insures the gold articles on his premises valued at £12,000 against burglary by paying a premium of £75. What is the rate of the premium?

39. A merchant insures his stock valued at £24,000 against fire. For \frac{3}{4} of it he pays at the rate of 1s. 6d. per cent., but the remainder is highly inflammable and for this he has to pay at the rate of 5s. 6d. per cent. Find the total premium paid.

40. A man on his 25th birthday takes out a policy under which at the age of 65, or at previous death, he or his estate receives £100 for each £1 13s. of annual premium. What is his annual premium if he takes out a policy for £2900? How much will he have paid altogether in premiums by the time he has paid his final premium on his 64th birthday?

41. A woman at the age of 60 is able to buy for £1000 an annuity of £73 12s. a year, paid half-yearly, the first payment being made when her age is 60\frac{1}{2}. How long must she live in order to receive more than she has paid?

APPENDIX

EX. 90 (a). Percentage Changes

1. Increase 360 by 32\frac{3}{4}\%.

2. Decrease 42 by 66\frac{2}{3}\%.

Find the number or quantity of which:

3. 60\% is 48.

4. 24\% is £1.

5. 62\frac{1}{2}\% is 3 tons.

6. What number when increased by 8\% becomes 243?

7. What number when decreased by 28\% becomes 270?

8. What sum of money when increased by 35\% becomes £10 16s.?

9. What sum of money when decreased by 15\% becomes £11 18s.?

10. In 2 years a boy's height increases from 5 ft. to 5 ft. 3 in.; find the increase per cent. for this period.

11. 44\% of the pupils in a school are boys. Find the number of pupils if there are 154 boys.

12. A man saves 15\% of his income; what percentage of it does he spend? Find his income if he spends £340 a year.

13. The duty £3 6s. on a watch is 24\% of its value. Find the value of the watch.

14. 54\% of the population of a town are females. Find the total population if there are 6670 males.

15. A man who spends £630 a year saves 16\% of his income. Find his income.

16. If 12\% of a bill is deducted, £55 remains to be paid. How much is the bill?

17. If a boy gives the answer to a question as 12s. when the correct answer is 12s. 6d., find the error per cent.

18. Find the error per cent. to the nearest whole number in saying that a distance which is actually 5 nautical miles is 6 statute miles, if 1 nautical mile = 6080 feet.

19. The population of a village increased by 8 per cent. between 1922 and 1932. It was 810 in 1932, what was it in 1922?

20. A piece of flannel 50 in. long, 40 in. wide, when washed loses 10\% of its length and 5\% of its width by shrinkage. How many square inches of area are lost? By what percentage is the area decreased?
*21. The price of an article was reduced by $1\frac{1}{2}$% to 6 guineas. How much was it reduced?

*22. A man's annual income from a loan was increased by £1 7s. 6d. owing to the rate of interest being raised from 3% to 3\frac{1}{2}%.
How much was the loan?

EX. 92 (a). Gain and Loss per cent.

Find the S.P. in the following:

1. C.P. £24; gain 12\frac{1}{2}%;
2. C.P. 1s. 8d.; gain 15%.
3. C.P. 6s. 8d.; loss 7\frac{1}{2}%;
4. C.P. £3 2s. 6d.; loss 14\frac{3}{8}%. 

Find the gain or loss per cent. in the following:

5. C.P. £56; loss £21.
6. C.P. 11\frac{1}{2}d.; gain 2\frac{1}{2}d.
7. C.P. 18s.; S.P. £1.
8. C.P. 7s.; S.P. 8s. 6d.
9. S.P. 10s.; gain 2s. 6d.
10. S.P. £3; loss 6s. 8d.
11. S.P. £76; gain £12.
12. S.P. £5 4s. 2d.; loss 20s. 10d.

Find the C.P. in the following:

13. S.P. £44; gain 10%.
14. S.P. £81; gain 12\frac{1}{2}%. 
15. S.P. 1 guinea; loss 16%.
16. S.P. 7s. 6d.; loss 6\frac{1}{2}%. 
17. Tea costing 1s. 9d. per lb. is sold at 2s. 6d. per lb.; find the gain per cent.
18. I bought an overcoat for ten guineas and sold it to a friend for £8 15s.; find my loss per cent.
19. A music dealer buys a gramophone for £3 10s. and sells it at a gain of 45 per cent. Find his profit.

20. By selling a horse for £35, a dealer made a profit of 40%; what did the horse cost?
21. If I sell my car for £198, I shall lose 28 per cent.; what did I pay for it?
22. A man gained 8 per cent. by selling an article for 18s.; what was his profit?
23. A golf professional buys 240 clubs for £160 and sells them at 17s. 6d. each; find his gain per cent.

24. A coal merchant buys anthracite at £4 a ton and sells it at 5s. 9d. per cwt.; find his gain per cent.
25. By selling a cottage for £252, a man gains 40 per cent.; what profit would he have made if he sold it for £300?
26. Tea is bought at £7 per cwt. and sold at 1s. 8d. per lb.; find the gain per cent.
27. A grocer buys eggs at 40d. per dozen and sells them at 50d. per dozen; find his gain per cent.
28. A man buys oranges at 1s. 1½d. per dozen and sells them at 8 for 1s.; find his gain per cent.
29. What did a man pay for 14 dozen eggs if by selling them at 8 for 1s. he gained 26 per cent.
30. Ink bought at 10s. 4d. per gallon is retailed at 6d. per quart.
31. A hen lays 3 eggs in 1 day. How many will she lay in 1 week?
32. A man gains 10% by selling a horse for £50; what did the horse cost?
33. A man loses 20% by selling a horse for £40; what did the horse cost?
34. A man gains 15% by selling a horse for £50; what did the horse cost?
35. A man gains 25% by selling a horse for £60; what did the horse cost?
36. A man gains 30% by selling a horse for £70; what did the horse cost?
37. A man gains 35% by selling a horse for £80; what did the horse cost?
38. A man gains 40% by selling a horse for £90; what did the horse cost?
39. A man gains 45% by selling a horse for £100; what did the horse cost?
40. A man gains 50% by selling a horse for £110; what did the horse cost?
41. A man gains 55% by selling a horse for £120; what did the horse cost?
42. A man gains 60% by selling a horse for £130; what did the horse cost?
43. A man gains 65% by selling a horse for £140; what did the horse cost?
44. A man gains 70% by selling a horse for £150; what did the horse cost?
45. A man gains 75% by selling a horse for £160; what did the horse cost?
46. A man gains 80% by selling a horse for £170; what did the horse cost?
47. A man gains 85% by selling a horse for £180; what did the horse cost?
48. A man gains 90% by selling a horse for £190; what did the horse cost?
49. A man gains 95% by selling a horse for £200; what did the horse cost?
50. A man gains 100% by selling a horse for £210; what did the horse cost?
A commercial traveller is paid £400 a year plus a commission of 15% on the amount by which his total sales exceed £1000. How much does he receive in a year in which his sales amount to £1800?

12. A man, who spends 82% of his income, saves £90 a year. How much does he spend in the year?

13. A merchant buys 800 lb. of tea for £93 6s. 8d. and sells it at a profit of 25%. What is the selling price per lb.?

14. The population of a town increased by 5% between 1922 and 1932. It was 15918 in 1932, what was it in 1922?

*15. Return fare is double single fare, but a week-end return ticket is only 1 3/4 times the single fare. By how much per cent, is expenditure on a return journey reduced by using a week-end ticket?

*16. A man gains £8 by selling a ton of apples at 6d. per lb.; find his gain per cent.

17. A dealer sells a chair for £1 16s. at a gain of 20 per cent.; what was his profit?

18. In an examination, it is necessary to get 35% of the total to pass. There are 3 papers; a boy gets 62 out of 120, and 35 out of 150 on the first two; how much must he get out of 180 on the third paper to pass?

*19. An electric light company formerly charged 6d. per unit, but has now lowered its prices 10%. I have two meters which read on 1st January, 3179 and 2575, and on 1st April, 3261 and 2613. How much is my bill for the period 1st January to 1st April?

*20. Find the value after 3 years of a car bought for £400, which loses each year 20% of its value at the beginning of that year.

21. A dealer obtains a commodity from the U.S.A. at a cost of 15 dollars per short ton (i.e. 2000 lb.). Taking £1 = 5 dollars, find the price per ton avoided by which he must sell it to gain 25 per cent.

*22. A shop manager is paid a salary of £150 a year and receives a commission of ¼ per cent. on the sales. What were the sales in a year in which he received £390 in all?

*23. In 1930 the gross income of a company was £325,000, which after the payment of working expenses left a balance of £12,500. In 1931 the gross income increased by 10% and the working expenses by 12%. What balance remained?

*24. A 10-gallon barrel is full of a wine which contains 8% of alcohol. Two gallons are drawn off and replaced by water. What is the percentage of alcohol in the mixture?

*25. In the course of a year, ¼ of my capital increases by 3%, ¼ of it decreases by 4%, and the rest increases by 6%. What is the percentage increase on the whole?

*26. 100 guineas is shared between A, B, C, so that B receives 50% more than A, and C receives 25% more than B. How much does each receive?

*27. A is 15% heavier than B, and C is 15% lighter than B. By how much per cent. is A heavier than C?

CHAPTER XIII
FURTHER AREAS AND VOLUMES

EX. 94 (a). Rectangles
1. Into how many squares, each of side 1 cm., can a square of side 1 dm. be divided? Express 1 sq. cm. in sq. dm.

2. Draw on a piece of squared paper ruled in inches and tenths of an inch a rectangle 2 1/2 in. long, 1 1/4 in. broad. How many small squares does it contain? What is the area of each small square and of the rectangle?

3. A rectangle is 1 1/2 cm. long, 1 3/4 cm. broad. Express its length and breadth in mm. Into how many squares, each of side 1 mm., can the rectangle be divided? What is the area of the rectangle in (i) sq. mm., (ii) sq. cm.?

4. A rectangle is 2 1/4 in. long, 3 1/2 in. broad. Into how many squares, each of side 3/4 in., can it be divided? What is the area of the rectangle?

Express:

5. 1 sq. m. in sq. dm.; 1 sq. dm. in sq. m.

6. 1 ac. in sq. ch., in sq. yd.    7. 1 sq. mi. in ac.

8. 12 sq. ft. in sq. ft.   9. 200 sq. mm. in sq. cm.

10. 4500 sq. cm. in sq. m. 11. 4840 sq. yd. in sq. ch.

Find the areas of the following rectangles:

12. 3 ft. by 1 ft. in sq. yd.
13. 1 dm. by 1 mm. in sq. cm.

14. 2 m. by 15 cm. in sq. dm.
15. 10 ch. by 10 ch. in ac.
EX. 95 (a). Rectangular Areas

Find the perimeters and areas of the following rectangles, giving the perimeters as compound quantities and the areas in terms of the unit indicated in brackets:—

1. 3 yd. 2 ft. by 2 ft. (sq. ft.).
2. 10 ft. 3 in. by 8 ft. (sq. ft.).
3. 15 ft. 4 in. by 9 ft. (sq. ft.).
4. 8-4 cm. by 5-6 cm. (sq. cm.).
5. 2-75 m. by 8 dm. (sq. m.).
6. 2$\frac{1}{2}$ ft. square (sq. ft.).

Find the lengths of the following rectangles:—

7. Area 35 sq. ft., breadth 2$\frac{1}{2}$ ft.
8. Area 36$\frac{1}{2}$ sq. m., breadth 4-2 m.
9. Area 10 sq. yd. 8 sq. ft., breadth 2 yd. 1 ft.
10. Area 32 sq. ft. 72 sq. in., breadth 3 ft. 9 in.

Find in acres the area of a field:

11. 34 ch. long, 18 ch. wide.
12. $\frac{1}{4}$ mi. long, 110 yd. wide.
13. A railway requires 4 ac. of ground for each mile length of track. Find the average breadth of the track.
14. A rectangular plot of area 1 ac. is enclosed by hurdles, and one side measures 110 yd. How many more square yards could be enclosed by the same hurdles if the enclosure is square?

Find the cost of the following carpets:—

15. 9 ft. by 5 ft. 6 in. at 3s. per sq. ft.
16. 11 ft. 4 in. by 6 ft. 3 in. at 2s. 6d. per sq. ft.

Find the cost per sq. ft., correct to the nearest penny, of the following:—

17. A Turkey rug, 6 ft. by 2 ft. 6 in., at 39s. 6d.
18. A Persian carpet, 14 ft. 3 in. by 10 ft., at £20.
19. A Bokhara rug, 4 ft. 10 in. by 3 ft. 9 in., at 11 guineas.

20. How many wooden blocks, each 9 in. by 4 in., are needed for the floor of a room 16 ft. 6 in. long, 13 ft. 8 in. wide?
21. How many planks, each 9 ft. 4 in. long, 7$\frac{1}{2}$ in. wide, are needed for a platform 140 ft. long, 25 ft. wide?
22. 5120 tiles, each 6 in. square, are needed to pave a rectangular court 40 ft. long. What is the width of the court?
23. A strip of carpet, 27 in. wide, 2 yd. long, costs 18s.; find the cost per sq. ft.
24. A sheet of cardboard weighs 61-3 gm.; a rectangular portion, 17-6 cm. by 13-5 cm., is cut out of it and found to weigh 48-7 gm. Find to the nearest sq. cm. the area of the original sheet.

EX. 96 (a). Papering and Carpentry

Find the total area of the four walls of a room:

1. 7 yd. long, 5 yd. wide, 10 ft. high.
2. 16 ft. 2 in. long, 12 ft. 6 in. wide, 9 ft. high.
3. 4 m. 80 cm. long, 3 m. 60 cm. wide, 2 m. 75 cm. high (in sq. m.).
4. Find the area of cardboard used for making a closed box, 8 cm. by 6 cm. by 2-5 cm.
5. Repeat No. 4 for a closed box, 2 ft. by 1 ft. 6 in. by 1 ft.
6. Find the total area of the external surface of an open tank, 4$\frac{1}{2}$ ft. square, 6 ft. high, external measurements.
7. Repeat No. 6 for an open tank, 2-5 m. long, 1-5 m. wide, 0-5 m. high.
Find the cost of the paper for the walls of the rooms, Nos. 8-10, if each "piece" of paper is 21 in. wide and 12 yd. long:

8. 13 ft. long, 8 ft. wide, 7 ft. 6 in. high, at 4s. per piece, allowing 63 sq. ft. for doors, etc.
9. 15 ft. 6 in. long, 12 ft. 6 in. wide, 8 ft. 6 in. high, at 3s. 6d. per piece, allowing 100 sq. ft. for doors, etc., and assuming that a whole number of pieces must be bought.
10. 18 ft. long, 14 ft. wide, 10 ft. 6 in. high, at 5s. 9d. per piece, allowing for a door 7 ft. by 3 ft., 2 windows each 6 ft. by 4 ft., a fireplace 4 ft. by 5 ft., and assuming that a whole number of pieces must be bought.

*11. Find the cost per sq. yd. of cloth 50 in. wide at 6s. 3d. per yard length.

*12. Find the cost per sq. ft. of material 27 in. wide at 6s. per yard length.

13. A room is 21 ft. long, 18 ft. wide. Carpet is bought in a strip 27 in. wide; what length is required to cover the floor? Find the cost if the strip costs 9s. 6d. per yard length.

14. Find the cost of a carpet 14 ft. by 10 ft., if the carpet is made up from a roll 30 in. wide at 8s. per yard.

*15. Find the cost of a carpet 13 ft. 6 in. by 11 ft. 8 in., if the carpet is made up from a roll 28 in. wide at 7s. 6d. per yard.

16. Find the cost of lining a box without a lid, 3 ft. long, 2 ft. wide, 8 in. deep, internal measurements, with material at 2s. 6d. per sq. ft.

*17. An open tank is 8 ft. long, 5 ft. wide, 4 ft. deep. Find the amount of paint required inside and out, if 1 lb. of paint covers 72 sq. ft. [Neglect the thickness of the material.]

*18. A tin box with a lid is a cube of edge 10 in. Find the number of sq. ft. of tin required to make it, if the rim of the lid is 3 in. wide.

**EX. 97 (a). Area of Borders**

1. Find the area of a path 5 ft. 6 in. wide running all round a lawn 30 yd. long, 70 ft. wide.

2. A photograph, 6 in. by 5 in., is mounted on a rectangular card so that there is a border 1 1/2 in. wide all round. What is the visible area of the card?

8. Find the area of a path running all round a lawn, 45 ft. long, 30 ft. wide, if the path is 5 ft. wide along the longer sides and 3 ft. wide along the shorter sides.

4. A picture is mounted on a card 1 ft. 2 in. high, 7 in. wide, and there is a margin 2 in. wide at the top and bottom, and 1 3/4 in. wide along the sides. Find the total area of the margin.

*5. A rectangular lawn 45 yd. by 22 yd. is surrounded by a path 4 1/2 ft. wide, and a wall 5 ft. high runs along the outer edge of the path. Find in sq. yd. (i) the area of the path, (ii) the area of the inner surface of the wall.

6. Find the shaded area in Ex. 97, No. 7, p. 183, if \(a = 3\frac{1}{2}, b = 2, c = 2\frac{1}{2}, d = 1\frac{1}{2}\), the units being inches.

*7. Find the area of the surface of the brickwork in Ex. 97, No. 8, p. 183, if \(a = 2\frac{1}{2}, b = 1\frac{1}{2}, c = 1\frac{1}{2}, d = 1\frac{1}{2}, e = 20, f = 13\), the units being feet.

8. Find the cost of carpeting the floor of a room 14 ft. long, 10 ft. wide, if a margin 9 in. wide is left all round, at 9s. 6d. per sq. yd.

9. Find the cost of the matting for the floor of a room, 6 m. 25 cm. by 5 m. 40 cm. at 7 fr. 20 c. per sq. m., if a border 20 cm. wide is left all the way round.

10. A walled courtyard is 18 yd. long, 15 yd. wide; a gravel path 5 ft. wide runs round the edge of it, and the rest of it is asphalted at 6d. per sq. ft. Find the cost of the asphalt.

11. A border 15 in. wide all round the edge of the floor of a room 15 ft. long, 13 ft. 6 in. wide is stained at a cost of 1 1/4d. per sq. ft. Find the cost.

*12. A courtyard 20 ft. square is paved with black and white tiles, each 5 in. square. There is a band 2 1/2 ft. wide of black tiles all round the edge, and the rest are white. How many tiles of each colour are used?

*13. A room, 30 ft. by 25 ft., is to have a carpet in the middle of the floor with a border 1 ft. 6 in. wide all round. If the carpet is made up from a roll 22 in. wide costing 3s. 6d. per yard length, and if 3d. per sq. yd. is charged for making it up, find the total cost.

*14. A rectangular grass park is closed in by a fence measuring 1800 yd., by 1100 yd. The whole is surrounded by a concrete pavement 3 yd. wide, and a roadway 25 yd. wide runs round outside the pavement. Find the area of (i) the pavement, (ii) the roadway.
EX. 98 (a). Cuboids

1. How many cubes, edge \( \frac{1}{3} \) in., are required for building up 1 cu. in.? What is the volume in cu. in. of a cube, edge \( \frac{1}{3} \) in.?

2. How many cubes, edge \( \frac{1}{4} \) in., are required for building up a cuboid, 2 in. by \( \frac{1}{3} \) in. by \( \frac{1}{2} \) in.? What is the volume in cu. in. of (i) each small cube, (ii) the cuboid? What is the value of \( 2 \times \frac{1}{4} \times \frac{1}{3} \)?

3. A cuboid measures 1-6 cm. by 1-5 cm. by 0-8 cm. Into how many cubes, each of edge 1 mm., can it be divided? What is the volume in cu. cm. of (i) each small cube, (ii) the cuboid? What is the value of \( 1-6 \times 1-5 \times 0-8 \)?

4. A cuboid measuring 4 in. by \( 2\frac{1}{2} \) in. by \( 1\frac{3}{4} \) in. is cut up into equal cubes. What is the largest possible edge of each cube; and what is its volume? How many such cubes can be obtained? What is the volume of the cuboid? What is the value of \( 4 \times 2\frac{1}{2} \times 1\frac{3}{4} \)?

Express:

5. 1 cu. yd. in cu. ft.; 1 cu. ft. in cu. yd.

6. 1 cu. cm. in cu. m.; 1 cu. mm. in cu. dm.

7. 3 cu. ft. in cu. yd. 8. 25 cu. dm. in cu. m.

9. 1000 c.c. in cu. m. 10. 500 c.c. in litres.

Find the volumes of the following cuboids:

11. 4 in. by \( 1\frac{1}{2} \) in. by \( \frac{1}{3} \) in. 12. 8 in. by \( 2\frac{1}{2} \) in. by \( \frac{1}{3} \) in.

13. 1 dm. by 1 cm. by 5 mm. 14. 1-5 dm. by 8 cm. by 6 mm.

EX. 99 (a). Volumes of Cuboids

1. Find in cu. in. the volume of a metal sheet, 1 ft. 8 in. long, 1 ft. 2 in. wide, \( \frac{3}{8} \) in. thick.

2. Find in cu. yd. the volume of air-space in a hall 24 yd. long, 15 yd. wide, 16\( \frac{2}{3} \) ft. high.

3. How many tiles, each \( 4\frac{1}{2} \) in. by 4 in. by \( \frac{3}{8} \) in., can be packed in a crate 2 ft. by 18 in. by 1 ft., internal measurements?

4. A tank 5 ft. long, 4 ft. 6 in. wide, 2 ft. deep, contains water to a depth of 4 in.; how many cu. ft. of water must be added to make the tank half-full?

5. Find the capacity in litres of a cistern, 2-5 m. by 1-5 m. by 1-2 m., internal measurements.

6. A tank 1-75 m. long, 1-5 m. wide, contains water to a depth of 32 cm. Find the volume of the water in litres.

*7. How many rectangular blocks, each measuring \( 1\frac{1}{2} \) in. by \( \frac{3}{4} \) in. by \( \frac{1}{4} \) in., can be cut from a rectangular wooden block 1 ft. 9 in. long, 1 ft. 8 in. wide, 5 in. high? How many cubes, edge \( 2\frac{1}{2} \) in., can be cut from this block, and how much wood remains over?

8. A notebook is 7-5 cm. long, 5 cm. wide, 4-5 mm. thick. How many can be packed in a box 3 dm. long, 2-5 dm. wide, 1-8 dm. high?

9. The volume of a rectangular beam 7\( \frac{1}{2} \) in. wide, 1-6 in. deep, is \( 1\frac{1}{2} \) cu. ft.; find its length.

10. The volume of a metal plate 1 ft. 6 in. long, 10 in. wide, is 155 cu. in.; find its thickness.

11. Find in lb. the weight of a sheet of plate-glass 9 ft. high, 10 ft. 8 in. wide, \( 1\frac{1}{2} \) in. thick, if the glass weighs 175 lb. per cu. ft.

12. Find in lb. the weight of a beam of timber 20 ft. long, 8 in. wide, 9 in. deep, if the timber weighs 48 lb. per cu. ft.

13. 20 bricks, each 8 in. by 6 in. by 3 in. are placed in a tank half-full of water whose base is 5 ft. by 2 ft. What is the height the water-level rises if all the bricks are submerged and no water overflows?

*14. A swimming-bath is 80 ft. long, 54 ft. wide. If water is pumped into it at the rate of 1\( \frac{1}{2} \) cu. ft. per second, find the rate in inches per minute at which the water-level rises after the whole of the floor is covered.

In Nos. 15–17, assume that 1 cu. ft. = 6\( \frac{1}{2} \) gall.

15. How many gallons of water will a tank 6 ft. long, 5 ft. wide, 1 ft. 8 in. deep hold?

16. 50 gallon of water are poured into an empty tank, 4 ft. long, 2 ft. 6 in. wide, 4 ft. deep. What is the depth of water in the tank?
*17. If 450 gal. of water are drawn out of a cistern, 4 ft. 6 in. long, 4 ft. wide, find how much the water-level falls.

18. The water in a tank 8 dm. long, 6 dm. wide, weighs 216 kg. Find the depth of the water. [1 c.c. of water weighs 1 gm.]

*19. Find the cost of gravel for a path 4 ft. wide running all round a lawn 77 ft. long, 38 ft. wide, if the depth of gravel is 3 in. and the gravel costs 10s. 6d. per cu. yd.

*20. A ream of paper consists of 480 sheets, each measuring 13½ in. by 8 in., and weighs 7½ lb. The paper weighs 50 lb. per cu. ft. How many sheets are there in a pile 1 in. deep?

*21. A piece of sheet-brass 9 in. square weighs 2 lb. 3 oz., and 1 cu. ft. of brass weighs 500 lb. Find, to the nearest 1/100 in., the thickness of the sheet.

*22. A closed rectangular tank, 4 ft. long, 2¾ ft. wide, contains 1½ tons of water. When the water freezes, the tank just bursts. Assuming that 1 cu. ft. of water weighs 1000 oz. and that water increases by 10 per cent. in volume when it freezes, find in feet the depth of the tank.

**EX. 100 (a). Volumes by Subtraction**

1. The outside measurements of a closed cigar box are 8½ in. by 5¼ in. by 2 in., and the wood is ¼ in. thick. Find the volume of the interior of the box.

   Find the volume of the wood required for making the closed boxes, Nos. 2-5:

2. External dimensions: 2 ft. by 1 ft. 3 in. by 1½ in.; wood ¼ in. thick.

3. External dimensions: 25 cm. by 18·5 cm. by 16 cm.; wood 1½ cm. thick.

4. Internal dimensions: 10 in. by 9·5 in. by 7 in.; wood ¼ in. thick.

5. Internal dimensions: 14 dm. by 9 cm. by 4 cm.; wood 5 mm. thick.

Find the volume of the wood required for making the open boxes, Nos. 6-8:

6. External dimensions: 12 in. long, 8½ in. wide, 6½ in. high; wood ¼ in. thick.

7. External dimensions: 17 cm. long, 10 cm. wide, 3 cm. high; wood 5 mm. thick.

8. Internal dimensions: 2·4 dm. long, 1·5 dm. wide, 9 cm. high; wood 1·25 cm. thick. (Ans. to nearest c.c.)

9. A closed rectangular box measures externally 3 ft. by 2 ft. 8 in. by 2 ft. 4 in., and is made of wood 1½ in. thick. Find in lb. the weight of the box if 1 cu. ft. of the wood weighs 48 lb.

*10. A corridor 14 ft. high, 4 ft. wide is built all round the outside of a rectangular room; the length of the corridor is 35 ft. in one direction and 28 ft. in the other, both measurements being from wall to wall. Find the volume of the air-space of the corridor.

*11. The base of an open wooden tee-box is 12 in. square and its height is 10 in., external measurements; the box is made of wood 1 in. thick, and is full of sand, level with the top. Find the total weight, to the nearest lb., if the wood weighs 0·35 oz. per cu. in. and the sand weighs 1·1 oz. per cu. in.

*12. A room, 37 ft. long, 33 ft. 6 in. broad, contains a concrete floor 6 ft. 6 in. wide all round the sides, and the central portion is occupied by a bath 8 ft. deep. Find (i) the number of cu. ft. of water the bath will hold, (ii) the cost of tiles, each 6 in. square, for the concrete floor at 3d. per tile.

**EX. 101 (a). Solids of uniform Cross-section**

Find the volume of a solid of uniform cross-section, given:

1. Area of cross-section = 48 sq. in., length = 1½ yd. (Ans. in cu. ft.)

2. Area of cross-section = 75 sq. mm., length = 7·2 m. (Ans. in c.c.)

Find the height of a can of uniform cross-section, given:

3. Volume = ⅓ cu. ft., area of cross-section = 40 sq. in.

4. Volume = 7½ litres, area of cross-section = 300 sq. cm.
Find the area of the cross-section, assuming it to be uniform, of a vessel, given:

5. Volume = 2\(\frac{3}{4}\) cu. ft., height = 1 ft. 3 in.
6. Capacity = 24 litres, height = 3-2 m.
7. An oil drum of uniform cross-section holds 10 gall. The area of its cross-section is 162 sq. in., find its height. [1 cu. ft. = 6\(\frac{3}{4}\) gal.]
8. A container, base area 15 sq. in., height 1 ft. 6 in., internal measurements, is full of spirits weighing 50 lb. per cu. ft. Find the weight of the spirits.

9. The dimensions of the T-shaped cross-section of a body, 1 ft. 8 in. long, are shown in inches in the diagram. Find (i) the volume of the solid, (ii) the weight if the material weighs 0-24 lb. per cu. in.

10. The rain over an area of 120 sq. yd. is collected in a tank 12 ft. long, 7 ft. 6 in. wide. What rise of level of the water in the tank is caused by half an inch of rain?

11. An empty cistern, 3 ft. deep on a base 4 ft. 6 in. square, receives the rainfall from roofs covering a horizontal area of 72 sq. yd. What depth of rainfall is necessary to fill the cistern?

12. A pond of surface area 8 ac. is frozen over with ice of total weight 2178 tons. Find the average thickness of the ice, if 1 cu. ft. of ice weighs 56 lb.

13. A river 30 ft. deep, 200 yd. wide, flows at 1 mi. an hour. Find to the nearest 1000 tons the weight of water which passes a given point in a quarter of an hour. [1 cu. ft. of water weighs about 1000 oz.]

14. A roll of copper-wire, 100 m. long, weighs 3-96 kg. Find the area of the cross-section of the wire if the copper weighs 8-8 gm. per c.c.

15. The internal cross-section of a pipe is 5-6 sq. in., and water is pouring out of it at the rate of 4\(\frac{1}{8}\) ft. per second. If the pipe remains full, find the number of gallons, to the nearest hundred, discharged by the pipe per hour.
CHAPTER XIV

CIRCLES AND CYLINDERS

EX. 103 (a). Circumference of Circle

[Take \( \pi \) to be 3.14 or \( \frac{22}{7} \), whichever is the more convenient. Give the answers as decimals, not vulgar fractions]

Find the lengths of the circumferences of the circles, Nos. 1-6:
1. Radius, 3.5 in.  2. Radius, 4.2 km.  3. Radius, 4 ft. 8 in.
4. Radius, 5 cm.   5. Diameter, 21 in.  6. Diameter, 6 yd.

Find the radii of the circles, Nos. 7-9:
7. Circumference, 5\( \frac{1}{2} \) ft.  8. Circumference, 10 in.
9. Circumference, 1 mi.

10. The equatorial diameter of the Earth is 7920 mi. Find the length of the Equator.

11. A piece of thread is wrapped 10 times round a reel, and when unwrapped is found to measure 121.5 cm.; find the diameter of the reel.

12. The driving-wheel of an engine is 6 ft. 5 in. in diameter. How many revolutions does it make per mile?

13. The length of a circular running-track is 400 m.; find the radius of the track.

14. A metal sheet is a quadrant of a circle of radius 6 in.; find its perimeter.

*15. The statement that the "gear" of a bicycle is 80 means that each revolution of the pedals makes the bicycle move a distance equal to the circumference of a circle of diameter 80 in. How many revolutions of the pedals are made per mile for this bicycle?

*16. Find the speed in miles per hour of a point on the rim of a turbine wheel, diameter 18 in., making 14,000 revolutions per minute.

EX. 104 (a). Area of Circle

Find the areas of the circles, Nos. 1-6:
Take \( \pi = 3\frac{1}{2} \) in Nos. 1-3, and take \( \pi = 3.14 \) in Nos. 4-6:
1. Radius, 3\( \frac{1}{2} \) in.   2. Diameter, 5-6 cm.  3. Radius, 10\( \frac{1}{2} \) ft.
4. Diameter, 10 cm.  5. Radius, 3 yd.   6. Diameter, 1 ft. 3 in.

Find the radii of the circles whose areas are given in Nos. 7-9, taking \( \pi = 3\frac{1}{2} \):
7. 154 ft.   8. 1386 sq. yd.  9. 6-16 sq. km.

[For the remainder of this exercise, take \( \pi = 3\frac{1}{2} \) or \( \frac{22}{7} \), whichever is the more convenient]

10. Find the area of the top of a circular table, diameter 4 ft. 8 in.

11. Find the area of a quadrant of a circle of radius 6 in.

*12. Circular discs, each 1 in. in diameter, are stamped out of a rectangular sheet of metal 6\( \frac{1}{2} \) in. long, 4\( \frac{3}{4} \) in. wide. How many discs can be made? What is the area of the metal that remains?

*13. Find the circumference of a circle of area 5544 sq. m.

*14. The area of one face of a semicircular protractor is 77 sq. cm.; find its total perimeter.

15. From a circular disc of diameter 2 in., two circles each of diameter 1 in. are cut away. Sketch the shape of what remains, and find its area.
16. A circular lawn of diameter 20 yd. is surrounded by a path 2 yd. wide. Find the area of the path.

17. Find the cost, to the nearest shilling, of paving a circular hall, 8 yd. in diameter, at 2s. 6d. per sq. ft.

18. If the perimeter of a quadrant of a circle is 100 yd., find the area of the quadrant.

19. A window is in the shape of a rectangle surmounted by a semicircle. The greatest height of the window is 7 ft. and the breadth of its base is 4 ft. Show the dimensions of the window in a sketch, and find its area.

20. A tray is made from a rectangle, 2 ft. by 18 in., by rounding each corner so that it becomes a quadrant of a circle of radius 3 in. Find the area of the tray.

21. Find the area of a sector of a circle of radius 5 in., if the angle of the sector is 150°. [See Ex. 104, No. 24, p. 200.]

22. Find the radius of a circle whose area is equal to the difference of the areas of two circles of radii 13 in., 12 in. respectively.

**EX. 105 (a). Surface of Cylinder**

Find the areas of the curved surfaces of the cylinders, Nos. 1–4:

1. Radius 10 cm., height 4 cm.
2. Radius 1 ft., thickness 1 in.
3. Diameter 3 1/2 ft., width 6 ft.
4. Diameter 6 mm., length 10 m.

Find the total areas of the surfaces of the solid cylinders, Nos. 5–8:

5. Radius 4 cm., length 1 dm.
6. Diameter 3 cm., thickness 2 mm.
7. Diameter 1 in., length 1 ft.
8. Diameter 3 ft. 4 in., width 1 ft. 10 in.

Find the total area of the external surface of a cylinder closed at one end and open at the other end, with the given external measurements, Nos. 9, 10:

9. Radius 4 cm., height 5 cm. 10. Diameter 1 yd., height 18 in.

11. The area of the curved surface of a cylinder 8 in. high is 176 sq. in.; find the radius of the cylinder.

**EX. 106 (a). Volume of Cylinder**

[Take 1 cu. ft. = 6 1/4 gall.; do not give results to more than 3 figures]

Find the volumes of the circular cylinders, Nos. 1–4:

1. Diameter 5 in., height 14 in.
2. Radius 6 cm., height 7 cm.
3. Radius 5 mm., length 10 m.

Find the heights of the circular cylinders, Nos. 5, 6:

5. Volume 198 c.c., radius 3 cm.
6. Volume 160 cu. in., radius 1 in.

Find the diameters of the circular cylinders, Nos. 7, 8:

7. Volume 39.6 cu. in., height 5.6 in.
8. Volume 2.64 litres, height 2.1 m.
9. A cylindrical tank is 2½ ft. in diameter and 3½ ft. high, internal measurements. How many gallons will it hold?

10. A cylindrical tin is 14 cm. high and 12 cm. in diameter. How many such tins can be filled with cocoa from a box containing 1 cu. m. of cocoa?

11. A cylindrical measure which holds 1 gallon is 8½ in. in diameter. Find its height.

Find the volume of metal in the hollow pipes, Nos. 12, 13:

12. Internal diameter 2 cm., metal 1 mm. thick, length 5 dm.

13. External diameter 4 in., metal ½ in. thick, length 6 ft.

14. An open cylindrical jar is 7 in. in diameter and 11 in. high, external measurements. The material is ½ in. thick. Find, in cu. in., the amount of water it will hold. Find also the volume of the material used in making the jar.

15. Find the weight of a cast-iron cylindrical pipe of 4 in. internal diameter, 4 ft. long, and of metal ½ in. thick, if 1 cu. in. of cast iron weighs ½ lb.

16. The cross-section of a barn is a rectangle with a semicircle above it. The width of the barn is 28 ft., its greatest height is 30 ft., and its length is 60 ft. Find its volume.

17. Water flows through a pipe, diameter 1 ft., at the rate of 5 ft. per second into a swimming-bath 60 ft. by 20 ft. By how much does the depth of water increase in 5 min. if the pipe remains full?

18. A bath holding 60 gallon can be filled in 3 min. by water from a pipe of cross-section 2 sq. in. Find, in ft. per sec., the speed of water in the pipe, if the pipe remains full.

19. The length of the rim of a cylindrical tin is 4 ft. and its height is 2½ ft., internal measurements. Find the capacity of the tin in gallons.
Find the unknown quantities in Nos. 6-10:

<table>
<thead>
<tr>
<th>Principal</th>
<th>Interest</th>
<th>Amount</th>
<th>Time</th>
<th>Rate</th>
<th>% p.a.</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.</td>
<td>£3 12s.</td>
<td>...</td>
<td>2 months</td>
<td>6</td>
<td>...</td>
</tr>
<tr>
<td>7.</td>
<td>£3000</td>
<td>£3810</td>
<td>6 years</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>8.</td>
<td>£2 11s.</td>
<td>...</td>
<td>2 years</td>
<td>2$4$</td>
<td>...</td>
</tr>
<tr>
<td>9.</td>
<td>£330</td>
<td>£77 19s. 3d.</td>
<td>...</td>
<td>5$4$</td>
<td>...</td>
</tr>
<tr>
<td>10.</td>
<td>£19 2s. 6d.</td>
<td>...</td>
<td>6 months</td>
<td>4$8$</td>
<td>...</td>
</tr>
</tbody>
</table>

11. At what rate % p.a. will 18s. amount to £1 in 16 mo.?
12. If the rate of interest is raised from 3$\frac{1}{3}$% p.a. to 4$\frac{1}{3}$% p.a., find the increase in 3 months’ interest on £120.

13. In the year 1248, moneylenders in Oxford were allowed to charge 2d. for a loan of £1 for a week. Find the rate % p.a., taking a year as 52 weeks.

14. Find the rate % p.a. if the interest on £212 10s. for 2 yr. is £6 7s. 6d., and find the interest on £850 for 2 mo. at the same rate.

15. In what time will £4500 amount to £5141 5s. at 3½% p.a.? To what will it amount at 6% p.a. for the same time?

16. A man borrowed £150 at 4½% p.a., and when he repaid it the interest was 12s. 6d., correct to the nearest penny. For how many days did the loan run?

17. When the rate of interest on a loan is lowered from 4½% p.a. to 3½% p.a., the quarterly interest (i.e. for 3 months) decreases by £1 10s. How much is the loan?

18. In what time does a sum of money increase by 5%, if invested at 7½% p.a. simple interest?

**EX. 115 (a). Miscellaneous Examples**

1. Find the amount of £540 for 8 mo. at 3½% p.a. simple interest.
2. A moneylender advances £35 at 7½% per month simple interest. How much must be paid at the end of 8 mo. to discharge the debt?
3. What percentage is equivalent to a discount of 9d. in the £?
4. The interest on a loan for 5 mo. at 7½% p.a. is £2 8s. What was the loan?
CHAPTER XVI
APPROXIMATIONS

EX. 117 (a). Contracted Multiplication
Evaluate correct to 2 significant figures:
1. $6.178 \times 8.423$. 2. $17.39 \times 0.4826$. 3. $3724 \times 8197$.
Evaluate correct to 3 significant figures:
4. $18.073 \times 24.126$. 5. $25947 \times 376.24$.
Evaluate correct to 2 places of decimals:
6. $0.82753 \times 9.71624$. 7. $(0.92614)^2$.
Evaluate correct to 3 significant figures:
8. $178.962 \times 0.316254$. 9. $0.84276 \times 0.651723$.
Evaluate correct to the number of places of decimals indicated in brackets:
10. $15.7124 \times 4.71628$ (1). 11. $0.7261344 \times 0.514362$ (3).
Evaluate correct to 4 significant figures:
12. $5173.8047 \times 72.61524$. 13. $(27.307254)^2$.
Evaluate:
*14. $0.82147 \times 0.31516 \times 0.24613$ correct to 3 decimal places.
*15. $190.467 \times 27.3162 \times 8.10634$ correct to 4 significant figures.
*16. $3.14159 \times (17.183)^2$ correct to the nearest integer.

EX. 118 (a). Contracted Division
Evaluate correct to 2 significant figures:
1. $7.309 \div 3.617$. 2. $8264 \div 9063$. 3. $0.04718 \div 0.9274$.
Evaluate correct to 3 significant figures:
4. $0.82476 \div 13.084$. 5. $48725 \div 37.152$.
Evaluate correct to 2 places of decimals:
6. $0.7208 \div 5.169$. 7. $173.26 \div 64.183$.

EX. 119 (a). Errors
Find to 1 significant figure the maximum absolute error, the relative error and error per cent. in Nos. 1–4:
1. A length of 2 ft. 8 in., correct to the nearest inch.
2. A profit of £1 4s. 9d., correct to the nearest threepence.
3. An area of 5 ac., correct to the nearest 100 sq. yd.
4. A journey takes 18 hr. 20 min., correct to the nearest 10 min.

Find to 1 significant figure the error per cent. in Nos. 5–10:
5. Calculated size of angle $22\frac{1}{2}$°, by measurement $23°$.
*7. $\pi^2$ is taken as 10 instead of 9.8696 . . .
8. Calculated area of circle of radius 5 in. is 78.540 . . . sq. in.;
by counting squares 78.42 sq. in.
9. 1 kg. is taken as 2$\frac{1}{2}$ lb. instead of 2.2036 lb.
10. 1 ft. is taken as 30.5 cm. instead of 30.4797 cm.
11. An angle is found by drawing and measurement to be 37°. If the maximum error per cent. is $3\%$, between what limits approximately does the true value lie?

12. A stop-watch reads correct to $\frac{1}{2}$ sec. A race is timed as 49$\frac{1}{2}$ sec. Find the maximum error per cent. to 1 significant figure.
*13. A weight is expressed in tons correct to 3 places of decimals. Show that the error is less than 18 oz.
*14. A man’s income is 24s. 8d. a day correct to the nearest penny. What is the maximum error in calculating his income for a year (365 days)?
EX. 122 (a). Miscellaneous Approximations

1. Find to 5 places of decimals the value of

\[ \frac{1}{2.3} \pm \frac{1}{2.34} - \frac{1}{2.345} \ldots \]

2. Given 1 ft. = 30.4797 cm., find in cm. the difference between 1 m. and 1.1 yd. to as many figures as the data justify.

3. An empty can weighs 12 oz. and when full of water weighs 3½ lb. If each measurement is correct to the nearest ½ oz., find the limits between which the weight of the water lies.

4. The record throw of a cricket ball is 140 yd. 2 ft.; express this in metres to 3 figures. [1 yd. = 0.9144 m.]

5. The Metropolitan Water Board supplies a daily average of 278,4 million gallons of water to a population estimated at 7,675,000. Calculate the average per head to 3 figures.

6. How many times (to 3 figures) is Madagascar (228,450 sq. mi.) as big as Great Britain (88,745 sq. mi.)?

7. In a bridge match of 300 hands in 1934, the United States scored 91650 points and Great Britain scored 88050 points. By what percentage, to 2 figures, of the winning score was Great Britain defeated?

8. If 12.45 litres of olive oil weigh 11.38 kg., find the weight in gm. of 1 c.c. of olive oil to 2 figures.

9. The volume of the Earth is \( 1.082 \times 10^{11} \) cu. km. and of the oceans is \( 1.412 \times 10^9 \) cu. km. Express the volume of the oceans as a percentage of that of the Earth correct to 2 figures.

10. Given 1 ton = 1016.0471 kg., express 1 kg. in lb. to 3 places of decimals.

11. The expenses of the General Election in 1931 amounted to £54,105 and 21,656,373 votes were recorded. Find the cost per vote to the nearest farthing.

12. Death duties for 1928 were £68,621,348 and for 1932 were £56,079,351. Find the decrease per cent., correct to 3 figures.

13. The population of England and Wales was 36,070,492 in 1911, and 39,947,931 in 1931. Find the increase per cent., correct to 3 figures.

14. Prove that the error in taking \( \pi^2 \) as 10 is about 1.3\%.

\[ \pi \approx 3.14159 \]

15. Given 1 m. = 39.3701 in., express 1 sq. m. in sq. yd. to 4 figures.

APPENDIX

16. A car has been driven at 231.7 mi. per hour. If the circumference of a wheel was 88 in., how many revolutions, to 2 figures, the wheel made per second.

17. Find to the nearest lb. the error made in converting 5000 kg. to lb. by the approximation 51 kg. = 3 cwt., given 1 kg. = 2.20463 lb.

18. Nurni ran 1 mi. in 4 min. 10 sec. Express his average speed in miles per hour to 3 figures.

19. Starting from A.D. 1800 every fourth year is a leap year, except that every hundredth year is not a leap year unless its number is divisible by 400. By taking a period of 400 yr., find the average number of days in a year to 4 places of decimals.

20. The coal output in the United Kingdom was 64,872,965 tons in 1929 by 903,914 miners and 54,061,222 tons in 1930 by 849,344 miners. Find, to the nearest integer, by what percentage the average number of tons worked by each miner in 1930 was less than in 1929.

CHAPTER XVII

MISCELLANEOUS EXTENSIONS

EX. 128 (a). Averages

Find the average of:

1. 47, 73, 29, 38, 28.
2. 6824, 6745, 6902, 6633.
3. £2 4s. 2d.; £3 11s. 8d.; £1 16s. 5d.; £4 3s. 7d.
4. 5 yd. 1 ft. 9 in.; 10 ft. 5 in.; 4 yd. 2 ft. 8 in.; 7 ft. 6 in.

Find the cost per lb. of the following mixtures:

5. 5 lb. at 2s. per lb.; 8 lb. at 1s. 9d. per lb.; 7 lb. at 1s. 4d. per lb.
6. 700 lb. at 6s. 8d. per lb.; 900 lb. at 11s. 4d. per lb.
7. In 1932, Sutcliffe scored 3336 runs in 52 innings, 7 times not out; find his average, to the nearest whole number.
8. A man buys 5 handkerchiefs at 2s. 8d. each and 9 at 1s. 6d. each. What is the average price?

What is the average price of 500 articles at 2s. 8d. each and 900 at 1s. 6d. each?
9. At Cherrapunji, the rainfall in one year (365 days) a little time ago was 551 in., of which 114 in. fell in 5 days. Find the average daily rainfall for the rest of the year, to \( \frac{1}{10} \) in.
10. Find, correct to the nearest penny, the average price of 35 articles at 2s. 6d. each and 24 articles at 1s. 8d. each.

11. A man travels 10 mi. at 10 m.p.h., then 10 mi. at 20 m.p.h., then 10 mi. at 30 m.p.h. Find his average speed for the whole journey.

12. A motorist drives at 24 m.p.h. for 2 hr. 20 min., and then drives 35 mi. farther at 28 m.p.h. Find his average speed for the journey, to 1/10 m.p.h.

13. If 20 lb. of tobacco costing 15s. 9d. per lb. are mixed with 30 lb. of tobacco costing 19s. 6d. per lb., find the cost price of the mixture per lb., and the price per lb. at which it must be sold to gain 25 per cent.

14. If 36 lb. of tea costing 2s. 6d. per lb. is mixed with 45 lb. of tea costing 1s. 9d. per lb., find the price per lb. at which the mixture must be sold to gain 28 per cent.

*15. A man’s average week-day expenditure is 26s. 6d. a day; and his average daily expenditure is 24s. a day. What is his average Sunday expenditure?

16. A motorist averages 24 m.p.h. per hour for a journey of 120 mi. If he averages 25 mi. per hour for the first 3 hr., find his average speed for the rest of the way.

*17. A man earned in seven consecutive weeks: £3 1s. 8d.; £4 4s. 7d.; £3 9s. 5d.; £7 7s. 4d.; £5 11s. 3d.; £1 15s. 9d.; £4 18s. 6d. In the next 5 weeks he earned enough to make his average for the 12 weeks £4 10s. 11d. What were his average earnings per week for the last 5 weeks?

18. An examination is taken by \( m \) boys and \( n \) girls; the average mark of all the candidates is \( b \), and the average mark of the boys is \( c \). Find the average mark of the girls.

*19. A man lends \( £x \) at \( p \) per cent. per annum and \( £y \) at \( r \) per cent. per annum, simple interest. What is the average rate of interest obtained on the total loan?

*20. For the first 6 days of a week, the average number of hours of sunshine was 11·2; and for the last 6 days of the same week it was 10·8. How many more hours of sunshine were there on the Sunday than on the Saturday in that week?

EX. 128 (a). Miscellaneous Examples

1. A can do a piece of work in 36 days, and B can do it in 45 days. How long will A and B take, working together?

2. A can do a piece of work in 30 days which A and B, working together, can do in 18 days. How long would B take, working alone?

3. A swimming-bath can be emptied by one pipe in 15 hours, by a second in 12 hours, and by a third in 10 hours. How long will it take if all three pipes are used?

4. A man rides from his house to a town at 10 m.p.h. and returns home at 15 m.p.h. Find his average speed for the double journey.

5. How much water must be added to 36 gall. of a liquid containing 65 per cent alcohol to obtain a liquid containing 45 per cent alcohol?

6. In what ratio must eggs costing 1s. 4d. per dozen be mixed with eggs costing 1s. 3d. per dozen so that a profit of 20 per cent is made by selling the whole at 2s. 1d. per score?

7. After a man has done \( 1/3 \) of a certain journey, he has 6 miles less to go than the distance already covered. Find the length of the journey.

8. A can do a piece of work in 84 days; B and C together can do it in 56 days; A and C together can do it in 63 days. How long will B take by himself?

9. A can do in 24 days what it takes B 30 days and C 32 days to do. How should a payment of £34 be shared between them if all are working together?

10. A and B together can do a piece of work in 60 days. They start together, but after 24 days B leaves, and A finishes by himself in 63 days more. In what time could each do the whole piece of work by himself?

11. A can do as much work in 2 days as B can do in 3 days, and B in 4 days as much as C in 5 days. A piece of work takes 20 days if all work together; how long would B take to do it by himself?
PART II—DRILL EXERCISES D 13–20

D 13. SIMPLE PRACTICE

Find by practice the value of:

1. £5 10s. × 46.  
2. £2 15s. × 28.  
3. £1 12s. × 145.  
4. £3 15s. × 265.  
5. £6 13s. × 382.  
6. £2 8s. 6d. × 139.  
7. 3s. 6d. × 2684.  
8. 15s. 7d. × 6285.  
9. 1s. 2¼d. × 8765.  
10. 12s. 3d. × 365.  
11. 8½d. × 2654.  
12. 7¾d. × 59623.  
13. 11¾d. × 38940.  
14. 1s. 8¾d. × 18541.  
15. 7s. 6½d. × 1846.  
16. £2 13s. 9d. × 472.  
17. £3 7s. 10d. × 261.  
18. £5 16s. 4d. × 83.  
19. £5 13s. 6½d. × 123.  
20. £7 15s. 7¾d. × 356.  
21. £2 9s. 2¼d. × 59.  
22. 13s. 7⅛d. × 24503.  
23. £2 18s. 8d. × 3668.  
24. £3 5s. 7¾d. × 17623.

25. Find cost of 46 qr. 5 bush. at 7s. 4d. per bush.  
26. Find cost of 3 cwt. 1 qr. 7 lb. at 3s. 10d. per lb.

D 14. COMPOUND PRACTICE

Find by practice the value of:

1. 3 cwt. 2 qr. 21 lb. at £55 10s. per cwt.  
2. 9 cwt. 3 qr. 21 lb. at £7 10s. 8d. per cwt.  
3. 17 cwt. 1 qr. 12 lb. at £1 19s. 8d. per cwt.  
4. 1 ton 7 cwt. 2 qr. 21 lb. at £2 per ton.  
5. 3 tons 1 cwt. 3 qr. 18 lb. at 16s. 4d. per cwt.  
6. 1 ton 3 cwt. 3 qr. 12 lb. at £18 7s. 6d. per ton.  
7. 3 gall. 2 cqt. 1 pt. at 18s. 6d. per gall.  
8. 7 qr. 3 cwt. 5½ pk. at £2 18s. 8d. per qr.  
9. 5 yd. 2 ft. 9 in. at £1 1s. 9d. per yd.  
10. 45 ac. 3 r. 20 p. at £111 1s. 4d. per ac.  
11. 20 ac. 3 r. 25 p. at £5 6s. 8d. per ac.  
12. 356 ac. 3 r. 39¼ p. at £2 13s. 4d. per ac.

D 15. PERCENTAGE

1. Express as fractions and as decimals: 35%; 7½%; 62¾%; 137¼%; 350%; 93¾%; 102¼%.
2. Express as fractions and as ratios: 8½%; 116⅛%; 83⅓%; 229¾%; 133⅓%; 54⅜%.

Express as percentages:

3. 1:2; ⅗; 20:1; 0:36; 0:085.  
4. ⅛; ¾; 1:7; 1:4; 0:045.  
5. The ratios, 5:3; 23:80.  

Find the values of:

7. (i) 20% of £65; (ii) 33⅓% of 70 yd.; (iii) 140% of 3·2 metres.  
8. (i) 5% of £3 10s.; (ii) 35% of 9 tons; (iii) 225% of 3 mi.  
9. 2¼% of £150.  
10. 37⅓% of 5 gall.  
11. 133⅓% of 5 ft.

Find correct to the nearest penny the values of:

12. 4% of £163.  
13. 3¼% of £84 12s.  
14. 5⅞% of £58 7s. 6d.  
15. 62½% of £108 5s.  
16. 24% of £264 10s.  
17. 0.1% of £1242.

Find, correct to the number of places of decimals stated in brackets, what percentage is:

18. 37 of 91 (1).  
19. 12⅔ of 38 (1).  
20. 95·7 of 3614 (2).  
21. 17½ of £1 (1).  
22. £1 11s. 4d. of £3 (1).  
23. 3s. 10d. of £32 (3).  
24. 25 lb. of 1 cwt. (1).  
25. 3 ch. of 1 mi. (2).  
26. 50 ac. of 1 sq. mi. (2).
27. 24% of a cost price is £31 10s.; what is 32% of the cost price?

28. If 12% is deducted from a bill, £154 is left; what is the bill?

29. The annual death rate in a district is 31%. If there are 51,298 deaths, what is the population?

30. How much, to the nearest penny, does it cost to insure goods worth £638 at 2½ per cent.?

D 16. PROFIT AND LOSS PER CENT

Find the gain or loss per cent. in the following transactions:

(C.P. = Cost Price; S.P. = Selling Price.)

1. C.P. £60; S.P. £75.
2. C.P. £75; S.P. £60.
3. C.P. £9; S.P. £9 9s.
4. C.P. 4s. 6d.; S.P. 5s. 6d.
5. C.P. 17s. 6d.; S.P. 13s. 9d.
6. C.P. 18s. 9d.; S.P. £1 10s.
7. C.P. £105; gain £3 10s.
8. S.P. £120; gain £30.
9. C.P. 3s. 6d.; loss 1s.
10. S.P. 3s. 6d.; loss 1s.

Find the S.P. in Nos. 11–14 and the C.P. in Nos. 15–20:

11. C.P. 18s.; gain 25%.
12. C.P. £17; gain 40%.
13. C.P. £9 10s.; loss 35%.
14. C.P. £11 5s.; loss 12%.
15. S.P. £3 10s.; gain 20%.
16. S.P. £4 10s.; gain 35%.
17. S.P. £19; loss 40%.
18. S.P. £66; loss 12%.
19. S.P. £477; gain 6%.
20. S.P. £696; loss 3½%.

21. If S.P. is £5 5s., the loss is 12½%. What S.P. gives gain 20%?

22. If S.P. is £37 2s., the gain is 6%. What S.P. gives gain 12½%?

23. I buy apples 15 for 1s. and sell them at 10d. a dozen; find my gain per cent.

24. A man buys 100 acres for £3500 and sells half for £1600. At what price per acre must he sell the rest to gain 4½%?

25. I buy 50 oranges at 2 a penny and 150 at 3 a penny and sell the whole at 5 for 2d. Find my gain or loss per cent.

D 17. SIMPLE INTEREST

Find the simple interest on:

1. £700; 3 yr.; 4%.
2. £150; 4 yr.; 3%.
3. £400; 2 yr.; 3½%.
4. £1200; 7 yr.; 4½%.
5. 900 francs; 5 yr.; 6%.
6. 650 dollars; 6 yr.; 4%.
7. £1800; 3½ yr.; 5%.
8. £800; 2½ yr.; 5½%.
9. £600; 5 mo.; 4%.
10. £480; 9 mo.; 2½%.
11. £1830; 4½ yr.; 2½%.
12. £644; 6 yr. 3 mo.; 2½%.
13. £6850; 5½ yr.; 3½%.
14. £330 12s. 6d.; 5½ yr.; 4%.

Find, to nearest penny, the simple interest on:

15. £350 17s. 6d.; 4 yr.; 3%.
16. £340 15s. 6d.; 2½ yr.; 4%.
17. £235 14s. 4d.; 3 yr.; 5%.
18. £547 2s. 4d.; 3½ yr.; 4½%.
19. £347 10s.; 219 days; 5%.
20. £126 10s.; 135 days; 3½%.

Find, to nearest penny, the amount of:

21. £575; 8½ yr.; 3½%.
22. £411 10s.; 3 mo.; 4½%.
23. £1643 7s. 5½d.; 4 yr.; 3½%.
24. 1895 guineas; 4½ yr.; 2½%.
25. £7500 from May 5 to Oct. 26 at 3½%.
26. £225 12s. 6d. from Sept. 29 to Dec. 25 at 3½%.

In what time will:

27. £150 amount to £165 15s. at 3%?
28. £285 amount to £364 16s. at 3½%?
29. £1275 produce £274 11s. interest at 3½%?
30. £450 produce £114 3s. 9d. interest at 3½%?
31. £142 10s. amount to £227 5s. 9d. at 3½%?

At what rate per cent. will:

32. £225 produce £81 interest in 9 yr.?
33. £620 produce £124 interest in 8 yr.?
34. £122 10s. produce £18 7s. 6d. interest in 4½ yr.?
35. £200 produce £4 16s. interest in 146 days?
36. £300 amount to £350 in 7 yr.?
37. £1850 amount to £2104 7s. 6d. in 5 yr.?
38. £5768 15s. produce £908 11s. 6½d. interest in 4½ yr.?
D 18. BANKER'S AND TRUE DISCOUNT

[Give answers correct to the nearest penny]

Find the banker's discount on bills due as follows:

1. £300 in 2 yr.; 4½%.
2. £250 in 6 mo.; 5%.
3. £1200 in 9 mo.; 3½%.
4. £660 in 4 mo.; 3½%.
5. £482 in 10 mo.; 3½%.
6. £845 in 8 mo.; 2½%.
7. £537 14s. in 3 mo.; 2½%.
8. £717 in 14 mo.; 4½%.

Allowing 3 days grace, find the banker's discount on a bill:

9. £150 drawn April 1 for 3 mo., discounted June 6; 4½%.
10. £800 drawn May 7 for 4 mo., discounted June 20; 3½%.
11. £1240 drawn July 10 for 2 mo., discounted July 20; 3%.
12. £550 drawn Aug. 8 for 1 mo., discounted Aug. 9; 5%.

Find the true discount on money due as follows:

13. £93 12s. in 8 mo.; 6%.
14. £1000 in 4 yr.; 5%.
15. £187 4s. in 1 yr.; 4½%.
16. £225 in 15 mo.; 3½%.
17. £55 in 146 days; 4½%.
18. £275 6s. 8d. in 18 mo.; 4½%.
19. £256 7s. 6d. in 3 yr.; 3½%.
20. £1380 7s. 6d. in 9 mo.; 3½%.

Find the true present worth of bills due as follows:

21. £813 9s. in 1½ yr.; 4½%.
22. £903 14s. in 2½ yr.; 3½%.
23. £3111 in 5½ yr.; 4½%.
24. £1905 10s. in 9 mo.; 4½%.
25. £676 13s. 4d. in 6 mo.; 3%.
26. £935 in 3 yr.; 4½%.
27. £2197 in 3 yr.; 4%, compound interest.

D 19. HARDER FRACTIONS

1. \( \frac{0.55 \times 0.0686}{0.0049} \)
2. \( \frac{0.0125 \times 0.9152}{0.0715} \)
3. \( \frac{32\sqrt{1052} - (0.5)^2}{0.128} \)
4. \( \frac{4.32(3.86)^2 - (2.14)^2)}{1 - 0.2 \times 0.7} \)
5. \( \frac{0.57}{0.019} \times 2\frac{3}{4} \)
6. \( \frac{\left( \frac{9}{13} \times \frac{6}{12} \right)}{\left( \frac{1}{2} \times \frac{1}{2} \right)} \times \left( \frac{\frac{14}{9} + \frac{12}{9}}{18} \right) \)
7. \( \frac{5\frac{1}{3}}{\left( \frac{28}{10} - 3\frac{2}{3} \right)} - 16\frac{1}{2} \times \frac{1}{3} + 3 \times 1 \frac{5}{3} \)

D 20. APPROXIMATIONS

Write down correct to the number of significant figures given in brackets:

1. 7·6986 (3).
2. 29·724 (2).
3. 0·08055 (2).
4. 86372 (3).
5. 349 (1).
6. 53·972 (3).
7. Express \( \sqrt{2} \) as a decimal, correct to 3 figures.
8. Express \( \frac{4}{3} \) as a percentage, correct to 2 figures.
9. The value of \( \pi \) is 3·14159265 ...; find, correct to 3 figures, the value of (i) \( \frac{\sqrt{2}}{\pi} \), (ii) 99% of \( \pi \).
10. Calculation shows that the length of a line is 4·72 in.; its measured length is 4·9 in.; find the error per cent. in the measurement, correct to 1 figure.
A rectangle is 7·8 cm. long, 5·3 cm. wide, both measurements being correct to the nearest mm. Between what limits does the perimeter of the rectangle lie?

A length is expressed in miles, correct to 3 places of decimals; show that the error is less than 32 in.

The time of 25 swings of a pendulum is measured as 18·4 sec., to the nearest $\frac{1}{4}$ sec. Between what limits does the time of 1 swing lie?

The numbers of new buildings in Great Britain for the 5 years 1926 to 1930 were respectively: 234670; 290882; 252848; 239780; 240044. Find the average number built per year, correct to the nearest hundred.

At an examination there were 1182 candidates, of whom 61·76% (to 4 figures) passed. How many passed?

Given 1 yd. = 91·44 cm., express 2½ in. in cm., correct to 3 figures.

Find, correct to 4 figures, the value of

$1 + \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \cdots$

The area of a rectangular field is 38,700 sq. yd. and its length is 232 yd., both measurements being correct to 3 figures. Find as accurately as the data justify the breadth of the field.

Find, correct to 3 figures, the values of:

$20.087 \times 0.31728$.

Find, correct to 2 figures, the values of:

$21. 52.73 + 13.04$.

Express 8,372,519 as a percentage of 26,018,732, correct to 2 figures.

The population of England and Wales increased from 32,527,843 in 1901 to 39,947,931 in 1931. Find the increase per cent., correct to 3 figures.

Find the increase per cent. to 2 places of decimals if expenditure rises from £137,842,500 to £148,207,300.

Find the decrease per cent. to 2 places of decimals if revenue falls from £125,463,700 to £117,845,600.
Paper A 28

1. (i) Simplify \( \left( \frac{\sqrt{3}}{4} + \frac{1}{\sqrt{2}} \right) \times (1 \frac{1}{2} - 1 \frac{3}{4}) \).
(ii) Find in L. s. d. the value of £3 60/9d., to the nearest farthing.

2. (i) A chest of tea weighs 84 lb. How many chests go to the ton?
(ii) Find the cost of 347 yd. at £2 14s. 8d. per yard.

3. (i) Find the ratio of 1 ft, 8 in. to 2 yd.
(ii) A boy obtains 37 marks out of 65. What percentage is this, to the nearest whole number?

4. A man saves 15\% of his income. (i) What is his income if he saves £72 a year? (ii) What is his income if he spends £150 a year?

5. A man measures the floor of a room as 16 ft. by 12 ft., but his foot-rule is half an inch short. What is the true area of the floor?

6. A bankrupt owes B £280, C £240, D £170, E £60. If his property is worth £210, how much will B and D receive?

Paper A 27

1. (i) Divide £3 16s. 11d. by 13.
(ii) Express 7s. 4\frac{3}{4}d. as a decimal of £1, to 4 places.

2. (i) Find the income tax at 2s. 9d. in the £ on an income of £235 if no tax is paid on the first £20.
(ii) Find the cost of 4 tons 13 cwt. 3 qr. at £4 13s. 4d. per ton.

3. (i) A car is worth £175 when new and £105 after 1 year. By how much per cent, has its value decreased in the year?
(ii) If a man’s salary is increased by 37\frac{1}{2}%, find the ratio of the increase to the new salary.

4. The return ticket from my home to London is 2s. 2d. and the quarterly ticket is £3 1s. 3d. How many times in the quarter must I make the journey to save money by taking a quarterly ticket?

5. A carpet 7\frac{1}{2} yd. long, 14 ft. wide, costs £21. What will a carpet of the same quality 5 yd. square cost?

6. Three men, paid at the same rate per hour, work in succession at a job and finish it in 70 hr. They are paid respectively £1 17s. 6d., £2 12s. 6d., £3 7s. 6d.; how many hours did each man spend on the job?

Paper A 29

1. (i) Find the H.C.F. of 7938 and 14112.
(ii) Which is the greater, \( \frac{1234}{1234} \) or \( \frac{6789}{6789} \)?

2. (i) Divide £7 18s. 9d. by 3\frac{3}{4}.
(ii) Find the cost of 317 lb. at 2s. 5\frac{3}{4}d. per lb.

3. (i) If 56% of a sum of money is 2 guineas, find the value of 72% of the same sum.
(ii) If the ratio of the cost price to the selling price of an article is 8 : 11, find the gain per cent.
512  APPENDIX

4. Taking 1 lb. = 0.4536 Kg., express 100 gm. in oz., to one place of decimals.

5. A cistern, 4 ft. by 2 ft. by 3 ft., holds 187 gall. of water weighing 16 cwt. 2 qr. 22 lb. Express 1 cu. ft. in gallons to 100 gall., and the weight of 1 cu. ft. of water in lb. to 1/0 lb.

6. The total area of 3 fields is 255 ac., and the areas of the two smaller fields are respectively ⅔ and ⅑ of the area of the largest. Find the area of each field.

Paper A 30

1. (i) Divide 1 ton 13 cwt. 45 lb. 1 oz. by 119.
   (ii) Find in acres the area of a field ¾ mi. long, 1/4 mi. wide.

2. (i) Find the rate payable on £126 at 14s. 7½d. in the £.
   (ii) How many payments of £1 6s. 9d. each can be made from £50, and what remains?

3. (i) Find the ratio of the price of 4 guineas a ton to a price of 3½d. per lb.
   (ii) The duty on a ring is £3 6s. and this is 24% of its value. Find its value.

4. A man buys 4 tons 5 cwt. of goods at £6 16s. a ton and is allowed to deduct 12½% for cash. What does he pay?

5. The velocity v ft. per second of water flowing in a pipe of diameter d in. delivering g gall. per minute is given by the formula
   \[ v = \frac{2g}{d^2} \]
   If a bath which holds 50 gall. is filled in 12 min. from a pipe of diameter ½ in., find the velocity of the water in the pipe.

6. The height of the barometer is recorded at hourly intervals on a certain day as follows:

<table>
<thead>
<tr>
<th>Time</th>
<th>9 a.m.</th>
<th>10 a.m.</th>
<th>11 a.m.</th>
<th>12</th>
<th>1 p.m.</th>
<th>2 p.m.</th>
<th>3 p.m.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height in.</td>
<td>29.65</td>
<td>29.80</td>
<td>29.87</td>
<td>29.80</td>
<td>30.00</td>
<td>29.82</td>
<td>29.25</td>
</tr>
</tbody>
</table>

Represent these readings by a graph and estimate (i) the height of the barometer at 9.36 a.m., 1.24 p.m.; (ii) the times when the height is 29.75 in., 29.55 in.; (iii) the times between which the height was more than 29.85 in.

Paper A 31

1. (i) Find the least integer by which 11340 must be multiplied to give a perfect square.
   (ii) What is the difference in feet per second between a speed of \(20\frac{1}{2}\) mi. per hour and a speed of 600 yd. a min.?

2. (i) Find to the nearest penny the cost of 5 mi. 1200 yd. at £3 17s. 8d. per mile.
   (ii) 105 reams of paper are sold for ten guineas and a ream contains 480 sheets; how many sheets are sold for a penny?

3. (i) If the price of an article is decreased by 35%, find the ratio of the decrease to the new price.
   (ii) A man's income is £400 a year; find the error per cent. in saying that his income is 22s. a day, regarding a year as 365 days.

4. In 1934, the population of Aberdeen was 167,258 and the rates amounted to £712,500. Find to the nearest penny the rate paid per head.

5. By selling an article for £14, a man would gain 12½%; what did it cost him? Find his gain per cent. if he sold it for £15.

6. £1260 is divided between A, B, C, D so that A's share is 7:6, B's share to C's share is 5:4, C's share to D's share is 3:2. How much does each get?

Paper A 32

1. (i) On what day of the month in 1934 did the 1500th hour of the year fall?
   (ii) Express \(\sqrt{50} + \frac{1}{6}\) as a decimal, correct to 2 places.

2. (i) Find to the nearest penny the cost of 3 yd. 2 ft. 7 in. at 17s. 8d. per yard.
   (ii) How many seconds would a torpedo travelling at 45 knots take to go 950 yd.? [1 knot = 6080 feet per hour.]

3. (i) In 7:23 lb. of gunpowder there are 5:42 lb. of nitre. What percentage is this, to the nearest whole number?
   (ii) To each half-crown a boy saves, his father adds one shilling. How much has the boy saved himself when he possesses 1½ guineas?
4. A man’s working hours a day were increased by 25%, and his wages per hour were increased by 20%. By how much per cent were his daily earnings increased?

5. Sheets of paper 20 in. by 13 in. are folded so as to make 8 leaves of a book. The leaves are then cut down to make pages $6\frac{1}{2}$ in. by $4\frac{3}{4}$ in. An unbound copy of the cut book weighs 8 oz. What is the weight of the paper wasted per copy?

6. The lengths of the rims of the wheels of A’s bicycle are 80 in. and of B’s bicycle are 85 in. Each bicycle was originally fitted with a correct cyclometer, but by mistake the cyclometers were interchanged, and so the readings for a tour which A and B made together differed by 66 mi. What was the length of the tour?

**PAPERS A 33-40 (Ch. I-XIV)**

**Paper A 33**

1. (i) Divide £10 1s. 3d. by 21.
   (ii) The duty on chutney is 5s. 4d. per cwt. How much is it per ton?

2. A writing-pad of 100 sheets, fitted with a cardboard base $\frac{1}{9}$ in. thick, is 0.86 in. thick. How thick is it when 25 sheets have been used?

3. How many tons (to the nearest hundred) of water fall on 1 sq. mi. of land during a rainfall of 0.3 in.? [Take 1 cu. ft. of water to weigh 62$\frac{1}{2}$ lb.]

4. 12% of a man’s income is deducted for taxes, and he spends $\frac{1}{3}$ of what then remains. Find his income if he saves £64 a year.

5. My study is 16 ft. long, 12$\frac{1}{2}$ ft. wide, 9 ft. high, and my bedroom is 15 ft. long, 10 ft. wide, 8 ft. high. By how much per cent does (i) the area of the walls, (ii) the volume of my study exceed that of my bedroom?

6. The radius of a wheel of a trolley is $10\frac{1}{2}$ in.; how many revolutions does the wheel make when the trolley travels $\frac{1}{2}$ mi.? [Take $\pi = \frac{\sqrt{6}}{2}$.]
5. A rectangular tank is 3·5 m. long, 1·8 m. broad, 1·5 m. deep; calculate in litres the amount of water it will hold. If 700 litres of water are drawn off, find in cm., to the nearest mm., how much the water-level sinks.

6. Find in c.c. the volume of a closed cylindrical tube, internal diameter 2 mm., internal length 35 cm. [Take \( \pi = \frac{22}{7} \).] It is filled with mercury which weighs 13·6 gm. per c.c.; find the weight of the mercury, correct to \( \frac{1}{10} \) gm.

**Paper A 36**

1. (i) A service cartridge weighs 415 grains. Find the weight, to the nearest oz., of 200 rounds (i.e. 200 cartridges).
   [1 lb. = 7000 grains.]
   (ii) How many articles at 1s. 7d. each can be bought for \( £10 \), and how much money remains over?

2. A man has weeded \( \frac{2}{5} \) of a field in 28 hr.; how long should he take to finish it, at the same rate?

3. By selling sugar at 4d. per lb., a tradesman calculates he will make 15% profit; but through faulty scales, he sells as 1 lb. what is really 17 oz.; what profit per cent. does he actually make?

4. X motors at a steady speed from A to B, a distance of 48 mi., leaving A at 9 a.m. and arriving at B at 11 a.m.; Y stays at B till 10 a.m. and then bicycles towards A along the same road at a steady speed of 10 m.p.h. Draw the travel graphs of X and Y, and find when and where the two men pass one another.

5. An open box 9 in. long, made of thin cardboard, has its two ends 8 in. square, and is fitted with a cardboard lid which covers the top, the front, and the two square ends. Find in sq. in. the area of cardboard used for making the box.

6. A circular running-track has an internal radius of 56 yd. and is \( \frac{3}{4} \) yd. wide. Find the area of the track, to the nearest 10 sq. yd. [Take \( \pi = \frac{22}{7} \).]

**Paper A 37**

1. (i) Divide 73 m. 5 fur. 200 yd. by 18.
   (ii) Find the cost of 256 tons at £1 18s. 8\( \frac{1}{2} \)d. per ton.

2. A piece of wood is 4 ft. 5 in. long, and a carpenter cuts it into equal pieces, each 4 in. long. Find the number of pieces that can be obtained and the length of what remains over if the saw-cut is (i) \( \frac{3}{8} \) in. wide, (ii) \( \frac{1}{4} \) in. wide.
APPENDIX

Paper A 39

1. A certain kind of wire weighs 135 lb. to the mile. Find (i) the length, to the nearest 10 yd., that goes to the cwt.; (ii) the weight in oz., to 3 decimal places, of 1 in. of the wire.

2. If goods are sent by lorry the charge is 2½d. per ton per mile, if sent by rail the charge is 2d. per ton per mile, but there is a total additional cost of 4s. per ton for transport to and from the station. What is the cost of each method for sending 100 tons (i) 80 mi., (ii) 120 mi.?

3. The rent of my house is £64 a year and the rateable value is ½ of the rent. During the year I have to pay two rates, one at 8s. 3d. in the £, the other at 6s. 9d. in the £, on the rateable value. My income is £750 a year. What percentage of my income do I spend on rent and rates combined?

4. If 4 men do as much work as 5 women, and if 7 men do as much work as 10 boys, in any given time, how should (i) £5 be shared between 1 man, 1 woman, and 1 boy who are working together; (ii) £108 be shared between 1 man, 2 women, and 4 boys who are working together?

5. A closed cistern, 75 cm. long, 28 cm. wide, external measurements, is made of metal 15 mm. thick and has a capacity of 27 litres. Find its external height.

6. 77 circular discs, each 1½ in. in diameter, are stamped out of a rectangular sheet 14 in. long, 9 in. wide. If the sheet is 0·8 in. thick, find the volume of the material remaining over, to the nearest cu. in. [Take \( \pi = \frac{42}{17} \).]

Paper A 40

1. (i) Express as a decimal of an acre, correct to 3 places of decimals, the area of a rectangular piece of ground 350 ft. long, 108 ft. wide. (ii) 1 ton 5 cwt. 8 lb. of a commodity costs £3 18s.; find the cost of 2 tons 3 cwt. 20 lb. at the same rate.

2. A housekeeper can buy candles 8½ in. long at 1s. 6d. per dozen, or candles 10½ in. long of the same quality and thickness at 1s. 10d. per dozen. Which kind is the more economical (i) if there is no waste, (ii) if the last ½ in. of each candle is useless?

PAPERS A 41-48 (Ch. I-XVII)

Paper A 41

1. Find the value of \( 3 \cdot \frac{1}{2} - \frac{1}{4} \cdot \frac{3}{8} \) correct to 6 places of decimals; to how many significant figures is it a correct approximation for \( \sqrt{8} \), given that \( \sqrt{8} = 2\cdot8284271 \ldots 

2. Simplify \( \left( \frac{1}{2} \times 1\frac{1}{2} + \frac{3}{4} \times 1\frac{1}{2} \right) \) of \( 2\frac{1}{2} \times 3\frac{1}{2} \).

3. Find correct to the nearest penny 63% of £3 10s.

4. An open tin box, 2 ft. 4 in. long, 1 ft. 6 in. wide, 8 in. high, is fitted with a tin lid which overlaps to a depth of 1 in. all round. What is the total area of tin sheeting used in making the box and lid?

5. If £333 10s. amounts to £383 10s. 6d. in 4 yr. at simple interest, find the rate per cent. p.a.

6. A man takes 110 steps a minute. If his stride is 30 cm. long, find his speed in km. per hour. What will it become if he increases his speed by 12½%?
Paper A 42

1. Ordinary bricks weigh 7 lb. each. How many go to the ton? Find in tons, cwt. the weight of 2000 bricks.

2. Find the error per cent. in taking the area of a field 3 mi. long, 100 yd. wide, as 9 ac.

3. In 1928 the rateable value of Merthyr Tydfil was £233,000, and rates were paid at 25s. 8d. in the £. How much was paid altogether? The population was 71,100; how much was paid per head, to the nearest sixpence?

4. Find to the nearest penny the simple interest on £250 for 35 days at 6% p.a. [Take 1 yr. = 365 days.]

5. A room, 18 ft. long, 15 ft. wide, has a carpet in the middle, leaving a margin 1 ft. 6 in. wide all round which is covered with linoleum at 3s. 6d. per sq. yd. Find the cost of the linoleum.

6. A man would gain 20% by selling an article for 9s. 6d. and 15% by selling a second article for 11s. 6d. Actually he sells the first article for 7s. 2d.; for what dose he sell the second article if there is no loss or gain on the two sales?

Paper A 43

1. Simplify \( \frac{7x + 1\frac{1}{2} - \frac{3x}{2}}{2x + \frac{1}{2} - 2y} \).

2. A man walks from his house to a town 6 mi. away at 4\( \frac{1}{2} \) mi. an hour and drives back at 18 mi. an hour. Find his average speed for the double journey.

3. If tea is quoted wholesale at a price which is equivalent to 1s. 7\( \frac{1}{2} \)d. per lb. correct to the nearest \( \frac{1}{4} \)d. per lb., between what limits does the wholesale price of 1 ton of this tea lie?

4. A rectangular zinc plate is 25 cm. by 15 cm. From each corner a square of side 2.5 cm. is cut away, and the remainder is then bent to form a lidless box. Find the volume of the box. [Neglect the thickness of the zinc.]

5. A man borrows £4 10s. and pays 3s. per month interest on it. Reckoning simple interest, to what rate per cent. p.a. is this equivalent?

6. £18 is paid for a piece of work which A can do in 15 days, B in 18 days, C in 22\( \frac{1}{2} \) days, working separately. If all work together, how long will it take and how much should each receive?

Paper A 44

1. Simplify \( \frac{\frac{3}{4} - \frac{1}{8}}{\frac{1}{8} - \frac{3}{8}} \).

2. The length and breadth of a rectangle are measured to be 6\( \frac{1}{4} \) cm., 3\( \frac{1}{2} \) cm., correct to the nearest mm. What is the greatest and least possible area of the rectangle? Find, to 1 significant figure, the maximum error per cent. in taking the area as \( 6\frac{1}{4} \times 3\frac{1}{2} \) sq. cm.

3. A motorist has 5 new tyres with a new car; he uses all about equally and discards them all when the car has run 12,000 mi. How many miles' running does he allow each tyre?

4. In what time will £320 amount to £342 at 2\( \frac{1}{2} \)% p.a. simple interest?

5. If a cylindrical tank, 5 ft. in diameter, contains 275 gall. of water, find the depth of the water, to the nearest inch. [Take \( 1 \text{ cu. ft. = 6} \frac{1}{2} \text{ gall. and } \pi = 3} \frac{1}{4} \) \( \pi \).]

6. By selling tea at 2s. 4d. per lb, a dealer makes a profit of 12%. If the duty on tea is reduced by 3d. per lb. [i.e., the dealer pays 3d. per lb. less for the tea] and if the selling price is also reduced by 3d. per lb., find the dealer's gain per cent.

Paper A 45

1. (i) Express 2s. 7d. as a percentage of 8s. 6d., correct to 2 figures.

(ii) 1 km. = 0.62137 mi.; express 100 m. in yd., ft., in., correct to the nearest inch.

2. In 1931 the population of Great Britain was 44,790,485, and the expenditure on alcoholic liquor was £113,850,000; find the expenditure per head, correct to the nearest sixpence.

3. The price of a quarterly season ticket (13 weeks) between two stations is £3 11s. 6d., and the ordinary return fare is 2s. 11d. A man makes 5 return journeys a week; how much does he save each quarter by taking a season ticket?

4. A brick is 10 in. long, 8\( \frac{1}{2} \) in. wide, 5 in. high; show that a cubical block of edge 7\( \frac{1}{3} \) in. has approximately the same surface area as the brick.
5. On what sum at 3\(\frac{1}{2}\)% p.a. is the simple interest £93 2s. for 10 months?

8. Divide £654 10s. among 4 people so that their shares are proportional to \(\frac{1}{2} : \frac{1}{3} : \frac{1}{6} : \frac{1}{8}\).

Paper A 46

1. (i) Simplify \((\frac{1}{2} + \frac{1}{3} - \frac{1}{4}) + (\frac{3}{5} of \frac{1}{2} of \frac{1}{4}) - (1 - 2 + 3\frac{1}{2}) \times \frac{1}{4}\).

   (ii) Find, correct to 1 figure, the error per cent. in taking the square root of 17 as 4\(\frac{1}{2}\). \((\sqrt{17} \approx 4.1231.)\)

2. A grocer buys cheese at £6 per cwt. What is the least price per lb. at which he can sell it so as to make a profit of not less than 15%?

3. The yield from income tax increased from £237,873,052 in 1929 to £255,339,304 in 1930. Find the increase per cent., correct to two figures.

4. If the rate of interest is reduced from 4\(\frac{1}{2}\)% to 3\(\frac{1}{2}\)% p.a., find the decrease in a half-year’s interest on £720.

5. Semicircles are described externally on each of the sides of a rectangle 8 cm. long, 6 cm. wide. Find (i) the perimeter of the figure, correct to the nearest mm.; (ii) the area of the figure, correct to the nearest sq. mm. \(\text{[Take } \pi = 3.1416.\]}

6. A man used to save 15% of his income. His income is now 20% higher and his expenditure 10% higher. What percentage of his income does he now save?

Paper A 47

1. How many \(\frac{1}{2}\)-lb. packets can be made up from 1 cwt. of tea and how much remains over?

2. I buy 30 books at 5s. 6d. each and 6 books at 10s. 6d. each; what is the average cost per book?

3. 10% of a man’s income is taken in tax, and 20% of the remainder is saved. This leaves him £450 to spend. How much is his income?

4. A lawn-tennis court, 78 ft. long, 36 ft. wide, is to be enclosed with wire-netting 12 ft. high, set up 6 ft. from each of the longer sides and 10 ft. from each end. The netting is 6 ft. wide and costs 14s. 9d. per roll of 25 yd. Find the least cost of the netting assuming that it is sold only in complete rolls.

5. What sum will amount to £4615 in 21 months at 6\(\frac{3}{4}\)% p.a. simple interest?

6. An open cylindrical jar is 5 in. in diameter and 8 in. high, external measurements. If the material is \(\frac{1}{2}\) in. thick, find, correct to \(\frac{1}{2}\) cu. in., the volume of the material used in making the jar. \(\text{[Take } \pi = 3.142.]\)

Paper A 48

1. In 1932 the tax receipts on 1,076,100 motor-cars amounted to £13,521,305. Find the average tax per car, to the nearest shilling.

2. The total number of revolutions of the engines of a ship are recorded by a counter. At 12 noon, the counter registered 101,182, the engines making 110 revolutions per minute; at 1.15 p.m. the rate was reduced to 95 revolutions per minute; and at 2.45 p.m. it was increased to 115 revolutions per minute. What should the counter register at 4 p.m.?

3. A man driving at 20 mi. an hour takes 6 sec. to cross a bridge. Find the length of the bridge. What would be his speed if he only took 5 sec.?

4. At what rate per cent. p.a. will the interest on £1305 amount to £17 8s. in 4 months?

5. Water is flowing at the rate of 110 gallon. per minute through a pipe whose cross-section is 16 sq. in. in area. If the pipe remains full, find the speed of the water in miles per hour. \(\text{[Take } 1 \text{ cu. ft. } = 6\text{ gal.]}\)

6. A merchant mixes 3 grades of tea costing respectively 1s. 7d. per lb., 1s. 10d. per lb., 2s. 1d. per lb. in the ratios 3 : 4 : 5, and sells the mixture at 2s. 6d. per lb. Find his gain per cent.
APPENDIX TO PART III

CHAPTER XVIII

SQUARE ROOT AND USE OF TABLES

EX. 129 (a). Square Roots of Integers

Find the square roots of:

1. 361.  
2. 1089.  
3. 2116.  
4. 3481.  
5. 5329.  
6. 8649.  
7. 11449.  
8. 13924.  
9. 16641.  
10. 25921.  
11. 4243600.  
12. 71289.  
13. 9486400.  
14. 116964.  
15. 162409.  
16. 259081.  
17. 1038661.  
18. 1456649.  
19. 5788836.  
20. 13010449.  
21. 9935104.  
22. 29571844.  
23. 19421649.

EX. 131 (a). Square Roots of Decimals and Fractions

Find the square roots of:

1. 7.84.  
2. 29.16.  
3. 0.4096.  
4. 0.0576.  
5. 0.005184.  
6. 0.024025.  
7. 0.07296.  
8. 0.00256.  
9. 16.0801.  
10. 4134.49.  
11. 0.0082944.  
12. 0.00091204.  
13. \(\frac{47}{100}\).  
14. \(\frac{4996}{10000}\).  
15. \(\frac{161}{16}\).  
16. \(\frac{824}{100}\).  
17. \(\frac{5041}{1000}\).  
18. \(\frac{10000}{10000}\).  
19. 4.22.  
20. 31.25.  
21. 10.8888.  
22. 30.8888.

EX. 132 (a). Approximate Square Roots

Find, correct to 3 significant figures, the square roots of:

1. 5.  
2. 50.  
3. 0.7.  
4. 3.7.  
5. 0.009.  
6. 0.06125.  
7. 0.5038.  
8. 0.2568.

Find, correct to the number of decimal places indicated in brackets, the square roots of:

10. 3.9 (2).  
11. 0.538 (3).  
12. 0.025 (3).  
13. 0.008 (3).  
14. 7.45 (1).  
15. 0.000404 (3).  
16. \(\frac{8}{3}\) (3).  
17. 11\(\frac{1}{2}\) (2).  
18. \(\sqrt{2}\) (3).  
19. \(\sqrt{5}\) (2).  
20. 29\(\frac{5}{3}\) (2).  
21. \(\sqrt{3}\) (3).
APPENDIX

Express the following fractions so that their denominators do not contain √ signs:

22. \( \frac{1}{\sqrt{12}} \)  
23. \( \frac{2}{\sqrt{6}} \)  
24. \( \frac{\sqrt{7}}{\sqrt{14}} \)  
25. \( \frac{\sqrt{15}}{\sqrt{20}} \)

Evaluate, correct to 3 significant figures:

26. \( \frac{1}{\sqrt{8}} \)  
27. \( \frac{2}{\sqrt{11}} \)  
28. \( \frac{\sqrt{5}}{\sqrt{3}} \)  
29. \( \frac{1}{\sqrt{0.1}} \)  
30. \( \frac{\sqrt{14}}{\sqrt{7}} \)  
31. \( \frac{\sqrt{10}}{\sqrt{0.2}} \)

EX. 136 (a). Pythagoras’ Theorem

[Give answers correct to 3 significant figures, unless otherwise stated]

Find the length of a side of a square of area:

1. \( 3\frac{3}{4} \) sq. in.  
2. 7 ac.  
3. 2175 sq. yd.

In Nos. 4–7, the length of the hypotenuse of a right-angled triangle is \( c \) in., and the lengths of the other sides are \( a \) in., \( b \) in.

4. If \( a = 3.74:3 \), \( b = 50.08 \), find \( c \).  
5. If \( a = 87.5 \), \( c = 104.6 \), find \( b \).  
6. If \( a = 4:5 \) and if \( c = 10 \), find \( a \).

7. If \( a : c = 2:3 \) and if \( b = 7 \), find \( c \).

8. A ladder 17 ft. 9 in. long leans against a wall so that its top is 15 ft. 6 in. above the ground. Find, to the nearest inch, the distance of the foot of the ladder from the wall.

9. Find, in yards, the length of the diagonal of a square field of area 3 ac.

*10. The length of a chord of a circle of radius 7.35 cm. is 12.6 cm.; find the distance of the chord from the centre of the circle.

*11. Find the height of an equilateral triangle, if the length of each side is 4.6 in.

12. The diagonal of a cube (i.e. the line joining two opposite corners) is 5 in. long. Find (i) the length of an edge, (ii) the total area of the surface of the cube.

*13. The base of a pyramid is a rectangle 7 in. by 5 in., and the length of each slant edge of the pyramid is 8 in. Find the height of the pyramid.

## APPENDIX

EX. 188 (a). Use of Tables

Find, to 3 significant figures, the values of:

1. \( \frac{4}{0.3903} \)  
2. \( \frac{9}{21.04} \)  
3. \( \frac{1}{\sqrt{2.733}} \)  
4. \( \frac{1}{(30.09)^2} \)  
5. \( \frac{5}{18.7 + 21.4} \)  
6. \( \frac{\sqrt{10}}{\sqrt{0.493} - \sqrt{0.648}} \)

7. If \( \frac{1}{u} + \frac{1}{v} = \frac{1}{f} \), find \( u \) if \( f = 3.74 \), \( v = 5.06 \).

8. If \( \frac{1}{a} + \frac{1}{v} = \frac{1}{f} \), find \( u \) if \( v = 0.874 \), \( f = 0.596 \).

9. If \( \frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2} \), find \( a \) if \( b = 24.3 \), \( c = 17.8 \).

Find, to 3 significant figures, the values of:

*10. \( \sqrt{(1 + \frac{1}{13}) + \frac{1}{\sqrt{17} - \sqrt{13}}} \)  
*11. \( \sqrt[3]{\left(\frac{107}{9}\right)^2 + \left(\frac{131}{4}\right)^3} \)

*12. \( \frac{1}{0.8304 + 0.0657} \)  
*13. Given 1 km. = 0.6214 mi., express \( \frac{1}{2} \) mi. in km.

*14. Given 1 yd. = 0.9144 m., express 1 sq. m. in sq. yd.

*15. Express 50 lb. as a percentage of 1 ton, correct to 4 figures.

*16. 16383 is the product of two consecutive odd integers; find the integers.

## CHAPTER XIX

LOGARITHMS

EX. 144 (a). Logarithms: Numbers 1 to 10

Find the value to 4 figures, as given by 4-figure tables, of:

1. \( 4.13 \times 2.06 \)  
2. \( 3.18 \times 2.7 \)  
3. \( 6.08 \times 1.43 \)

4. \( 2.076 \times 1.907 \)  
5. \( 7.604 \times 1.008 \)  
6. \( 8.052 \times 1.104 \)

7. \( 9.43 \times 2.76 \)  
8. \( 8.07 \times 3.29 \)  
9. \( 6.58 \times 6.37 \)

10. \( 5.174 \times 3.806 \)  
11. \( 9.403 \times 3.075 \)  
12. \( 8.006 \times 3.708 \)

*13. \( 3.17 \times 1.83 \times 1.42 \)  
*14. \( 4.36 \times 2.87 \times 6.09 \)

*15. \( 1.371 \times 9.328 \)  
3.427 \times 2.104 \)

*16. \( 4.329 \times 7.182 \)
EX. 147 (a). Logarithms: Positive Characteristics

Find the value to 4 figures, as given by 4-figure tables, of:

1. \(39.6 \times 28.7\).
2. \(7.09 \times 308\).
3. \(5470 \times 20.7\).
4. \(763 + 2.94\).
5. \(4280 \div 60.7\).
6. \(8540 \div 17.8\).
7. \(21.73 \times 158.2\).
8. \(3.107 \times 72.05\).
9. \(4083 \times 5107\).
10. \(863.9 \div 97.06\).
11. \(7923 \div 65.073\).
12. \(82068 \div 91.04\).
13. \(1000 \div 75.18\).
14. \(11.06 \div 1.008\).
15. \(90080 \div 99.08\).
16. \((74.02)^2\).
17. \((8.06)^2\).
18. \((15.07)^2\).
19. \(\sqrt{8437}\).
20. \(\sqrt[3]{10800}\).
21. \(\sqrt[5]{500.6}\).
22. \(\sqrt[3]{1000}\).
23. \(\sqrt[4]{105}\).
24. \(6030 \times 6003\).
25. \(7005 \div 70.5\).
26. \(\sqrt[8]{807}\).
27. \(28317 \div 90.714\).
28. \(\sqrt[8]{10.08^2}\).
29. \(\sqrt[5]{2^8 + 3^8}\).

EX. 148 (a). Logarithms: Positive Characteristics

Find the value to 4 figures, as given by 4-figure tables, of:

1. \(2.073 \times 5.164 \times 14.06\).
2. \(2.007 \times 2070 \times 27\).
3. \(86.04 \times 17.92\).
4. \(6347 \times 52.08\).
5. \(5612\).
6. \(8372\).
7. \(9076\).
8. \(6237 \times 405.4\).
9. \(7216 \times 807.5\).
10. \(8947 \times 99.03\).
11. \(10.03 \times 19.4\).
12. \(\sqrt[8]{153.4 \times 70.4\times 28.05 \times 46.7}\).
13. \(\sqrt[8]{63238\times 47.15}\).
14. \(\sqrt[8]{7030\times 24.81}\).
15. \(\frac{3}{290}\).
16. \(\frac{1}{590}\).
17. \(45.07 \times \frac{1}{8.164}\).
18. \(23 \times 284 \times (16.2 \times 107)\).
19. \(\sqrt[8]{(4-109)^3} \div 2\).
20. \(\sqrt[8]{1628 \times 451.3}\).
21. \(\sqrt[8]{6174 \times 35.02}\).
22. \(3.16^2 + (47.2)^2\).
23. \(\frac{3}{471} + \frac{813}{155}\).
24. \(\frac{3}{39.4}\).
25. \(\frac{7}{(2.73)^{\frac{1}{2}} + (3.16)^{\frac{1}{2}}}\).

EX. 149 (a). Use of Logarithms

[Take \(\log \pi = 0.4971\). Give answers to 3 significant figures]

1. Given 1 mi. = 1.609 km., express 1 in. in cm.
2. The record for running 220 yd. is 20.6 sec.; express the average speed in miles per hour.
3. Captain Cuttle's time for the Derby (1\(\frac{1}{2}\) mi.) in 1922 was 2 min. 34\(\frac{3}{4}\) sec.; express the average speed in miles per hour.
4. A packet of paper containing 1000 sheets weighs 8.937 kg.; each sheet measures 32 cm. by 20.5 cm. Find in grams the weight of 1 sq. m. of the paper.
5. Losses by fire in the United Kingdom decreased from £9,016,000 in 1930 to £7,945,000 in 1931. Find the decrease per cent.
6. Investments in Savings Certificates increased from £284,995,518 in 1920 to £371,602,258 in 1930. Find the increase per cent.
7. The volume of a metal tube, \(l\) in. long, outer radius \(r\) in., thickness \(t\) in., is \(V = \pi l t (2r - t)\). (i) Find \(V\) if \(l = 8.54\), \(t = 1.25\), \(r = 6.15\); (ii) find \(l\) if \(V = 200\), \(r = 5.35\), \(t = 1.45\).
8. The volume of a segment of a sphere of radius \(r\) in., height of segment \(h\) in., is \(V = \pi h^2 (r - \frac{1}{3}h)\). Find \(V\) if \(r = 5.42\), \(h = 2.58\).
9. The volume of a sphere of radius \(r\) in. is \(V\) cu. in. where \(V = \frac{4}{3} \pi r^3\) and \(r = \sqrt[3]{\frac{10V}{4\pi}}\). (i) Find \(V\) if \(r = 3.055\); (ii) find \(r\) if \(V = 100\).
10. If \(\pi r^2 = 45\), find the value of \(r\); (ii) \(2\pi r\). Interpret the question geometrically.
11. The surface \(S\) sq. in. and the volume \(V\) cu. in. of a sphere are connected by the formula, \(S^2 = 36\pi V\). (i) Find \(S\) when \(V = 50\); (ii) find \(V\) when \(S = 20\).
12. If coal costs 48s. 6d. a ton, express the price in francs per 1000 kg., given 1 kg. = 2.205 lb. and £1 = 78.5 francs.
18. Find the weight of a solid metal cylinder, height 11.4 cm., base-diameter 6.3 cm., if the metal weighs 8.35 gm. per c.c.

*14. The total area of the surface of a cone, base-radius \( r \) in., height \( h \) in., is \( \pi r (r + \sqrt{r^2 + h^2}) \) sq. in. Find the total area if \( r = 2.15 \), \( h = 4.3 \).

15. The distance of the horizon from a point \( h \) ft. above the Earth's surface is \( \sqrt{\frac{2gh}{g}} \) mi., where \( r \) is the radius of the Earth in miles. Taking \( r = 3950 \), find the distance of the horizon from the top of a tower 117 ft. high.

*16. Given 1 cu. m. = 35.32 cu. ft., express 1 in. in cm.

*17. If 1 cu. cm. of gold weighs 19.3 gm., find the weight in oz. of 1 cu. in. of gold. [1 in. = 2.54 cm., 1 oz. = 28.35 gm.]

*18. Evaluate \( \frac{a^b + h^a}{e^{c/d}} \) when \( a = 4.028 \), \( b = 3.704 \), \( c = 5.174 \), \( d = 3.916 \).

19. The muzzle velocity of a shell of diameter \( D \) in., weight \( w \) lb., is \( \sqrt{\frac{644 \times 2240 \times \pi D^4 \times 1}{4W}} \) ft. per second, where the charge-pressure is \( P \) tons per sq. in. and the length of barrel is \( L \) ft. Find the velocity if \( D = 12 \), \( w = 850 \), \( L = 45 \), \( P = 11.5 \).

*20. A square lead plate 3.7 mm. thick weighs 1.425 kg. Find the length of the plate, given that 1 c.c. of lead weighs 11.3 gm.

**EX. 152 (a). Logarithms: Negative Characteristics**

Find the value to 4 figures, as given by 4-figure tables, of:

1. \( 0.765 \times 0.809 \)  
2. \( 0.0236 \times 0.817 \)  
3. \( 0.15 \times 0.0073 \)  
4. \( 0.0162 \times 0.703 \)  
5. \( 0.417 \times 0.05106 \)  
6. \( 683.2 \times 0.07154 \)  
7. \( 28.6 \times 0.923 \)  
8. \( 0.0616 \times 0.487 \)  
9. \( 0.514 \div 62.7 \)  
10. \( 7.825 \div 0.8162 \)  
11. \( 436.3 \div 0.00101 \)  
12. \( 1 \div 248.3 \)  
13. \( 0.827 \)  
14. \( 0.0904 \)  
15. \( 0.203 \)  
16. \( 0.102 \)  
17. \( 0.3072 \)  
18. \( 0.06107 \)  
19. \( 0.3704 \)  
20. \( 0.892 \)  
21. \( \sqrt[3]{7.364} \)  
22. \( \frac{1}{3}(0.9197) \)  
23. \( \frac{1}{2}(0.0614) \)  
24. \( \frac{1}{2}(0.6597) \)  
25. \( \frac{1}{2}(0.0808) \)  
26. \( \frac{1}{2}(0.7247) \)  
27. \( \sqrt[3]{0.783} \)  
28. \( \sqrt[3]{0.0103} \)  

**EX. 153 (a). Logarithms: Negative Characteristics**

Find the value to 4 figures, as given by 4-figure tables, of:

1. \( 0.0726 \times 0.814 \times 0.697 \)  
2. \( 0.1738 \times 0.3142 \times 0.0092 \)  
3. \( 0.3528 \times 0.6132 \times 0.9693 \)  
4. \( 0.07316 \times 0.6109 \times 0.3048 \)  
5. \( 0.3159 \times 0.7214 \times 0.3705 \)  
6. \( 86.15 \times 0.504 \times 0.7316 \)  
7. \( 3.907 \times 0.8624 \times 0.07136 \)  
8. \( 0.0364 \times 0.1072 \times 0.0578 \times 0.08044 \times 1.66 \)  
9. \( \frac{1}{2}(0.6254) \times 5.68 \times 0.9725 \)  
10. \( \sqrt[3]{0.0356} \times 0.647 \times 0.1932 \times 0.7246 \)  
11. \( \sqrt[3]{0.0628} \times 0.01 \)  
12. \( \sqrt[3]{0.08735} \)  
13. \( \frac{1}{3}(0.0808) \)  
14. \( \sqrt[3]{0.0417} \)  
15. \( \sqrt[3]{0.9165} \)  
16. \( \sqrt[3]{0.5005} \)  
17. \( 0.07) \times 0.07 \)  
18. \( 0.706-b \)  
19. \( 0.94-\sqrt{0.0048} \)  
20. \( 0.07 \cdot 0.07 \)  
21. \( 70.6 \)  
22. \( 0.94-\sqrt{0.0048} \)  
23. \( 0.048 \)  

Find the value to 3 figures, of:

24. \( \sqrt[3]{0.0962} \times 0.1345 \times \sqrt[3]{187.3} \)  
25. \( \sqrt[3]{0.723} + \sqrt[3]{0.846} \)  
26. \( \sqrt[3]{0.1865} \div 0.2167 \)  
27. \( \sqrt[3]{0.1865} + \sqrt[3]{0.2167} \)  

**EX. 155 (a). Use of Logarithms**

[Give answers to 3 figures. Take \( \log \pi = 0.4971 \)]

1. Out of 100,000 males born, 80,550 reach the age of 30, and 58,800 reach the age of 60. What percentage of men 30 years old die before they are 60?

2. In 1929 the numbers of pupils in England and Wales at secondary schools were as follows:

   Boys: under 12 years' old, 48,279; 12 or over, 160,659.
   Girls: under 12 years' old, 46,271; 12 or over, 134,976.

What percentage of the total number were boys?
The maximum volume of a cone of total surface S sq. ft. is \[ \frac{1}{3} \sqrt{\frac{S^3}{4\pi}} \] cu. ft. Calculate the maximum volume when \( S = 3 \).

If \( a^2 = b^2 + c^2 \), find \( a \) when \( b = 0.763 \), \( c = 0.542 \).

Evaluate \( a \sqrt{\left( \frac{b}{c^2} \right)} \) when \( a = 47.25 \), \( b = 0.3413 \), \( c = 0.2655 \).

Evaluate \( \left( a^2 + b^2 - c^2 \right) / 2bc \) when \( b = 0.764 \), \( c = 1.083 \), \( a = 0.825 \).

*12. If \( \frac{4\pi}{3} \times 6.8 \times r^3 = 29.3 \), evaluate \( 4\pi r^3 \).

13. Evaluate \( (37.9)^{\frac{2}{3}} \times (0.73 \times 0.864)^{-\frac{1}{3}} \).

14. Evaluate \( \frac{\log 3}{\log 4} - \frac{3}{4} \log 3 - 4.22 \).

15. Evaluate \( (1.246)^3 - \frac{1}{3} (1.246)^3 + 1 \).

16. 10,500 short tons of ore yielded a profit of £4920. Find the average profit in dollars per 1000 kg. of ore. [1 short ton = 2000 lb., 1 lb. = 453.6 gm., £1 = 5.15 dollars.]

17. Find the least integral value of \( n \) such that \((0.9)^n\) is less than 0.1.

18. The sluice area A sq. ft. that must be opened to allow Q cu. ft. of water to enter a dock in T sec. is given by \( A = 2Q / \left( 0.7T \right) \sqrt{2\pi h} \), where \( h \) ft. is the rise of the water. Find \( A \) when \( Q = 300,000 \), \( h = 26 \), \( T = 300 \), \( g = 32.2 \).

19. The loss of horse-power by friction of a wheel \( c \) ft. in circumference, making \( n \) revolutions per minute in atmosphere of density \( w \) lb. per cu. ft., is given by \( c b n^3 w / (1011 \pi)^5 \). What power is lost when \( c = 35 \), \( n = 200 \), \( w = 0.24 \) ?

20. The following formula occurs in wireless telephony: \( k = 1 / \left( d + (n^2 + 9r^2)^{-1} \right) \). Find \( k \) when \( n = 100 \pi \), \( c = 1.3 \times 10^{-9} \), \( r = 5 \times 10^6 \).

21. If \( 2\pi r = 10 \), find \( \pi r^2 \).

22. Evaluate \( 46.5 k d^4 \) when \( k = 1.37 \), \( d = 0.724 \).

23. Find the area in sq. yd. of a rectangular court, 25.4 m. long, 16.5 m. wide. [1 ft. = 0.3048 m.]

24. If \( 1 \) sq. mi. = 2.589 sq. km., express \( 1 \) cu. yd. in cu. m.

25. The maximum volume of a cone of total surface \( S \) sq. ft. is \( \frac{1}{3} \sqrt{\frac{S^3}{4\pi}} \) cu. ft. Calculate the maximum volume when \( S = 3 \).

26. If \( a^2 = b^2 + c^2 \), find \( a \) when \( b = 0.763 \), \( c = 0.542 \).

27. Evaluate \( a \sqrt{\left( \frac{b}{c^2} \right)} \) when \( a = 47.25 \), \( b = 0.3413 \), \( c = 0.2655 \).

28. Evaluate \( \left( a^2 + b^2 - c^2 \right) / 2bc \) when \( b = 0.764 \), \( c = 1.083 \), \( a = 0.825 \).

29. If \( 3\pi \times 6.8 \times r^3 = 29.3 \), evaluate \( 3\pi r^3 \).

30. Evaluate \( (37.9)^{\frac{2}{3}} \times (0.73 \times 0.864)^{-\frac{1}{3}} \).

31. Evaluate \( \frac{\log 3}{\log 4} - \frac{3}{4} \log 3 - 4.22 \).

32. Evaluate \( (1.246)^3 - \frac{1}{3} (1.246)^3 + 1 \).

33. 10,500 short tons of ore yielded a profit of £4920. Find the average profit in dollars per 1000 kg. of ore. [1 short ton = 2000 lb., 1 lb. = 453.6 gm., £1 = 5.15 dollars.]

34. Find the least integral value of \( n \) such that \((0.9)^n\) is less than 0.1.

35. The sluice area \( A \) sq. ft. that must be opened to allow \( Q \) cu. ft. of water to enter a dock in \( T \) sec. is given by \( A = 2Q / \left( 0.7T \right) \sqrt{2\pi h} \), where \( h \) ft. is the rise of the water. Find \( A \) when \( Q = 300,000 \), \( h = 26 \), \( T = 300 \), \( g = 32.2 \).

36. The loss of horse-power by friction of a wheel \( c \) ft. in circumference, making \( n \) revolutions per minute in atmosphere of density \( w \) lb. per cu. ft., is given by \( c b n^3 w / (1011 \pi)^5 \). What power is lost when \( c = 35 \), \( n = 200 \), \( w = 0.24 \) ?

37. The following formula occurs in wireless telephony: \( k = 1 / \left( d + (n^2 + 9r^2)^{-1} \right) \). Find \( k \) when \( n = 100 \pi \), \( c = 1.3 \times 10^{-9} \), \( r = 5 \times 10^6 \).
APPENDIX

Find the volume of a right pyramid, given:
5. Slant edge, 17 in.; rectangular base, 18 in. by 24 in.
6. Slant edge, 7 cm.; rectangular base, 4 cm. by 6 cm.
7. The volume of a right pyramid, 5 in. high, standing on a square base, is 50 cu. in. Find the length of a side of the base and of a slant edge.

Find the total area of the surface of a right pyramid, given:
8. Height, 8 in.; rectangular base, 30 in. by 12 in.
9. Slant edge, 10 cm.; rectangular base, 6 cm. by 8 cm.

*10. On the sides of a square, side 16 in., as bases, four isosceles triangles are drawn externally, the equal sides being each 17 in. long. The triangles are folded about their bases so as to form a pyramid. Find the volume of the pyramid.

*11. A tin is in the shape of a truncated right pyramid. The base is a rectangle 5 cm. by 4 cm., and the top is a rectangle 10 cm. by 8 cm., and the depth is 6 cm. Find the amount of water the tin will hold. [It can be proved that this truncated pyramid is part of a right pyramid whose base is a rectangle 10 cm. by 8 cm., and whose height is 12 cm. Assume this fact and illustrate it by a free-hand sketch.]

*12. A tent is in the shape of a right pyramid of height 12 ft., its base being a regular hexagon, side 8 ft. Find its volume, and prove that the area of the canvas used in making the tent is twice the area of the base of the tent. [A regular hexagon is a 6-sided figure with all its sides equal and all its angles equal.]

EX. 159 (a). Circles and Cylinders

[Give answers to 3 figures; \( \log \pi = 0.4971 \); 1 gal. = 277.3 cu. in.]
1. Find (i) the circumference, (ii) the area of a circle of radius 7.63 dm.
2. Find (i) the radius, (ii) the area of a circle whose circumference is 2.6 ft. 6 in.
3. Find in metres (i) the radius, (ii) the circumference of a circle whose area is 1 sq. km.
4. Find the weight of petrol in a cylindrical can, diameter 25 cm., height 30 cm., if 1 l. of petrol weighs 0.74 kg.

APPENDIX

5. A piece of metal is 6 cm. square and 2 mm. thick and has a circular hole of radius 2 cm. punched in it. Find the weight of 100 such pieces if the metal weighs 6.75 kg. per cu. dm.

*6. Find the diameter of a semicircular plate whose perimeter is 15 cm.

7. A cylindrical tank is 7 ft. 6 in. in diameter and contains 1000 gal. of water. Find the depth of the water.
8. A cylindrical tank is 7 ft. 6 in. in diameter and contains 1000 gal. of water. Find the depth of the water.
9. Water is flowing out of a pipe of diameter 3 in. at the rate of 48 gal. per minute. Find the speed of the water in the pipe in inches per second if the pipe is full.

10. An inkpot is formed by hollowing out a cylindrical hole, 1 1/2 in. deep, 1 3/4 in. diameter, in a glass tube, edge 2 1/2 in. Find its weight if 1 cu. in. of glass weighs 2.35 oz.

11. A swimming-bath which holds 25,000 gal. is filled by water flowing at 3 mi. per hour through a pipe of diameter 3 in. Find in hours the time taken to fill the bath.

12. The external diameter of a glass tube is 3 3/4 in. and the glass is 3/8 in. thick. If the glass weighs 156 lb. per cu. ft., find in inches what length of tube weighs 1 lb.

13. A long strip of paper is rolled on a cylinder 1 in. in diameter and the total diameter of the roll is 3 in. What would be the total diameter if the same strip is rolled on a cylinder of diameter 3 in.?

14. What length of a solid lead cylinder of 2 in. diameter must be taken to be cast into a hollow cylinder, external diameter 1 ft., 1 1/2 in. thick, 15 in. long?

15. A cylindrical vessel 8 in. in diameter contains water 2 ft. deep. A cylindrical rod, 2 ft. long, is thrust vertically into the water till its end rests on the bottom of the vessel. If the water-level rises 1 in., find the diameter of the rod.

EX. 160 (a). Cones

[Give answers to 3 figures; \( \log \pi = 0.4971 \)]
Find the volume of a circular cone, given:
1. Height, 7.35 in.; radius of base, 3.45 in.
2. Slant length, 8 cm.; radius of base, 6 cm.

*3. Slant length, 10 cm.; perimeter of base, 30 cm.
Find the area of the curved surface of a circular cone, given:
4. Slant length, 4.26 in.; radius of base, 3.18 in.
5. Slant length, 9.4 cm.; height, 7.5 cm.
6. Height, 6 cm.; perimeter of base, 20 cm.
7. Find the base-radius of a cone whose height is 6.75 cm. and whose volume is 95 cu. cm.
8. Find the volume of a cone if the area of the curved surface is 120 sq. in. and if the perimeter of the base is 30 in.
9. The height of a conical tent is 8 ft. and the length of its slant edge is 10 ft.; find (i) the volume of the tent, (ii) the area of the canvas.
10. A piece of paper in the form of a sector of a circle of radius 6 in., angle of sector 120°, has its two straight edges joined to form a hollow cone. Find the height of the cone.

**EX. 161 (a). Spheres**

*Give answers to 3 figures: log π = 0.4971*

1. Find the volume and area of the surface of a sphere of diameter 4.76 in.
2. Find the radius of a sphere of volume, 50 cu. in.
3. Find the radius of a sphere whose surface is 1 sq. ft.
4. Find the volume of a sphere whose surface is 1 sq. m.
5. How many solid shot, each of diameter \(\frac{\sqrt{5}}{2}\) in., can be made out of a circular lead disc, diameter 10 in., and 2.5 in. thick?
6. A cylinder, radius 2 in., height 6 in., containing water to a depth of \(\frac{3}{4}\) in. How many lead spheres, diameter 1 in., can be placed in it without causing the water to overflow?
7. A cylindrical jar of height and diameter 13 cm., is half-full of water. A solid metal sphere, diameter 13 cm., is then placed in it. Find in c.c. the amount of water that overflows.
8. Find in inches the diameter of a solid iron sphere weighing 16 lb., if 1 cu. ft. of iron weighs 748 lb.
9. Find in lb. the weight of a hollow metal ball, external diameter 6 in., thickness 0.2 in., if 1 cu. ft. of the metal weighs 625 lb.

**EX. 163 (a). Miscellaneous Mensuration**

*Give answers to 3 figures. Logarithm tables should be used where suitable*

1. Two roads meet at right angles, and a triangular plot of land has a frontage of 395 ft. on one road and 288 ft. on the other. Find the value of the plot at £350 per acre.
2. ABCD is a field with the side AD perpendicular to the sides AB, DC, with a path 2 yd. wide running along three sides; dimensions are shown in yards. Find (i) the length of fencing all round the edge of the field, (ii) the area in acres of the part which can be cultivated.
3. The diagram represents the front of a wooden dog-kennel with a rectangular opening; the sloping edges of the roof are of equal length. Find the area of the front surface.
4. The diagram represents a triangle ABC enclosed in a rectangle. Find the area of \(\triangle ABC\), and the distance of A from BC.
5. Southampton Water is a channel about 31 81 mi. long, with an average breadth of about 14 15 mi. If the water rises 6 ft. and weighs 656 lb. per cu. ft., find in tons the water that has entered the channel.
6. The section of a railway cutting is a trapezium with parallel sides of lengths 36 ft., 56 ft., and 25 ft. apart. If the cutting is 250 yd. long, find the number of cu. yd. of earth removed in making it.
7. The circumference of a circle is 350 yd. Find the area of the circle.
8. A cylindrical hole of diameter 1 in. is bored centrally through a solid wooden cube, edge 2 in., the axis of the cavity being perpendicular to the base of the cube. Find in oz. the weight of the block, if the wood weighs 37-5 lb. per cu. ft.

9. The tin-sheeting used to make a closed cylindrical can of given volume is least when the height equals the diameter. What is the most economical diameter for a cylindrical can which holds 1 cu. ft.?

10. A hollow metal cylindrical pipe, with open ends, is 2 ft. long and 2 ft. 6 in. in circumference, outside measurements; the metal is 1 in. thick. Find its weight if 1 cu. ft. of the metal weighs 486 lb.

*11. Find the area of the curved surface of a circular cone, of volume 12 cu. in. and base area 8 sq. in.

12. A solid metal sphere, diameter 1 1/2 ft., is dropped into a tank 4 ft. long, 3 ft. wide, 5 ft. deep, half-full of water. Find in inches the rise in the water-level.

*13. A solid glass sphere weighs twice as much as a solid iron sphere, and iron is 3 times as heavy as glass. Find the ratio of the radius of the glass sphere to that of the iron sphere in the form \( \pi : 1 \).

*14. The area of a square field is 2 1/4 ac. Along the four sides of the field and inside it there is a belt of trees of uniform width which occupy 1284 sq. yd.; find the width of this belt.

*15. A square cloister is 120 ft. each way, outside measurements. Round the inside there is a walk 18 ft. broad, and straight across the centre there are two paths, each 6 ft. broad, parallel to the sides; the remainder of the central portion is sown with grass. The walk and the paths are paved with stones, each 3 ft. by 2 ft., costing 4s. each, and the cost of sowing is 1s. per sq. yd. Find the cost of (i) the paving, (ii) the sowing.

16. A rectangular block of marble 3 ft. long, 1 ft. broad, 6 in. deep, is hollowed out to form a basin holding 6 gall. of water. When the basin is full of water, the total weight of the basin and the water is 150 lb. Find in lb. the weight of the marble per cu. ft. [1 gall. of water weighs 10 lb., 1 cu. ft. of water weighs 82-3 lb.]

*17. A cylindrical vessel 9 in. in diameter, 1 ft. high, and axis vertical, is half-full of water. A solid cylinder 4 in. in diameter, 1 ft. high, is lowered into it with its axis vertical, till its base is 2 in. above the bottom of the vessel. Find how much the water-level rises.

*18. A cube has an edge of 2 in. A triangle is formed by joining the centre of one face of the cube to the ends of one of the edges of the opposite face. Find the area of this triangle, and its perimeter.

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CHAPTER XXI

SPECIFIC GRAVITY

EX. 164 (a). Density and Specific Gravity

[Use the sp. gr. table, p. 331. Give answers to 2 figures]

1. Find the weight of a rectangular block of wood, 15 cm. by 12 cm. by 8 cm.; sp. gr. 0-55.

2. The density of a metal is 427 lb. per cu. ft.; find its specific gravity.

3. 30 c.c. of a liquid weigh 22.5 gm.; find its specific gravity.

4. A rectangular vessel, 9 cm. by 6 cm., contains mercury to a depth of 12 cm.; find the weight of the mercury.

5. A bar of cast iron weighs 50 lb.; find its volume in cu. in.

6. Find the weight of 1 km. length of copper wire, sp. gr. 8-95, if the diameter of the wire is 0-35 cm.

7. 13 1/4 cu. ft. of granite weigh 1 ton. Find its sp. gr.

8. Find the weight of a rectangular sheet of lead, 9 in. by 8 in., 1/2 in. thick.

9. A solid cylinder of copper is 9 cm. long and 5 cm. in diameter. Find its weight.

10. An ebony snuff-box is a hollow cylinder closed at both ends, external diameter 9-5 cm., external height 2-7 cm.; the thickness is 3 mm. Find the weight of the box.

11. Find the thickness of a rectangular cast-iron plate which weighs the same as a block of ebony 4 in. thick, of the same length and breadth as the iron plate.

12. A thread of mercury, 15-7 cm. long, is run out of a straight tube of uniform bore and found to weigh 0-108 gm. Find the diameter of the tube.

13. A lump of metal weighs 88.5 gm. When it is totally immersed in a cylinder of radius 3 cm., the height of the water-level in the cylinder rises 0.45 cm. Find the sp. gr. of the metal.
14. A bottle weighs 9 oz. when empty and weighs 22$\frac{1}{2}$ oz. when full of water. What will it weigh when full of glycerine?

*16. If 1 kg. of a liquid, sp. gr. 0.75, is mixed with 1 kg. of a liquid, sp. gr. 0.6; find the sp. gr. of the mixture, if there is no chemical action.

*17. An alloy is made containing 2 oz. of platinum for each oz. of silver. What is the sp. gr. of the alloy?

*17. A cylindrical jar of capacity 1 pt. is $5\frac{1}{2}$ in. high. If a cylindrical jug, 11 in. high with a diameter 3 times that of the jar, is filled with milk, the weight of the milk is 372 oz. Find its sp. gr.

EX. 165 (a). Principle of Archimedes

[Give answers to 2 figures; 1 cu. ft. of water weighs 62-3 lb.]

1. A block of wood floats in water so that the volume of the submerged portion is 18 cu. in.; find the weight of the block.

2. A wooden block of volume 75 cu. in. and sp. gr. 0.65 is floating in water; find the volume of water displaced.

3. A body of sp. gr. 0.8 is floating in water. What fraction of the body is below the surface?

4. Find the apparent weight in water of a cuboid of aluminium, sp. gr. 2.65, which measures 6 cm. by 4 cm. by 3 cm.

5. Find the apparent weight in oil, sp. gr. 0.9, of a metal cylinder 6 cm. long, 0.82 cm. in diameter, if the sp. gr. of the metal is 8-4.

6. A lump of metal, sp. gr. 8.5, appears to weigh 250 gm. in water. Find its weight in air.

7. A lump of zinc weighs 62.8 gm. in air and 56.6 gm. in a liquid of sp. gr. 0.7. Find the sp. gr. of the zinc.

8. A wire of uniform circular cross-section and 122.6 cm. long weighs 12.16 gm. in air and 10.76 gm. in water. Find (i) the diameter of its cross-section, (ii) its apparent weight in oil of sp. gr. 0.88.

CHAPTER XXII

COMPOUND INTEREST

EX. 166 (a). Compound Interest by Calculation

[Give answers correct to the nearest penny]

Find the compound interest (payable yearly) on the following sums for the stated periods and rates of interest:—

1. £420; 2 yr.; 3%.
2. £780; 2 yr.; 6%.
3. £291; 3 yr.; 4%.
4. £173 15s.; 3 yr.; 5%.
5. £417 13s. 10d.; 3 yr.; 3%.
6. £249 7s. 4d.; 3 yr.; 4%.
7. £517 9s.; 2 yr.; 4$\frac{2}{3}$%.
8. £461 3s. 5d.; 2 yr.; 2$\frac{2}{3}$%.
9. £308 10s. 9d.; 3 yr.; 5$\frac{1}{2}$%.
10. £269; 2$\frac{1}{2}$ yr.; 5%.
11. £872; 2$\frac{1}{2}$ yr.; 3$\frac{1}{4}$%.
12. £596; 2$\frac{1}{4}$ yr.; 3%.

Find the amount at compound interest of:

13. £3654 15s. for 2$\frac{1}{2}$ yr. at 5$\frac{3}{4}$%, payable yearly.
14. £176 for 1 yr. at 3$\frac{3}{4}$% p.a., payable half-yearly.
15. £1275 for 1$\frac{1}{2}$ yr. at 5$\frac{1}{2}$% p.a., payable half-yearly.
16. £856 for 1 yr. at 5% p.a., payable quarterly.

Find the difference between the simple and compound interest, payable yearly, on the following sums for the stated periods and rates of interest:—

17. £1260; 3 yr.; 6%.
18. £634 8s.; 3 yr.; 3$\frac{1}{4}$%.

*19. A moneylender charges 60% p.a. compound interest, payable monthly. If a man borrows £100, how much does he owe at the end of 3 months?

*20. A man borrows £300 at 5% p.a. compound interest and repays it by two equal instalments at the end of the first and second years. Find the size of each instalment to the nearest shilling.
EX. 168 (a). Compound Interest Formula

Find (i) by logarithms, (ii) by using the compound interest tables, the approximate amounts of the following sums, at compound interest for the stated periods and rates:

1. £1200; 8 yr.; 4%. 2. £735; 9 yr.; 6%.
3. £826; 20 yr.; 2½%. 4. £3625; 12 yr.; 3½%.

Find by logarithms the approximate value of the compound interest on the following sums for the stated periods and rates:

5. £1050; 7 yr.; 5%. 6. £5240; 11 yr.; 3%.
7. £2345; 15 yr.; 3½%. 8. £1475; 24 yr.; 2½%.

Find by logarithms the approximate sum of money which amounts at compound interest to:

9. £525 in 12 yr. at 5%. 10. £1350 in 20 yr. at 3½%.

Find by logarithms the approximate rate per cent. at compound interest if:

11. £400 amounts to £605 in 14 yr.
12. £335 amounts to £1500 in 16 yr.

In what time, at compound interest, will:

13. £661 amount to £1000 at 3½% p.a.?
14. £1278 amount to £5000 at 4¾% p.a.?

15. Find the difference between the amount of £100, if invested (i) for 13 yr. at 3% p.a. compound interest, (ii) for 8 yr. at 5% p.a. compound interest.

*16. A certain sum of money at compound interest amounts in 2 yr. to £811 4s. and in 3 yr. to £843 13s. Find the sum of money.

*17. A steel ball is let fall from a height of 15 ft., and after each impact on the floor rises ⅔ of the height from which it fell. How high does it rise after hitting the floor for the eighth time?

*18. Use the compound interest table on p. 342 to draw graphs showing the amount of £100 at any time in the first 10 yr. at (i) 3½%, (ii) 6%. Draw on the diagram the corresponding simple interest graphs.

*19. The value of a machine decreased from £180 to £86 in 8 yr. Find the yearly decay factor, assuming it to be constant.

CHAPTER XXIII

HARDER PERCENTAGE

EX. 169 (a). Harder Percentages

1. An article was sold for £1 15s. at a profit of 25%; what did it cost? What would be the profit per cent. if it was sold for 2 guineas?
2. A grocer gains 20% by selling tea at 2s. 3d. per lb. Find his gain per cent. if he raises the price 3d. per lb.
3. If an article is sold for £27 12s. 6d., the loss is 15%; what is the selling price if there is a gain of 20%?
4. By selling 5 similar things for 9s. 7d., a tradesman gains 15%. What would he gain or lose per cent. if he were to sell 12 of these same things for a guinea?
5. By selling articles at 9s. per hundred a dealer gains 20%; find the gain per cent. if he sold them at 150 for 12s.
6. The wholesale price of an article is 17s. 6d.; a retailer is allowed 10% discount off the wholesale price and sells the article for 1 guinea. Find his gain per cent. to the nearest whole number.

*7. A man would gain 20% by selling a chair for 47s. 6d. and would gain 15% by selling a table for 57s. 6d. He sells the chair for 36s.; what is the least price for which he must sell the table to avoid any loss on the two together?
8. A man who weighs 18½ stone undergoes a treatment which claims to reduce his weight every period of 8 mo. by 20% of the amount at the beginning of that period. What ought his weight to be in 2 years time?
9. A sells a horse to B at a profit of 20%; B sells it to C for £60 at a profit of 38%. What did the horse cost A? What profit per cent. would A have made if he had sold it direct to C for £69?

*10. Every 6 mo. the price of a commodity increases by 25%. A shopkeeper finds that he has to pay £45 more for a certain quantity than he did last year. Find its present price.

*11. A sold a field to B at a profit of 10%; later on B sold it back to A at a profit of 10%, thereby gaining £5 10s. How much did A pay for the field originally?
12. An article on its way from the manufacturer to the consumer passes through the hands of two middlemen. If the manufacturer and each middleman make a profit of 20%, and if the consumer pays £1 1s. for it, find the cost of manufacture.

13. A shopkeeper offers goods at prices which give him a profit of 40%, but he gives customers 10% discount. What is his net profit per cent.?

14. A man's income increases by 20% and in the same time his income tax increases from 4s. in the £ to 5s. in the £. Find the percentage change in his net income.

15. When the price of an article is reduced by 15% the sales increase by 35%. Find the percentage change in the total receipts.

16. The cost of running a factory is made up as follows: rent, 20%; labour, 50%; material, 25%; sundries, 5%. If rent is reduced by 12%, labour increased by 8%, material increased by 20%, and sundries are unchanged, find the percentage change in running costs.

17. In 1930 a tradesman found that on every £100 of goods sold he made a gross profit of £40 and, after deducting trade expenses, a net profit of £18. In 1931 he handled 50% more goods, but the price at which he bought them had risen 80% and his other trade expenses had doubled. If he increases the price at which he sells the goods by 70%, find the increase or decrease per cent. of (i) his gross profit, (ii) his net profit.

18. In 1930 an establishment used 41,100 cu. ft. of gas; and 1 cu. ft. of gas supplied 550 units of heat, and the gas cost 10d. per therm. In 1931 the number of cu. ft. of gas consumed diminished by 10%, but the gas contained 572 units of heat per cu. ft. and cost 8d. per therm. By what percentage did the expenditure alter? [1 therm = 100,000 units of heat.]

EX. 170 (a). Mixtures

1. A tobacconist buys 14 lb. of tobacco at 13s. per lb. and 14 lb. at 12s. per lb.; he mixes them and sells the mixture at 1s. per oz. Find his gain per cent.

2. A retailer buys 60 lb. of tea at 1s. 6d. per lb. and mixes it with 10 lb. at 2s. 3d. per lb. He sells the mixture at 2s. 14d. per lb. Find his gain per cent.

3. 10 lb. of coffee at 2s. 7d. per lb. are mixed with 8 lb. of chicory at 8d. per lb. At what price per lb. must the mixture be sold to gain 33 1/3%?

4. A grocer mixes some tea at 1s. 6d. per lb. with one-sixth of that amount at 1s. per lb., and sells the mixture at 1s. 6d. per lb. Find his gain per cent.

5. 5 gal. of a mixture of two liquids A, B in the ratio 7: 5 are added to 10 gal. of a mixture of A, B in the ratio 2: 1. Find the ratio of the amounts of A, B in the resulting mixture.

6. A 36-gall. cask contains 30 gall. of spirit and 6 gall. of water. 12 gall. are drawn out, and the cask is then filled up with spirit; 12 more gallons are withdrawn and the cask is then filled up with water. What is the final percentage of spirit in the cask?

7. Tea at 1s. 4d. per lb. is blended with tea at 2s. 6d. per lb. so that a profit of 25% is made by selling the mixture at 2s. 6d. per lb. In what ratio are the teas mixed?

8. How many lb. of tea at 2s. 7½d. per lb. must be added to 100 lb. of tea at 3s. 3d. per lb. so that a profit of 33 1/3% is made by selling the mixture at 3s. 11d. per lb.?

9. An alloy contains 24% of tin by weight. How much more tin to the nearest lb. must be added to 100 lb. of the alloy so that the percentage of tin may be doubled?

10. Two kinds of tea are blended in the ratio 2: 1, and the mixture is retailed at 2s. per lb. at a profit of 20%. The first kind costs £10 per cwt.; what is the price per cwt. of the second kind?

11. 12½% of a bankrupt's assets are first taken for legal expenses, and the creditors received 6s. 9d. in the £. The assets were £5780 14s.; what were the liabilities?

12. A man bought a number of similar articles for £5; 3 of them were broken, but by selling each of the rest at 12½% above cost price he made a profit of 8s. How many did he buy?

13. Three boys worked out 150 examples; A did 14 out of 15 right, B 9 out of 10 right, C 4 out of 5 right. What is the least number that were done right by all of them?

14. A and B provide £52,500 and £17,500 for business capital. B receives 20% of the gross profits as manager; interest is then paid at 4% on the capital, and the rest of the profits are divided equally between them. If A receives £3934, what does B receive?
APPENDIX

*15. The normal work in an office was done by A, B, C, each working 6 days a week. A did 42% of the work and B did 40% of it. In a certain week, the work was suddenly doubled; C did no more than usual. A increased his daily amount of work by 150% for 4 days and then fell ill and could do no more. By how much per cent. did B have to increase his week's work to get everything done?

CHAPTER XXIV
SHARES AND STOCKS

EX. 172 (a). Shares

Find the cost of and the income from the following preference shares:—
1. 80 Lever 8% (£1) shares at 30s.
2. 240 Debenhams 7% (10s.) shares at 9s. 4d.
3. 320 Madame Tussaud 6% (£1) shares at 8d.
4. 1500 British Milk 6% (10s.) shares at 7s. 10d.

Find the number of shares that can be bought and the income obtained by investing:
5. £100 in (10s.) shares at 8s., paying 3%
6. £250 in (2s.) shares at 2s. 6d., paying 7%
7. £1650 in (£1) shares at 23s., paying 10%

Find the sum of money obtained by selling:
8. 1200 Rowntree 6% (£1) shares at 13s.
9. 160 Smith's Crisps (5s.) shares at 21s. 3d.
10. 180 Daily Mirror (5s.) shares at 16s. 6d.

Find the yield obtained by investing money in:
11. (£1) shares at 33s., paying 20%
12. (5s.) shares at 16s., paying 24%
13. (10s.) shares at 37s. 6d., paying 35%
14. (2s.) shares at 9d., paying 13½%.

APPENDIX

15. A man bought 280 (10s.) shares for £105. At what price did the shares stand?
16. A man sold 1750 (5s.) shares for £748. At what price did the shares stand?
17. A man who owns 1250 (£1) shares receives from them a dividend of £43 15s. What is the rate % of the dividend?
18. A man who owns 2400 (2s.) shares receives from them a dividend of £18. What is the rate % of the dividend?
19. By investing £1250 in (£1) shares paying 4%, a man obtained a dividend of £80; at what price did the shares stand?
20. By investing £350 in (5s.) shares paying 2½%, a man obtained a dividend of £12 10s.; at what price did the shares stand?

EX. 173 (a). Share Transactions

1. A man sells 360 Phillips (£1) shares at 33s. and invests the proceeds in Newnes (10s.) shares at 27s. How many Newnes shares does he buy?
2. A man sells 640 Kia-ora (10s.) shares at 8s. 9d. and invests the proceeds in Maple (£1) shares at 25s. How many Maple shares does he buy?
3. A man sells 440 Leyland (£1) shares at 22s. 6d. and with the proceeds can just buy 1200 Home and Colonial (4s.) shares. At what price do the (4s.) shares stand?
4. A man bought 250 (10s.) shares at 11s. 6d. and sold them when they had risen to 12s. 4d. How much did he gain?
5. A man bought 4000 (2s.) shares at 3s. 2d. and sold them when they had fallen to 8d. How much did he lose?
6. A man bought 300 (5s.) shares at 7s. 4d.; he sold 100 of them when they had risen to 9s. and the remainder when they had fallen to 6s. 6d. What did he gain or lose by the whole transaction?
7. A man sold 350 (10s.) shares, paying 4%, at 8s. and invested the proceeds in (5s.) shares, paying 8%, at 7s. How many (5s.) shares did he buy and what was the change of income?
8. A man sold 720 (10s.) shares, paying 9%, at 18s. 9d. and invested the proceeds in (2s.) shares, paying 3%, at 1s. 6d. How many (2s.) shares did he buy and what was the change of income?

9. A man sold 20 (£100) shares, paying 4½%, at 103½ and lent £2000 of the proceeds on mortgage at 5%, and put the balance on deposit at 2½%. Find the change of income.

*10. A man bought some (£1) shares at 16s. 6d. and held them for 5 years, receiving during that time a yearly dividend of 5%. He then sold them at 15s. 9d. Find the net yield per cent. per annum on his investment.

*11. A man bought 100 (£1) shares at 2¼ and received a dividend of 21¼% and a bonus of one additional share (on which no dividend was paid) for every 5 shares held by him. He then sold all his shares at 1½. Find his total profit.

*12. The issued capital of a company consisted of 750,000 (£1) shares. At the end of 1930, the profit-balance was £254,978; a dividend of 30% was paid and the rest was carried forward to 1931; also each shareholder was given a bonus of 2 shares for every 3 shares he already held, the new shares ranking for dividend in 1931 but not in 1930. How many shares of the company were due for dividend in 1931? The profits earned in 1931 were £347,054, and a dividend of 25% was paid on all the issued shares. What was the total amount of money that remained to be carried forward to 1932?

Find, allowing for brokerage, (i) the cost of buying, (ii) the proceeds from selling:

18. 160 (5s.) shares at 14s. 9d., brokerage 1¾d. per share.
14. 1200 (2s.) shares at 1s. 10d., brokerage ½d. per share.
15. 135 (10s.) shares quoted at 19s. to 19s. 6d., brokerage 2d. per share.
16. 3600 (5s.) shares quoted at 3s. 7d. to 3s. 9d., brokerage ½d. per share.
17. A man bought 1200 (10s.) shares at 11s. 8d.; he sold 400 of them at 12s. 4d. and the remainder at 9s. 10d. Find his total loss, allowing brokerage 1d. per share on each transaction.

EX. 175 (a). Stocks

[Give answers correct to the nearest penny]

Find the cost of and the income derived from the following:

1. £700 Brazil 5% stock at 74.
2. £1240 L.M.S. 4% preference at 80.
3. £362 14½. 9d. Southern Railway 5% preference at 109.

How much stock can be bought, and what is the income obtained, by investing:

4. £1800 in Bank of England stock at 360, paying 12%?
5. £350 in Conversion 5% stock at 116½?
6. £610 in Burnley 4½% stock at 114½?

Find the proceeds obtained by selling:

7. £675 London Transport 5% stock at 126.
8. £840 Norwich 3% stock at 93.
9. £518 4s. 10d. Indian 3½% stock at 90½.

Find, in the form £ s. d. per cent., the yield from the following stocks:

10. Japan 6% stock at 90.
11. New Zealand 3½% stock at 115.
12. Walsall 4½% stock at 112½.
13. Japan 5½% stock at 82½.
14. Find the price of a 5% stock if it yields 6½% on an investment.
15. Find the price of a 6½% stock if it yields 4½% on an investment.

16. By investing £5760 in a 5½% stock a man obtained an income of £270. Find the price of the stock.
17. A man buys £850 Funding Loan 4½% stock at 112 and £1100 L.N.E.R. 4½% first preference at 68. Find the total cost and the average yield on his investment.

*18. Units of £5 War Loan 4½% stock were issued at a discount of 8d. each. Find the cost of £175 of War Loan and the income obtained from the investment.
*19. Which investment gives the higher net yield: a 4% stock at 95 free of income tax, or a 6% stock at 105 from the dividends of which there is a deduction of 5s. in the £ income tax?

20. A man buys £660 of 3½% stock at 82½ and 250 (£1) shares at 4½, paying 30%. Find his total income from the investment.

*21. A man invested £900 in a 6% stock and received a yearly return of £56 after income tax at 5s. in the £ had been deducted. Find the price of the stock.

Find, allowing for brokerage, (i) the cost of buying, (ii) the proceeds from selling:

22. £2500 stock at 65, brokerage ¼ per cent.
23. £640 stock at 103¼, brokerage ½ per cent.
24. £850 stock at 114¼, brokerage ½ per cent.
25. £1200 stock quoted at 78½—79, brokerage ¼ per cent.
26. £360 stock quoted at 112½—113¼, brokerage ½ per cent.
27. £3500 stock quoted at 83½—84½, brokerage ¼ per cent.
28. A man paid £732 for £600 stock. At what price did the stock stand if brokerage was ¼ per cent?
29. A man received £875 from the sale of £1250 stock. At what price did the stock stand if brokerage was ¼ per cent?
30. By investing £847 in a 7% stock a man obtained an income of £55. Find the price of the stock if brokerage was ¼ per cent.

EX. 176 (a). Stock Transactions

1. A man sells £650 of a 4% stock at 96 and invests the proceeds in a 4½% stock at 104. How much 4½% stock does he buy?
2. A man sells £2750 of a 3½% stock at 108 and invests the proceeds in 4% stock at 41¼. How much 4% stock does he buy?
3. A man invested £9000 in 7% stock at 120 and sold out when it had fallen to 97½. What did he lose?
4. A man invested £1200 in 3½% stock at 96 and sold out at par. What did he gain?
5. A man sold £22,000 of 4% stock at 92 and invested the proceeds in 5% stock at 110. Find the change of income.

6. A man invested £2300 in a 4½% stock at 115. When the stock had risen to 125, he sold out and re-invested in a 3½% stock at par. Find the change of income.

7. A man invested £450 in 3½% stock at 90. When the stock had risen to 95, he sold out and re-invested in 5% stock at 98. Find the change of income.

8. A man invested £8034 16s., partly in 5% stock at 99 and the rest in 3½% stock at 61. His income from the 5% stock is £200. Find his total income.

9. A man sold £15,000 of 2½% stock at 57 and invested £4100 in 4½% stock at 102½ and the rest of the proceeds in 5% stock at 89. Find the change of income.

10. A man sold £2600 of a 3½% stock at 57 and invested the proceeds in a 4½% stock, thereby increasing his income by £7 10s. Find the price of the 4½% stock.

*11. A man invested £10,000 as follows: he bought £4500 of 3½% stock at 93, and £5500 of 5% stock at 102, and put the remainder on deposit at his bank at 2½ per annum. At the end of 1 year, when the dividends had been paid, he sold the 3½% stock at 97 and the 5% stock at 97. What did he gain or lose on the whole transaction, taking account of the interest received?

*12. A man invested £4004 in a 3½% stock at 89½, and after receiving 1 year’s dividend sold his holding. His total gain was £302 8s.; at what price did he sell the stock?

*13. A man invested in a stock at 140, and when he had received dividends, less 6s. in the £ income tax, the net yield on his investment was 33¾%. What dividend was paid on the stock?

*14. A man invested £5647 10s. in a 5% stock at 93½. After receiving one half-year’s dividend, less tax at 6s. in the £, he sold out at 94. If brokerage is reckoned at ¼ per cent, on each transaction, find how much he gained or lost, taking account of interest received.

*15. A man sold £10,000 of 4½% stock at 87 and invested the proceeds in a stock at 116, thereby increasing his income from this investment by 12½ per cent. What dividend was paid on the second stock?

*16. A man invested a certain sum, partly in 3½% stock at 90 and the rest in 4½% stock at 108, so that he received equal amounts of the two stocks. His total income was then £135. What would have been his income if he had invested the same sum so as to get equal incomes from the two stocks?
CHAPTER XXV

RELATIVE VELOCITY, MISCELLANEOUS EXAMPLES

EX. 178 (a). Relative Velocity

1. A man sculls at 6 m.p.h. in still water. How long will he take to scull 1 mi. upstream and back again, if the stream flows at 2 m.p.h.?

2. A man allowed himself 20 min. to get to a station 2½ mi. away. After cycling for ¾ mi. at 7 m.p.h., he is delayed 5 min. At what speed must he do the rest of the journey to arrive in time?

3. A train 146 m. long travelling at 60 km. per hour passes a train 204 m. long travelling at 40 km. per hour in the opposite direction. How long do they take to pass one another?

4. A train 132 yd. long travelling at 49 m.p.h. passes a man walking by the side of the railway in 6 sec. Find the speed at which the man is walking, and the direction.

5. A goods train 192 yd. long passes completely through a station of length 391 yd. in 33 sec. It meets a passenger train 123 yd. long and passes it completely in 7½ sec. Find the speed of the passenger train in m.p.h., and the time it takes to pass completely through the station.

6. A man walking in a fog caught up a cart going the same way at 3 m.p.h. If the cart was just visible at 55 yd., and if the man was within sight of it for 5 min. altogether (on either side of it), find the rate at which the man was walking.

7. A man walking at 5 m.p.h. passes a milestone at 3 p.m.; and at 3.14 p.m. a cyclist going the same way at 12 m.p.h. passes the same milestone. At what time does he overtake the pedestrian?

8. A train approaching a station at 60 m.p.h. blows a whistle every 5 sec. If sound travels at 1100 ft. per sec., find the intervals between the sounds of the whistles as heard by a man on the station platform.

*9. A passenger on a steamer 264 ft. long notices that the waves travelling in the same direction as the steamer pass the stern at intervals of 21 sec. and that each wave takes 36 sec. to travel the whole length of the vessel. Find the distance between the crests of successive waves.

*10. A steamer leaves New York on Saturday at 10 a.m. (New York time) and travels at 25 m.p.h. to Liverpool, 3060 mi. Another steamer leaves Liverpool the same Saturday at noon (Greenwich time) and travels at 30 m.p.h. to New York by the same route. Greenwich time is 5 hr. later than New York time; find the Greenwich time when they meet.

EX. 180 (a). Miscellaneous Examples

1. A man buys strawberries at 8d. per lb. He sells ¾ of them at 1s. per lb. and the rest at 8 lb. for 4s. Find his gain per cent.

2. A train from Plymouth to London took 123½ min. for the 128½ mi. from Plymouth to Bristol, where it stopped for 3½ min., and then took 59½ min. for the 118½ mi. from Bristol to London. Find, to 3 figures, the average speed (i) from Plymouth to Bristol, (ii) for the whole journey.

3. A metal pipe has a bore of 5 cm. and is 2 mm. thick. Find, to 2 figures, the weight per metre length of the pipe if the specific gravity of the metal is 7-75.

4. A cylindrical bore-hole, diameter 8½ in., was sunk into oil-bearing strata where the oil rises at the rate of 350 ft. per day. How many gallons per day, to 3 figures, does the bore-hole yield? (1 gal. = 277-3 cu. in.)

*5. If the retail price of an article is 25% greater than the wholesale price, by how much per cent. is the wholesale price less than the retail price?

6. The diameter of the Earth is 7926 mi., and the Earth rotates on its axis once every 23 hr. 56 min. What is the speed, to the nearest mile per hour, of a point on the Equator?

7. How much capital have I, if it will amount in 4 years at 3% p.a. simple interest to £3024?

8. Find, to 1 figure, the error per cent. in taking the weight of a rainfall of 1 in., as 100 tons per acre. (1 cu. ft. of water weighs 62-3 lb.)
9. The engine of a car makes 44 revolutions to each revolution of the road wheel, whose diameter is 87-5 cm. Find, to 3 figures, the number of revolutions per minute of the engine when the car is travelling at 40 m.p.h. [1 ft. = 30-48 cm.]

10. At the beginning and end of a quarter, the gas-meter reads 139,400 and 148,800 cu. ft. respectively, and the water-meter reads 38,500 and 52,000 gall. respectively, and the electricity-meter reads 102 and 130 units respectively. Gas costs 5s. per 1000 cu. ft., water costs 1s. 8d. per 1000 gall., electricity costs 8d. per unit. For the gas and electricity there is a discount of 24% per cent. Find the total cost, to the nearest penny.

*11. The diameter of an iron sphere is measured as (3-71 ± 0-01) cm. and its weight as (209-1 ± 0-1) gm. Find the weight of 1 cu. cm. of iron, correct to as many figures as the data justify.

*12. A 6-in. diameter pipe takes water from 2 other pipes of diameters 3 in., 5 in., in which water is running at 2 ft. per sec., 3-6 ft. per sec. respectively. Find the speed of flow in the 6-in. pipe if all are kept full.

13. A cylindrical oil drum is 2½ ft. long and is 3 ft. in diameter, internal measurements. How many gallons, to 3 figures, does it hold? [1 gall. = 277-3 cu. in.]

*14. Two cars are travelling the same way; the faster, going at u m.p.h., starts c mi. behind the slower, going at v m.p.h. When t min. have elapsed, the cars are d mi. apart. Find d in terms of t, c, u, v, (i) when the slower car is in front, (ii) when the faster car is in front.

15. Linen loses 12½% of its length in bleaching; a merchant buys linen, bleaches it, and sells it at 1s. 8d. per yard at a profit of 25%. Find the cost per yard of the unbleached linen.

16. The area of the curved surface of a cylinder, 5 in. high, is ½ sq. ft.; find the area of its base in sq. in., correct to 2 figures.

17. The weight of a hollow cylindrical shaft, inside diameter 32 in., is the same as that of a solid cylindrical shaft of the same height and material and of diameter 16 in. Find the thickness of the hollow shaft in inches, to 3 figures.

*18. A, B, C own a business in which A has invested £20,000, B £3000, and C £2000. Half the yearly profits are divided equally between B and C as their salaries, the other half is divided between A, B, C in proportion to their invested capitals. If one year C receives £870, how much do A and B receive that year?

*19. A wooden cylinder, 1 in. in diameter, 6 in. high, floats with axis vertical in water contained in a cylindrical vessel B, 2 in. in diameter, 6 in. high. The top of A is 1 in. above the top of B and the water is 5 in. deep. If A is pressed down to touch the bottom of B, its axis still being vertical, how much will the water-level rise?

20. A body of specific gravity 1·6 weighs 240 gm. Find its apparent weight if suspended in a liquid of specific gravity 0·72.

21. A man starts from X at 10 a.m. to walk to Y, 12 mi. away, at 4 m.p.h. After 1½ hr. he meets a bus coming from Y to X where it waits 10 min. and then returns to Y; the bus travels at 10 m.p.h. Find from a graph when and where the bus passes the man on its return journey.

22. A man borrows £280 to be repaid in two equal instalments, one after 1 year and the other 1 year later. Reckoning compound interest at 8% per annum, find the amount of each installment, to the nearest £.

23. How many cu. ft. of concrete are required to make an open tank in the shape of a rectangle with semicircular ends with inside dimensions, total length 15 ft., width 5 ft., depth 6 ft., if the walls are all 6 in. thick and the floor is 1 ft. thick. (Answer to 3 figures.)

*24. A lump of metal weighs 5 lb. in air and 4·4 lb. in water. Find (i) its specific gravity, (ii) its weight in a liquid of specific gravity 0·6.

*25. When the tax on tobacco is raised by 1s. 10d. per lb., a retailer finds that he must raise his prices by 1¼d. per oz. to obtain the same percentage of profit as before. What was that percentage?

*26. A cyclist starts at 10 a.m. to ride to a place 11 mi. away. He rides at 12 m.p.h. till one of his tyres bursts. After 10 min. delay, he walks on at 4 m.p.h. and arrives at his destination at 11·35 a.m. Find from a graph how far he had ridden when the burst occurred.
PART III—DRILL EXERCISES. D 21-26

D 21. SQUARE ROOT AND USE OF TABLES

Calculate the square roots of:
1. 18496.  2. 94864.  3. 1·0609.  4. 157609.
5. 0·651249.  6. 30·1401.  7. 0·136161.  8. 0·092416.
9. 3·16.  10. 43·58.  11. 132·4.  12. 266·7.

Calculate, correct to 3 figures, the square roots of:
13. 4·6.  14. 0·46.  15. 0·0108.  16. 0·8124.

Calculate, correct to 3 figures, the values of:
17. \(\sqrt[3]{9}\).  18. \(\sqrt[3]{6}\).  19. \(\sqrt[3]{4}\).  20. \(\frac{\sqrt[2]{2}}{1+\frac{1}{2}}\).

Find to 4 significant figures, as given by 4-figure tables, the values of:
21. 7·2142.  22. 0·3916.  23. 0·0704.  24. \(\sqrt{0·9086}\).
25. \(\sqrt{37·32}\).  26. \(\sqrt[3]{0·03732}\).  27. \(\sqrt[3]{(5·613^2 + 7·096^2)}\).
28. \(\frac{1}{1·26^2}\).  29. \(\frac{1}{0·72^6}\).  30. \(\frac{1}{0·926^2}\).  31. \(\frac{1}{0·904^3}\).

Use tables to evaluate to 3 figures:
32. \(\frac{1}{\sqrt[9]{27}} + \frac{1}{\sqrt[9]{29}}\).  33. \(\frac{1}{\sqrt[7]{98}} + \frac{1}{\sqrt[8]{82}}\).

D 22. LOGARITHMS (EXCLUDING ROOTS)

Evaluate as accurately as your tables permit:
1. 3·28 \times 7·94.  2. 84·5 \times 50·7.  3. 37·4 \times 190.
4. 0·861 \times 0·726.  5. 0·0057 \times 6·28.  6. 117 \times 0·716.
7. 3·472 \times 9·714.  8. 29·07 \times 40·16.  9. 0·8663 \times 3·207.
10. 62·74 \times 0·3005.  11. 0·0713 \times 0·9109.  12. (0·006836)^2.
13. (0·4783)^2.  14. (20·08)^2.  15. (0·8005)^2.
16. 49·7 \div 18·3.  17. 1250 \div 0·777.  18. 37·3 \div 0·863.

\[ D 23. MISCELLANEOUS LOGARITHMS \]

Evaluate as accurately as your tables permit:
1. \(\sqrt[6]{9·4}\).  2. \(\sqrt[5]{14·7}\).  3. \(\sqrt[8]{(0·72)^2}\).  4. (29·3)^{2/5}.
5. \(\sqrt[5]{0·784}\).  6. \(\sqrt[6]{0·0465}\).  7. \(\sqrt[5]{0·821}\).  8. \(\sqrt[5]{0·0763}\).
9. \(\sqrt[5]{0·388}\).  10. \(\sqrt[5]{0·564}\).  11. (0·809)^{2/3}.  12. \(\sqrt[5]{0·0621}\).
13. \(\sqrt[5]{0·2975}\).  14. \(\sqrt[5]{0·0072}\).  15. \(\sqrt[5]{0·5327}\).  16. (0·007356)^{3/4}.
17. \(\sqrt[25]{0·004}\).  18. \(\sqrt[25]{0·472}\).  19. \(\sqrt[25]{0·7072}\).  20. \(\sqrt[25]{0·06084}\).
21. \(\sqrt[25]{(0·0072)^2}\).  22. \(\sqrt[25]{1\div\sqrt[25]{0·873}}\).  23. (14·5)^{3/2}.  24. \(\sqrt[25]{1\div\sqrt[25]{32·87}}\).
25. \(\sqrt[25]{(0·329)^2 \div \sqrt[25]{148·5}}\).  26. \(\sqrt[25]{0·065 \times 964\div 28}\).
D 24. MENSURATION

[Give all answers correct to 3 significant figures]

1. Find circumference of circle, radius 4.3 in.; 7.2 cm.; 100 yd.
2. Find diameter if circumference of circle is 1 ft.; 3.5 m.; 1 mi.
3. Find area of circle if (i) radius is 8.14 in.; (ii) diameter is 6.3 cm.; (iii) circumference is 440 yd.
4. Find area between 2 concentric circles, radii 3.6, 5.4 in.
5. Find volume of circular cylinder, given (i) height 7.5 in., radius 2.4 in.; (ii) height 1.5 cm., diameter 9 cm.; (iii) height 4.8 in., girth 10 in.; (iv) height = girth = 6.283 cm.
6. Find area of curved surface of cylinders in No. 5.
7. Find total area of surface of a closed cylinder, radius 5 cm., height 4 cm.
8. Find weight of cylindrical metal bar, 18 in. long, 1.5 in. diameter, density 4.75 oz. per cu. in.
9. Find area of internal surface of an open cylindrical tank, diameter 10 ft., height 9 ft. The tank is full of water weighing 62.3 lb. per cu. ft.; find in tons the weight of the water.
10. A metal pipe with open ends is 4.5 ft. long; internal and external diameters are 4.2 in., 3.8 in. Find volume of metal.

APPENDIX

28. $\sqrt{523 \times 42.78}$
29. $\sqrt{523 \times 42.78}$
30. $(0.76)\times 0.047$
31. $\sqrt{(17.41)^2 + (0.382)^2}$
32. $2 \times 142$
33. $(0.76)\times 0.047$
34. $0.076 \times \sqrt{924.58}$
35. $0.076 \times \sqrt{924.58}$
36. $(0.76)\times 0.047$
37. $(1.33)^2$
38. $(0.543)^2$
39. $(1.33)^2$
40. $(0.543)^2$

11. Find area of total surface of the pipe in No. 10.
12. A cylindrical well, diameter 8 ft. 6 in., contains 10,000 gallons of water. Find the depth of the water. [1 cu. ft. = 6.3 gal.]
13. Find area of fence 5 ft. 3 in. high enclosing a circular courtyard, diameter 160 ft.
14. A trough, 12 ft. long, is 10 in. wide at the top, 7 in. wide at the bottom, and 6 in. deep. How much water will it hold?
15. The volume of a coil of wire is 7.65 cu. in.; what is its length if the diameter of the cross-section is 0.35 in.?
16. The volume of a coil of wire, 4 ft. long, is 4.87 cu. in. Find the diameter of the cross-section.
17. An oil drum holds 48 gall.; its height is 2 ft.; find its diameter. [1 cu. ft. = 6.3 gal.]
18. Find the volume of a circular cone, given (i) height 9 cm., base-area 10 sq. cm.; (ii) height 3.5 in., base-radius 2.8 in.; (iii) height 12 in., slant height 13 in.
19. Find area of curved surface of circular cone, given (i) height 6 in., base-radius 8 in.; (ii) height 3 in., slant height 5 in.; (iii) base-area 10 sq. cm., volume 15 cu. em.
20. A sector of a circle, radius 4 in., angle of sector 150°, is bent into the shape of a circular cone. Find (i) area of sector; (ii) base-radius of cone; (iii) height of cone.
21. Find volume and surface-area of sphere, radius 5.34 in.
22. Find radius of sphere, volume 1 cu. ft.
23. Find volume of metal in a hollow metal spherical shell, external diameter 5.5 cm., 3 mm. thick.
24. How many spherical shot, diameter 0.2 in., can be made from a lead disc; radius 10 in., $\frac{1}{4}$ in. thick?
25. Find weight of disc in No. 24; sp. gr. of lead 11.4; 1 cu. ft. of water weighs 62.3 lb.
26. Find weight of solid sphere, diameter 5.2 cm., sp. gr. 8.8.
27. A cylindrical steel bar, sp. gr. 7.8; diameter 2.75 cm., weighs 17.82 Kg. Find its length.
D 25. COMPOUND INTEREST

Find, to nearest penny, the amount at compound interest of:

1. £300; 2 yr.; 4%.
2. £250; 2 yr.; 5%.
3. £80; 3 yr.; 5%.
4. £750; 3 yr.; 4%.
5. £500; 3 yr.; 3%.
6. £245; 2 yr.; 6%.
7. £430; 3 yr.; 4%.
8. £3850; 3 yr.; 3%.
9. £271; 3 yr.; 3%.
10. £934; 3 yr.; 3%.
11. £168 10s.; 2 yr.; 4%.
12. £93 15s.; 3 yr.; 5%.
13. £318 4s. 6d.; 2 yr.; 3%.
14. £107 12s. 6d.; 3 yr.; 4%.
15. £20 17s. 5d.; 3 yr.; 4%.
16. £71 9s. 7d.; 3 yr.; 5%.
17. £160; 3 yr.; 2½%.
18. £500; 3 yr.; 4½%.
19. £240; 3 yr.; 3½%.
20. £800; 3 yr.; 2½%.
21. £2500; 3 yr.; 3½%.
22. £1200; 2 yr.; 2½%.
23. £935; 3 yr.; 3½%.
24. £538 10s.; 2 yr.; 4½%.
25. £640; 1½ yr.; 4%.
26. £425; 1½ yr.; 6%.
27. £85 12s.; 2½ yr.; 4½%.
28. £10,000; 2½ yr.; 4½%.
29. £3689 8s. 3d.; 2½ yr.; 4½%.
30. £846 11s. 6d.; 2½ yr.; 3½%.

Find, to nearest penny, the compound interest, payable half-yearly, or:

31. £600; 1½ yr.; 4½%.
32. £760 10s.; 2 yr.; 8%.
33. £317 16s.; 2½ yr.; 6%.
34. £160; 3 yr.; 3½%.
35. £270; 2 yr.; 2½%.
36. £420 15s.; 3 yr.; 6%.

D 26. STOCKS AND SHARES

[Give answers correct to the nearest penny]

Find the amount of stock and the income obtained by investing:

1. £360 in 3½% stock at 80.
2. £480 in 3½% stock at 96.
3. £1440 in 4½% stock at 108.
4. £1260 in 6½% stock at 168.
5. £588 5s. in 3½% stock at 90½.
6. £3995 in 3½% stock at 99½.
7. £2000 in 3½% stock at 88½.
8. £5250 in 3½% stock at 102½.
9. £500 in 4½% stock at 94.
10. £3097 in 5% stock at 105½.

Find the proceeds from the sale of:

11. £1000 of 4½% stock at 82½.
12. £10,000 of 3% stock at 93½.
13. £4000 of 3½% stock at 97½.
14. £1250 of 4½% stock at 106.
15. £755 of 2½% stock at 83.
16. £560 of 3% stock at 91.

Find the income from, and the cost of:

17. £2000 of 3½% stock at 95.
18. £450 of 2½% stock at 72.
19. £1240 of 4½% stock at 125.
20. £280 of 3½% stock at 105.
21. £1125 of 2½% stock at 82.
22. £960 of 4½% stock at 112.

Find the yield, £ s. d. per cent., from investing in:

23. 7½% stock at 154.
24. 2½% stock at 96.
25. 3½% stock at 95½.
26. 4½% stock at 117½.
27. 5½% stock at 136.
28. 3% stock at 93½.

Find the price of the stock if the income derived by investing:

29. £630 in 3½% stock is £27.
30. £240 in 4½% stock is £10.
31. £595 in 3½% stock is £24 10s.
32. £393 in 6½% stock is £24.
33. £821 10s. in 5½% stock is £34 2s.
34. £285 12s. in 2½% stock is £8 10s.

Find the change of income if a man sells:

35. £1000 of 3½% stock at 72 and buys 4% stock at 90.
36. £340 of 6½% stock at 175 and buys 3½% stock at 119.
37. £4700 of 3½% stock at 98 and buys 3½% stock at 94.
38. £3750 of 3½% stock at 96 and buys 2½% stock at 75.
39. £1000 of 3½% stock at 96 and buys 5½% stock at 108.
40. £5000 of 6½% stock at 196 and buys 4½% stock at 141.
41. £6800 of 3½% stock at 85 and invests half the proceeds in 4½% stock at 102 and the other half in 3½% stock at 72.
42. A man invests £18,150 in 3½% at 90½ and later sells at 91 and buys 3½% stock at 97½. Find the change of income.

Find the cost of, and the income obtained from,

43. 150 (10s.) 6½% shares at 12s. 4d.
44. 400 (5s.) 8% shares at 8s. 3d.
45. 750 (2½s.) 4½% shares at 1s. 8d.
APPENDIX

562

Find the number of shares that can be bought, and the net income obtained after deduction of income tax at 4s. in the £, by investing:

46. £250 in (2s.) shares at 2s. 6d., paying 61%.
47. £720 in (6s.) shares at 4s., paying 34%.
48. £840 in (10s.) shares at 48s., paying 35%.

Find the yield obtained by investing money in:

49. (10s.) shares at 12s., paying 51%.
50. (2s.) shares at 1s. 8d., paying 33%.

Find the gain or loss if a man buys:

51. 250 (1£) shares at 2£ and sells them at 2£.
52. 400 (5s.) shares at 3s. 6d. and sells them at 2s. 9d.
53. 900 (2s.) shares at 2s. 6d. and sells them at 3s. 1d.
54. 135 (1£) shares at 2£ and sells them at 3£.

REVISION PAPERS

PAPERS A 49–56 (Ch. I–XX)

Paper A 49

1. (i) Find the integer whose cube root lies between 4·19 and 4·2.
(ii) Use tables to find, correct to 3 figures, \( \sqrt[3]{375} \).

2. Use logarithms to evaluate, correct to 3 figures:
   (i) \( \sqrt{\frac{997 \times 0.43}{2271 \times 1.685}} \);
   (ii) \( (0.814)^{0.6} \).

3. A piece of cotton which is wound exactly 250 times round a reel measures 92 ft. 3 in. Find the diameter of the reel, correct to 1½ in.

4. Find, correct to the nearest penny, the simple interest on £760 for 80 days at 3½% p.a. [Take 1 year = 365 days.]

5. Find in acres, correct to 3 figures, the area of a triangular plot of ground, if the lengths of its sides are 945 yd., 1025 yd., and 1190 yd. [Use the expression, \( \sqrt{s(s-a)(s-b)(s-c)} \).]

6. A gallon measure is in the form of a cylinder. The internal diameter must not differ by more than 5% from the depth. Find the greatest and least possible diameters, to the nearest 1/50 in. [1 gal. = 277·3 cu. in.]

Paper A 50

1. (i) How many equal payments of 15s. 6d. each can be made out of £20, and how much will be left over?
(ii) Calculate the square root of 0·034969.

2. Use logarithms to evaluate, correct to 3 figures:
   (i) \( \sqrt[3]{\frac{997 \times 0.43}{2271 \times 1.685}} \);
   (ii) \( (0.814)^{0.6} \).

3. A piece of cotton which is wound exactly 250 times round a reel measures 92 ft. 3 in. Find the diameter of the reel, correct to 1½ in.

4. Find, correct to the nearest penny, the simple interest on £760 for 80 days at 3½% p.a. [Take 1 year = 365 days.]

5. Find in acres, correct to 3 figures, the area of a triangular plot of ground, if the lengths of its sides are 945 yd., 1025 yd., and 1190 yd. [Use the expression, \( \sqrt{s(s-a)(s-b)(s-c)} \).]

6. A gallon measure is in the form of a cylinder. The internal diameter must not differ by more than 5% from the depth. Find the greatest and least possible diameters, to the nearest 1/50 in. [1 gal. = 277·3 cu. in.]

Paper A 51

1. (i) Divide 435 tons 6 cwt. 84 lb. by 282.
(ii) Simplify \( \frac{\frac{3}{4} - \frac{2}{3} + \frac{1}{8}}{\frac{1}{4} + \frac{3}{5} + \frac{1}{8}} \).

2. Use logarithms to evaluate, correct to 3 figures:
   (i) \( (0.2079)^{0.3} \times \sqrt{(41.36)} \);
   (ii) \( 10^{0.8} \times 10^{0.9} \).

3. In 1930, 7305 persons were killed and 177,895 were injured in road accidents in Great Britain; in 1931, 6691 persons were killed and 202,119 were injured. Find, correct to 3 figures, (i) the decrease per cent. in the number killed, (ii) the increase per cent. in the number injured.

4. A swimming-bath can be emptied by 3 pipes in 3½ hr.; one pipe would take 7½ hr. by itself, another 11½ hr. by itself; how long would the third take by itself?
5. The area of the cross-section of a cylindrical tank is \(1\frac{1}{3}\) sq. m., and the tank contains 1200 litres of water. Find the depth of the water in cm. If, owing to a fall of temperature, ice forms on the surface to a depth of 5 cm, find, to the nearest mm., the depth of the water below the ice. [In freezing, any volume of water expands in the ratio 1:087 : 1.]

6. A solid cylinder 7\(\frac{1}{2}\) in. long is surrounded by a solid cone of height 4 in., the base diameter of the cone and the diameter of the cylinder being each 6 in. Find, correct to 3 figures, (i) the total surface of the solid in sq. in.; (ii) the weight in lb. of the solid if the material weighs 475 lb. per cu. ft.

Paper A 52

1. (i) Multiply \(3s\) 10\(\frac{3}{4}\) d. by 4000.
   (ii) Find without using tables the square root of \(3\frac{1}{4}\), correct to 2 places of decimals.

2. Use logarithms to evaluate, correct to 3 figures:
   (i) \(\sqrt[3]{45.08 \times \frac{320 \times 114}{62.3 \times (1 + 0.45)}}\)
   (ii) \(613(1.2)^{1.3} - 1\) \(\times\) 8917 given log 1.2 = 0.0791812.

3. The area of Manitoba is 47,250,000 ac. If 36\(\frac{2}{3}\)% is under wheat, and if the yield is 9.48 bush. per acre, find, correct to 2 figures, the number of bushels of wheat produced.

4. The simple interest on £280 for 1\(\frac{1}{4}\) years is 14 guineas; find the rate \% p.a.

5. The weight of a coil of wire is 36\(\frac{2}{3}\) gm. and its length is 6\(\frac{1}{2}\) m.; the wire weighs 7.46 gm. per c.c.; find, correct to 2 figures, the diameter of the wire in mm.

6. A hollow spherical steel shell has an external diameter of 4 in. and is 0\(\frac{1}{2}\) in. thick. Find its weight in oz., correct to 3 figures, if the steel weighs 485 lb. per cu. ft.

Paper A 53

1. (i) Given 1 yd. = 0.9144 m., express 2 ft. 4\(\frac{3}{4}\) in. in metres, correct to 3 figures.
   (ii) A man saves 28\% of his income; this leaves him £756 a year to spend. What is his income?

2. Use logarithms to evaluate, correct to 3 figures:
   (i) \(\sqrt[1.3]{\frac{a(s-a)}{(b+c)-(s-b)}}\) when \(a = 351\), \(b = 41,\) \(c = 320,\) \(s = \frac{1}{2}(a+b+c).
   (ii) the value of \(x\) if \(\log x = 2 - \log \sqrt[3]{3}.

3. Coal was quoted for delivery in France at 47s. 6d. per ton. Find, correct to 3 figures, the equivalent quotation in francs per quintal, taking \(1 = 83.4\) francs and 1 quintal = 100 lb.

4. If the rate of interest on a loan is raised from 4\(\frac{3}{4}\)% p.a. to 5\(\frac{1}{2}\)% p.a., the annual interest is increased by £2 13s. 6d. Find the amount of the loan.

5. Find in lb., correct to 3 figures, the weight of an open iron pipe 10 ft. long, external diameter 7 in., internal diameter 5\(\frac{1}{2}\) in., given that 1 cu. ft. of iron weighs 452 lb.

6. 12 balls, each 1\(\frac{1}{2}\) in. in diameter, are packed in a box 6 in. by 4\(\frac{3}{4}\) in. by 1\(\frac{1}{2}\) in. internal measurements. Find, correct to 3 figures, what percentage of the contents of the box is air-space.

Paper A 54

1. (i) Find the value, correct to 4 places of decimals, of \(\frac{1}{4} + \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} + \ldots\)
   (ii) Find 6\(\frac{3}{4}\) per cent. of 2 tons 10 cwt.

2. Use logarithms to evaluate, correct to 3 figures:
   (i) \(\frac{0.6285 \times \sqrt[3]{0.68}}{6\times 7}\);
   (ii) \((0.042)^{-0.8}\).

3. A train leaves Waterloo at 10.50 a.m. and reaches Tressmer (232 mi.) at 5.57 p.m. Allowing 30 min. for stops, find, to the nearest mile per hour, the average speed of actual running.

4. After deducting 35\% from the price of an article, the price becomes £74 15s. What was the original price? What would the price have become if the original price had been increased in the ratio 5 : 7?

5. There are 47 teeth on the gear-wheel of a bicycle and 19 on the small cog-wheel fixed on the axle of the back wheel. The diameter of the back wheel is 28\(\frac{1}{2}\) in. How many revolutions, correct to 3 figures, must the pedals make when the bicycle is ridden 1 mi.?
APPENDIX

6. The angle of a sector of a circle is 216°, and the radius is 5 in.; find the area of the sector, correct to \( \frac{1}{8} \) sq. in.

If the sector is bent to form the curved surface of a circular cone, find the height of the cone.

Paper A 55

1. (i) Express \( 8\frac{3}{4} \) of £1,572 in s. d., correct to nearest penny.

(ii) Simplify \( \frac{10}{5} \times \frac{11}{3} + \frac{4}{3} \).

\( \frac{8}{5} \times \frac{11}{5} + \frac{4}{3} = \frac{11}{3} \)

2. Use logarithms to find, correct to 3 figures:

(i) the value of \( \sqrt{r} \) if \( 2r = 7 \);

(ii) the value of \( n \) if \( 153(1.2)^n = 300 \).

3. Exports of manufactured goods from the United Kingdom decreased in value from £440,042,000 in 1930 to £292,029,000 in 1931. Find, correct to 3 figures, the decrease per cent.

4. A swimming-bath, 60 ft. long, 24 ft. wide, is 9 ft. deep at one end and 3 ft. deep at the other, the floor sloping uniformly. Find the number of gallons the bath contains when (i) it is full, (ii) just half the floor is covered. [Take 1 cu. ft. = 6\( \frac{1}{2} \) gallons.]

5. A man used to save 28% of his income; but when his income increased, he increased his expenditure by \( 12\frac{1}{2} \)%, and then saved 25% of his income. Find the percentage increase in his income.

6. The height of the conical bowl of a winglass is equal to the diameter of the rim (internal measurements). If a pint of wine is just sufficient to fill the glass 9 times, find the height in inches, correct to 3 figures. [1 gallon = 277.4 cu. in.]

Paper A 56

1. (i) Multiply £2 17s. 9d. by 2\( \frac{1}{2} \).

(ii) Find the fraction which, when increased by 20%, becomes \( \frac{4}{5} \).

2. (i) Evaluate, correct to 3 figures, \( (0.08072)^{-2} \).

(ii) From the formula, \( 4m^2t = gr^4 \), find the value of \( l \), correct to 2 figures, if \( t = 1.20 \), \( g = 981 \), \( \pi = 3.142 \).

APPENDIX

3. An analysis of 0.8572 gm. of a Lee-Enfield bullet casing showed that it contained 0.6695 gm. of copper, 0.1835 gm. of nickel, the rest being iron. Find, correct to 3 figures, (i) the percentage of copper in the casing, (ii) the ratio of copper to nickel in the form \( n : 1 \).

4. An article is marked for sale at £5 10s., off which the shopkeeper took \( 2\frac{1}{2} % \) for cash. His net profit was then \( 10\% \); what did the shopkeeper pay for the article?

5. Find, correct to \( \frac{1}{5} \) in., the length of the edge of a cubical cistern which contains as much as a cistern 5 ft. 3 in. long, 4 ft. 6 in. wide, 3 ft. 9 in. high.

6. A cylinder, 9 in. high, 6 in. in diameter, external measurements, is fitted with a hemispherical lid. Find the area of the total external surface, correct to the nearest sq. in. If the sides, base and lid are all \( \frac{1}{3} \) in. thick, find the air-space inside the closed vessel, to the nearest cu. in.

PAPERS A 57-64 (Ch. I-XXV)

Paper A 57

1. (i) Taking £1 = 73-625 francs, express 23s. 10d. in francs, correct to 4 figures.

(ii) By how much per cent. does a price of 4\( \frac{1}{2} \)d. per dozen exceed a price of 3s. per 100?

2. Evaluate, correct to 3 figures:

\[ \frac{0.6792}{\sqrt[3]{0.8455}} \]

\[ \frac{114}{5} \times \frac{1}{2} \]

3. A manufacturer's catalogued prices are 25% above cost, but buyers are allowed a discount of 8% off these prices. What is the manufacturer's net gain per cent.?

4. A cylinder, 3\( \frac{1}{2} \) cm. in diameter, 12 cm. high, internal measurements, is filled with mercury, specific gravity 13-6. Find in kg., correct to 3 figures, the weight of the mercury.

5. Find, correct to the nearest penny, the compound interest on £268 for 1\( \frac{1}{3} \) yr. at 5\( \frac{1}{4} \)% p.a., payable half-yearly.

6. Find the income obtained from investing £3000 in \( 5\frac{1}{2} \% \) stock at 84. What capital profit is made if the stock is sold afterwards at \( 87\frac{1}{2} \% \)?
APPENDIX

Paper A 58

1. (i) By how much per cent. does 2:1 exceed 1:5, and by how much per cent. is 0:4 less than 0:45?
   (ii) Find, without using tables, the square root of 0:7, correct to 2 figures.

2. Find, correct to 3 figures:
   (i) the value of $t(1+t)$ if $t^2 = 5$
   (ii) the value of $n$ if $160(1.045)^n = 183$.

3. 41 cu. ft. of teak weigh 1 ton 15 lb.; find the specific gravity of teak, correct to 2 figures. [1 cu. ft. of water weighs 62.5 lb.]

4. A solid metal cylinder, 17/2 cm. high, 5-7 cm. in diameter, is melted down and recast in the form of a cube surmounted by a right pyramid whose height equals the edge of the cube. Find the edge of the cube, correct to 3 figures.

5. A merchant bought a 200-litre cask of wine at 15 fr. 10 c. per litre, and also paid 40 fr. for the cask. Carriage and import duties amounted to 36. 9d. per gallon; the rate of exchange was 81:35 fr. to the £. Find the lowest selling price per gallon which would ensure a profit of 35% on his outlay. Answer to a shilling. [Take 1 litre = 1.761 pt.]

6. A man sells £450 Corporation 43/4% stock at 1023/8 and invests the proceeds in 34/5% stock at 68 1/2. Find the increase in his income.

Paper A 59

1. (i) How much money is obtained from a rate of 23/4d. in the £ on a rateable value of £76,400?
   (ii) How long does a train 180 yd. long, travelling at 44 mi. an hour, take to pass completely a train 150 yd. long, travelling at 36 mi. an hour in the opposite direction?

2. (i) Find, correct to 3 figures, the value of $x$ if $x^2 + 17.39^2 = 21.42^2$.
   (ii) Find, correct to 3 figures, the area in acres of a circular enclosure whose circumference is 400 m. long. [Take 1 m. = 1.094 yd.]

3. In 1930, 3580 tons of ore were taken from a mine and yielded 8645 lb. of tin. In 1931 the output of ore was increased by 15% and the yield of tin per ton of ore was increased by 12%. Find in lb., correct to 3 figures, the amount of tin obtained from the mine in 1931.

4. Find, correct to the nearest penny, the compound interest on £460 for 1½ yr. at 5½% p.a., payable half-yearly.

5. A hemispherical bowl has an internal radius of 11.5 cm. and external radius of 12.5 cm., and is made of metal of specific gravity 7.45. It is filled with a liquid of specific gravity 1.15. Find the total weight of the bowl and its contents in kg., correct to 2 figures.

6. A man bought 350 (£1) shares at 34s. 6d. If a dividend of 10% was paid on the shares, find (i) the dividend he received after an income tax of 4s. 6d. in the £ had been deducted; (ii) the percentage yield on his investment, correct to 1½%, taking account of the deduction.

Paper A 60

1. (i) Taking 1 mi. = 1-609343 km., express 5 fur. 150 yd. in km., correct to 4 places of decimals.
   (ii) The ratio of A to B equals 1½ : 1½, find by how much per cent. B exceeds A.

2. Evaluate, correct to 3 figures:
   (i) $\frac{\log (0.0707 \times 48)}{2[(0.2416) \times 10^{0.04}]}$;
   (ii) $\frac{11 + \log 37.24}{\log (\sqrt{3})}$.

3. The number N of gallons of water discharged per minute from a reservoir through a pipe of diameter D in. is given in the following table:

<table>
<thead>
<tr>
<th>D</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>5</td>
<td>37</td>
<td>117</td>
<td>262</td>
<td>364</td>
<td>484</td>
<td>624</td>
<td>805</td>
</tr>
</tbody>
</table>

Represent these facts by a graph and find (i) the number of gallons discharged per minute from a pipe of diameter 5 in.; (ii) the diameter in cm. of a pipe which would discharge 1½ cu. m. per minute. [Take 1 cu. m. = 220 gall., and 1 in. = 2.54 cm.]
APPENDIX

4. A sells an article to B at a profit of 32%; B sells it to C at a profit of 25%; C sells it to D at a profit of 16%. What would have been A's profit per cent. if he had sold the article direct to D for the price D actually paid?

5. A body weighs 76.3 gm. in air and 57.6 gm. in water. Find its specific gravity, correct to 2 figures.

6. A man invests £600 in a 3% stock at 95$\frac{3}{4}$. Later on he sells it at 95$\frac{1}{4}$ and reinvests the proceeds in 2$\frac{1}{2}$% stock at 85$\frac{3}{4}$. Reckoning brokerage at $\frac{3}{4}$ per cent. on each transaction, find the change in income.

Paper A 61

1. (i) Find, correct to the nearest penny, the cost of 2 tons 13 cwt. 3 qr. at £1 16s. 4d. per ton.
   (ii) Find the principal, if the simple interest for 3$\frac{3}{4}$ yr. at 6% p.a. is £28 2s. 6d.

2. (i) If $2mr=25r$ and if $f^2=r^2+16$, find, correct to 3 figures, the value of $\sqrt{r}$. Interpret the data and the result geometrically.
   (ii) Evaluate, correct to 3 figures, $(\frac{2}{3})^{-0.7}$.

3. A bottle when empty weighs 14$\frac{3}{4}$ oz. and when full of water weighs 37$\frac{1}{4}$ oz., and when full of another liquid 31$\frac{3}{4}$ oz. Find the specific gravity of the liquid.

4. A tradesman retails an article for 6d. and makes a profit of 20 per cent. When he has sold $\frac{3}{4}$ of his stock, the retail price is advanced to 7$\frac{1}{4}$d., and he sells the rest of his stock at this price. What profit per cent. does he make on the whole transaction?

5. The walls of a room, 20 ft. long, 15 ft. wide, 13 ft. high, are to be papered all over except for a panel on each of the four walls, 5$\frac{1}{2}$ ft. wide, 12 ft. high. If the paper is 21 in. wide, find the total length used, and the cost of the paper if it is sold only in rolls of 12 yd. each at 8s. 6d. per roll.

6. A man invests half of his capital in 6% Housing Bonds at 114 and the other half in a South American Railway 3$\frac{1}{2}$% stock at 23$\frac{1}{4}$. What percentage does he obtain on the money he invests (i) if both dividends are paid in full, (ii) if the Housing dividend is paid in full, but the Railway dividend is only paid at the rate of 1$\frac{1}{2}$%?

Paper A 62

1. (i) Express 0.32846 ton in cwt., lb., oz., correct to the nearest oz.
   (ii) The numbers of university students in England, Scotland and Wales in 1930 were respectively 34,960, 11,650, 3070. Find, correct to 3 figures, what percentage of the total were at an English university.

2. (i) If $d=\frac{2}{3}(6V+n)$, find the value of $d$, correct to 3 figures, if $V=4$.
   (ii) Find, correct to 4 places of decimals,
   $\frac{1}{5} \cdot \frac{2}{5} \cdot \frac{2}{5} \cdot \frac{3}{5} \cdot \frac{6}{5} \cdot \frac{7}{5} + \ldots$

3. A sells an article to B at a profit of 20 per cent.; B sells it to C at a profit of 15 per cent. C paid 15s. 10d. more than it cost A. What profit did A make?

4. At what rate per cent. per annum compound interest does £520 amount to £573 6s. in 2 yr.?

5. In what ratio must tea costing 1s. 7$\frac{1}{4}$d. per lb. be mixed with tea costing 2s. 10$\frac{1}{4}$d. per lb. so that a profit of 10% is made by selling the mixture at 2s. 9d. per lb.?

6. By selling a certain amount of 2$\frac{1}{2}$% stock at 55$\frac{1}{4}$ and investing the proceeds in 6% stock at 87$\frac{3}{4}$, a man increases his income by £100 a year, brokerage being $\frac{3}{4}$ per cent. on each transaction. How much stock did he sell?

Paper A 63

1. (i) Express a price of 12,500 francs per hectare in £ per acre, correct to 3 figures, taking £1 = 79.65 fr. [1 are = 119.6 sq. yd.]
   (ii) The sides of a rectangle are measured as 12.4 cm., 10.6 cm., correct to the nearest mm. Between what limits does the area of the rectangle lie, to the nearest sq. cm.

2. Evaluate, correct to 3 figures,
   \[
   \frac{(11.7)^2 \times 0.000624 \times \sqrt{2}}{0.101(1 + a \times b)}
   \]
   where $a=46\frac{1}{2}$, $b=5\frac{1}{2}$.

3. A cuboid of lead, specific gravity 11.4, measures 10 cm. by 6 cm. by 4 cm. It is melted down and made into spherical pellets, each of radius 5 mm. Find, correct to 3 figures, how many pellets can be made and the weight of each pellet in grams.
APPENDIX

4. A tradesman fixes the selling price of goods at 30% above cost. He sells half his stock at this price, one-quarter of his stock at a discount of 15% on this price, and the rest at a discount of 30% on it. What is his gain per cent. on the whole transaction?

5. A's income is 60% of B's income, and A's expenditure is 70% of B's expenditure. If A's income is 75% of B's expenditure, find the ratio of A's savings to B's savings.

6. A person invests £3500 in 3% stock at 105 and £2389 10s. in 2½% stock at 59. If income tax is deducted at the rate of 5s. in the £, find the net percentage, to the nearest ½%, he obtains on the money invested.

Paper A 64

1. (i) Express £1 14s. 6d. as a percentage of £3 7s. 6d., correct to 3 figures.
   (ii) Find the value of \( x \), correct to 3 figures, if \[
   \frac{1}{x} = 1097 \left( \frac{7}{10} - \frac{1}{\sqrt{11}} \right).
   \]

2. Every afternoon at 1 o'clock a man walking into a town meets a bus, travelling at 15 m.p.h. in the opposite direction, at a certain corner. One day the man starts later than usual and the bus passes him at 3 min. past 1 o'clock. If the bus was travelling to time, and if the man always walks at 3 mi. per hour, how late was the man in starting?

3. A thermometer has a straight cylindrical stem of internal diameter \( \frac{1}{4} \) in., and a bulb at the end which is a sphere of internal diameter \( \frac{1}{2} \) in., which is filled with mercury. For each degree rise in temperature, this mercury expands by \( \frac{8}{5} \) of its volume. Find, to the nearest \( \frac{1}{100} \) in., how much the mercury rises in the stem for a rise of 10 degrees in temperature.

4. A manufacturer sells goods to a retailer at a profit of 35%, but the retailer goes bankrupt and pays at the rate of 11s. 3d. in the £. Find the manufacturer's loss per cent. At what rate in the £ must the retailer be able to pay his debts if the manufacturer is not to suffer any loss? Answer to a farthing.

5. A machine costs £245 and each year its value depreciates by 8% of its value at the beginning of that year. Find, correct to the nearest £, its value after 5 yr.

6. How much stock at 97½ must I sell out to realise £1300, brokerage \( \frac{1}{2} \)%? If by investing this £1300 in 4½% stock at 101\( \frac{3}{4} \), brokerage \( \frac{1}{4} \)% I increase my income by £4 3s. 4d., find the rate of interest on the first stock.
ANSWERS TO PART I

EXERCISE 1

Page 1
15. 7.  16. 7.  17. 4.  18. 5.  19. 7.  20. 5.

Page 2
1. 1st, 3rd, 4th, 6th.  2. 1st, 3rd, 4th.  8. 2nd, 3rd, 5th.
4. 1st, 2nd, 4th, 5th.  5. 1st, 2nd, 4th, 6th.  6. 1st, 2nd, 4th, 5th, 7th.
7. 1st, 3rd, 4th, 6th.  8. 1st, 3rd.  9. 1st, 3rd.  10. 1st, 2nd, 4th.
11. 2, 5, 8; 1, 4, 7; 0, 3, 6, 9.  12. 2, 6; 1, 3, 5, 7, 9; 0, 2, 4, 6, 8.
13. 3; 2; 0, 9.  14. 8; 3; 3.  15. 2, 7; 0; 00, 25, 50, 75.

EXERCISE 3

Page 4
1. 2ª, 3.  2. 2ª.  3. 57.  4. Prime.
5. 3, 5ª.  6. 5ª.  7. Prime.  8. 3, 37.
13. Prime.  14. 3ª, 23.  15. 2ª, 7ª.  16. 2ª, 3, 5ª.
17. 3ª, 5, 7.  18. 2, 3ª, 73.  19. 2, 2ª, 5ª.  20. 2ª, 3, 11.
21. 2ª, 3ª, 7ª, 22. 2ª, 3ª.  23. 3ª, 7, 11.  24. 2, 13, 29.
25. 3ª, 7, 31.  26. 11, 101.  27. 2ª, 3ª, 11, 13.  28. 2ª, 3ª, 7ª.
29. 7ª, 11.  30. 2ª, 13, 29.

EXERCISE 4

Page 5
1. 14, 24.  2. 21, 30.  3. 5, 24.  4. 5, 30.  5. 18.  6. 200.
18. 12.  14. 100.  15. 24.  16. 4.  17. 80.  18. 2400.
19. 60.  20. 18.  21. 8.  22. 20.  23. 15.  24. 55.
26. 49.  28. 21.  27. 28.  28. 36.  29. 10.  30. 3.
31. 10.  32. 3.  33. 4.  34. 4.  35. 24.  36. 25.  37. 3.

EXERCISE 5

Page 6
1. 3ª.  2. 5ª.  3. aª.  4. 2ª.  5. 3ª.
6. 5ª.  7. 6ª.  8. 2ª.  9. 3ª.  10. 5ª.
11. 7ª.  12. 2ª.  13. 3ª, 14. 5ª.  15. 10ª.
16. 7.  17. 2ª, 18. 3ª, 5ª.  19. 5ª, 7ª.  20. aª, bª.
21. 2ª, 3.  22. 3ª, 5.  23. 2ª, 3ª.  24. 2ª, 3ª.  25. 2ª, 3ª.
26. 27. 3ª.  27. 2ª, 5ª.  28. 2ª, 3ª.  29. 2ª, 3ª.  30. 2ª, 3ª, 7ª.
Page 7
EXERCISE 6
1. 28, 36, 54; 58, 78, 112. 2. 36; 58, 78; 59, 72, 119.
3. 28; 48, 36. 4. 28, 58, 78, 28, 52, 78.
5. 28; 4, 34, 52, 7. 6. 6, 40, 8. 62, 9, 12.
10. 14, 27, 12, 42, 18, 54, 14, 63, 15, 75.
16. 98, 17, 112, 18, 224, 19, 231, 20, 14, 21, 28.
22. 45, 23, 60, 24, 220, 25, 6, 26, 3, 45.
27. 77, 4312, 29, 6, 42, 29, 154, 924, 30, 12, 12, 31, 18, 3000.
32. 75, 30, 33, 252, 84, 34, 24, 3, 5, 35, 29, 47, 36, 25, 85.
37. 28, 3.

Page 9
EXERCISE 7
1. 25, 26, 3, 2. 4, 2, 5, 7, 6, 9, 7, 6, 8, 1, 9, 20, 10, 1.
11. 7, 12, 1, 13, 24, 32, 14, 23, 15, 28, 3, 5.
16. 29, 52, 17, 29, 13, 8, 5.

Page 10
EXERCISE 8
1. 6, 2, 21, 3, 35, 4, 63, 5, 5, 6, 6, 13, 7, 54, 8, 42, 9, 55, 10, 112, 11, 63, 12, 252.
13. 35, 14, 12, 15, 21, 16, 19, 17, 31, 18, 19.

Page 11
EXERCISE 9
1. 24, 2, 30, 3, 20, 4, 40, 5, 12, 6, 18, 7, 14, 8, 36, 9, 60, 10, 24, 11, 12, 12, 12, 13, 24, 14, 120.
15. 28, 35, 16, 28, 34, 17, 24, 32, 18, 24, 34, 19, 24, 38.

Page 12
EXERCISE 10
1. 25, 3, 2, 22, 5, 32, 5, 5, 22, 5, 34, 7, 11, 6, 2, 3, 5, 8, 29, 32, 5.
5. 38, 34, 78, 11, 6, 23, 5, 72, 9, 24, 3, 7.
10. 28, 34, 24, 5, 13, 17, 28, 34, 5, 24, 34, 5, 12, 25, 34, 5.
14, 28, 34, 7, 11, 15, 32, 13, 22, 34, 71, 18, 32, 11, 13.
19, 28, 34, 5, 22, 28, 34, 7, 11, 17.
22, 28, 34, 7, 11, 17.

Page 13
EXERCISE 11
1. 72, 2, 2890, 3, 104, 4, 7, 6, 5, 40.
6. 90, 7, 5, 36, 8, 90, 11, 21.
12. 90, 12, 420, 20, 20, 14, 30, 15, 40, 16.
17. 2, 1, 36, 26, 20, 18, 3, 13, 26, 17, 2, 26, 15.
20. 1, 2, 36, 26, 20, 21, 30, 15, 40, 17.
22, 2, 28, 22, 2, 4.

Page 14
EXERCISE 12
2. 15, 3, 24, 4, 36, 5, 14, 6, 144, 28, 9.
7, 28, 9, 6, 9, 25, 9, 10, 11, 9.
12, 10 sq. in. 13, 36, 9 in. 14, 21, 9 in. 15, 16 sq. in.
16, 64, 9 in. 17, 100 sq. ft. 18, 20 sq. ft. 19, 80 sq. ft.
20, 49 sq. ft. 21, 81 sq. ft. 22, 6 sq. ft. 23, 35 sq. ft.
24, 32 sq. ch. 25, 80 sq. ft. 26, 364 sq. ft. 27, 20 sq. ch.

Page 15
EXERCISE 13
1. 120 sq. in. 2, 6000 sq. yd. 3, 150 sq. ft. 4, 225 sq. yd.
5, 221 sq. ft. 6, 110 sq. yd. 7, 100 sq. ft. 8, 360 sq. in.
9, 360 sq. in. 10, 1250 sq. ft. 11, 24 sq. ft. 12, 360 sq. ft.
13, 240 sq. in. 14, 750 sq. ft. 15, 18 sq. ft. 16, 4840 sq. yd.
17, 6 in. 2, 4 in. 18, 9 yd., 42 yd. 19, 5 sq. ft. 20, 6 yd.
21, 2 ft. 2. 22, 3 sq. ft. 23, 9 ft. 24, 34. 900 sq. yd.
25, 5 in. 35 sq. in. 26, 286 yd. 27, 80 ft.

Page 16
EXERCISE 14
1. 2 ac. 2, 2 ac. 3, 160 ac. 4, 20, 100, 1 sq. ch.
5, 24200, 484000 sq. yd. 6, 1250, 320, ac. 7, 242, 684, yd.
8, 14 sq. in. 9, 49 sq. in. 10, 66 sq. in. 11, 50 sq. in.
12, 44 sq. ft. 13, 50 sq. ft. 14, 248 sq. ft. 15, 94 sq. ft.
16, 105 sq. ft. 17, 72 sq. ft. 18, 162 sq. ft. 19, 40 sq. yd.
20, 360 sq. in. 21, 348 sq. yd. 22, 176 sq. ft. 720 sq. ft.
23, 512 sq. ft. 24, 350 sq. ft. 25, 740 sq. ft. 26, 184 sq. ft.
27, 375 sq. sq. 28, 396 sq. in. 29, 160 sq. ft. 30, 468 sq. in.
31, 580 sq. in. 32, 79 sq. ft. 33, 2h(+/6) sq. ft.
34, (2h+/2h+b+b) sq. ft.

Page 17
EXERCISE 15
1. 280, 2, 288, 3, 27, 4, 13824.
5, 252, 6, 1080, 7, 296 ft. 8, 24 ft. 4, 9.
9, 6 cu. ft. 10, 240 cu. in. 11, 216; 8, 12, 8 in. 20 in. 16 in.
13, 1 yd.; 2 yd.; 9 yd. 14, 144 cu. ft. 15, 27 in.
16, 7200 cu. ft. 17, 1728. 18, 2 ft.
### EXERCISE 16

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 9, 7, 5 in.; 315 cu. in.</td>
<td>2. 7, 4, 9 in.; 252 cu. in.</td>
</tr>
<tr>
<td>3. 894 cu. in.</td>
<td>4. 10, 16 cu. in.</td>
</tr>
<tr>
<td>5. 300 cu. in.</td>
<td>6. 172 cu. in.</td>
</tr>
<tr>
<td>7. 32 sq. in.; 320 cu. in.</td>
<td>8. 264 cu. in.</td>
</tr>
</tbody>
</table>

### EXERCISE 17

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 6s., 10s.</td>
<td>2. 2s., 3s.</td>
</tr>
<tr>
<td>3. 480, 1200 yd.</td>
<td>4. 110 yd.</td>
</tr>
<tr>
<td>5. £1.</td>
<td>6. 1s. 3d.</td>
</tr>
<tr>
<td>7. 50.</td>
<td>8. 50.</td>
</tr>
<tr>
<td>9. 180 yd.</td>
<td>10. 15.</td>
</tr>
<tr>
<td>11. 25 oz.</td>
<td>12. 90 min.</td>
</tr>
<tr>
<td>13. 125 mi.</td>
<td>14. 30 mi.</td>
</tr>
<tr>
<td>15. 2s. 6d.</td>
<td>16. 17.</td>
</tr>
<tr>
<td>17. 18.</td>
<td>18. 30 lb.</td>
</tr>
</tbody>
</table>

### EXERCISE 18

<table>
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<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 1.4d.</td>
<td>2. 75 min.</td>
</tr>
<tr>
<td>3. 120 mi.</td>
<td>4. 45.</td>
</tr>
<tr>
<td>5. 7s. 9d.</td>
<td>6. 15s.</td>
</tr>
<tr>
<td>7. £2. 5s.</td>
<td>8. 9s. 6d.</td>
</tr>
<tr>
<td>9. 20.</td>
<td>10. 3, 9 mi.</td>
</tr>
<tr>
<td>11. 2, 10 in.</td>
<td>12. 18, 10 yd.</td>
</tr>
<tr>
<td>15. 2 oz.</td>
<td>16. 91 lb.</td>
</tr>
<tr>
<td>17. 23 sq. ft.</td>
<td>18. 4 sq. mi.</td>
</tr>
</tbody>
</table>

### EXERCISE 19

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 24, 8, 2 hr.</td>
<td>2. 48, 24, 16 min.</td>
</tr>
<tr>
<td>3. No answer.</td>
<td>4. 8 hr.; 80 min.; no answer.</td>
</tr>
<tr>
<td>5. No answer.</td>
<td>6. 9 days.</td>
</tr>
<tr>
<td>7. No answer.</td>
<td>8. 45.</td>
</tr>
<tr>
<td>9. No answer.</td>
<td>10. 48 days.</td>
</tr>
<tr>
<td>11. No answer.</td>
<td>12. 4 wk.</td>
</tr>
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</table>

### EXERCISE 20

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 30.</td>
<td>2. 5 hr.</td>
</tr>
<tr>
<td>5. No answer.</td>
<td>3. 9 days.</td>
</tr>
<tr>
<td>6. 9 days.</td>
<td>4. 40 days.</td>
</tr>
<tr>
<td>7. 45.</td>
<td>8. 27.</td>
</tr>
<tr>
<td>9. No answer.</td>
<td>10. 42 min.</td>
</tr>
<tr>
<td>11. 4.</td>
<td>12. 48 days.</td>
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### EXERCISE 21

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<th>Question</th>
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</tr>
</thead>
<tbody>
<tr>
<td>1. 10s.</td>
<td>2. 12 days.</td>
</tr>
<tr>
<td>5. 18 days.</td>
<td>3. 16 wk.</td>
</tr>
<tr>
<td>8. 5 sq. mi.</td>
<td>4. 72 sec.</td>
</tr>
<tr>
<td>10. 4s. 6d.</td>
<td>5. No answer.</td>
</tr>
<tr>
<td>11. 15 oz.</td>
<td>6. 4 gal.</td>
</tr>
<tr>
<td>12. 126 oz.</td>
<td>7. 90 lb.</td>
</tr>
<tr>
<td>13. 18.</td>
<td>8. 90 lb.</td>
</tr>
<tr>
<td>14. No answer.</td>
<td>9. 90 lb.</td>
</tr>
<tr>
<td>15. 20 hr.</td>
<td>10. 25°.</td>
</tr>
<tr>
<td>16. 4 gal.</td>
<td>17. 90 lb.</td>
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</table>

### EXERCISE 22

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<th>Question</th>
<th>Answer</th>
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</thead>
<tbody>
<tr>
<td>1. 3d.</td>
<td>2. 4d.</td>
</tr>
<tr>
<td>3. 2d.</td>
<td>4. 9d.</td>
</tr>
<tr>
<td>5. 8d.</td>
<td>6. 10d.</td>
</tr>
<tr>
<td>7. 8 oz.</td>
<td>8. 4 oz.</td>
</tr>
<tr>
<td>9. 2 oz.</td>
<td>10. 8 oz.</td>
</tr>
<tr>
<td>11. 12 oz.</td>
<td>12. 12 oz.</td>
</tr>
<tr>
<td>13. 12 min.</td>
<td>14. 6 min.</td>
</tr>
<tr>
<td>15. 15 min.</td>
<td>16. 24 min.</td>
</tr>
<tr>
<td>17. 24 min.</td>
<td>18. 24 min.</td>
</tr>
<tr>
<td>19. 27. 10s.</td>
<td>20. 5s.</td>
</tr>
<tr>
<td>21. 30.</td>
<td>22. 30.</td>
</tr>
<tr>
<td>27. 30.</td>
<td>28. 30.</td>
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### EXERCISE 23

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<tbody>
<tr>
<td>1. 1/4</td>
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<td>4. 1/4</td>
</tr>
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<tr>
<td>15. 1/4</td>
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<tr>
<td>17. 1/4</td>
<td>18. 1/4</td>
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<td>30. 1 lb. 8 oz.</td>
<td>31. 3 lb. 8 oz.</td>
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<td>12. 11(\frac{1}{4})</td>
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<td>17. 10(\frac{1}{2})</td>
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<td>32. 2(\frac{1}{2})</td>
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<td>26. 6</td>
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<td>32. 1(\frac{1}{2})</td>
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<tr>
<td>7. 0</td>
<td>8. 24</td>
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<td>13. 3 (\frac{1}{2})</td>
<td>14. 2 (\frac{3}{4})</td>
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<td>20. 3 (\frac{3}{4})</td>
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<tbody>
<tr>
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<td>2. 4(\frac{1}{2})</td>
</tr>
<tr>
<td>4. 3 (\frac{1}{2}) in.</td>
<td>5. 14</td>
</tr>
<tr>
<td>10. 21 mi.</td>
<td>11. 24 tons</td>
</tr>
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<td>13. 3 (\frac{1}{2})</td>
<td>14. 27 tons</td>
</tr>
<tr>
<td>16. 5 (\frac{1}{2})</td>
<td>17. 80 tons</td>
</tr>
<tr>
<td>19. 64 gall.</td>
<td>20. 6 (\frac{1}{2})</td>
</tr>
<tr>
<td>22. 24 days</td>
<td>21. 6 min.</td>
</tr>
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### EXERCISE 34
ANSWERS

EXERCISE 34

Page 44

1. 14 in. 2. 2½ in. 3. 3 in. 4. 2s. 8d. 5. 4s.
6. 4s. 6d. 7. 7½ lb. 8. 15 lb. 9. 17 lb. 10. 14 lb.
16. 44. 17. 9. 18. 7½ 19. 3½d. 20. 2¾d.
21. 4½d. 22. 15. 3d. 23. 9d. 24. 1s. 4d. 25. 1½ in.
26. 1¾ in. 27. 1 in. 28. 1½ oz. 29. 1¾ oz. 30. 2 oz.
31. 2½ 32. 2½ 33. 3½ 34. 1. 35. 3½b.
36. 3½.

Page 45

EXERCISE 35

1. 3½ 2. 3½ 3. 1½ 4. 2½ 5. 5. 6. 6.
13. 6 14. 6 15. 15 16. 3½ 17. 4 18. 4½.

Page 46

EXERCISE 36


Page 47

EXERCISE 37

1. 3½ 2. 3½ 3. 3½ 4. 3½ 5. 5. 6. 9.
25. 1½.

Page 48

EXERCISE 39

1. 3½ 2. 3½ 3. 3½ 4. 1½ 5. ½ 6. ½.

Page 50

EXERCISE 40

1. 6 2. 6 3. 6 4. 10 7½ lb.
5. 1½ 6. 12 sq. ft. 7. 1½ 8. 3½.
13. 3½ 14. 3½ 15. 18 cu. in. 16. 23.
17. 16 ft. 18. 1st; 2½ 19. 18 days. 20. 15 min.
21. 3½ 22. 15½s. 23. 15½ 24. 3135 tons.
25. 200 gall. 26. 26 jugs; 2½ gal. 27. 13 pieces; 3½ in.
28. 35 articles; 3½d. 29. 132 yd.
30. youngest, 3½; other, 10½.
31. 2½ 32. 3½ 33. £7254.
34. 3½ 35. 3½.

Page 59

EXERCISE 44

31. 3½ 32. 3½ 33. 3½ 34. 3½
35. 2½ 36. 2½ 37. 2½ 38. 2½.
39. 2½ 40. 2½ 41. 2½ 42. 2½.
43. 6000 m. 44. 90 m. 45. 65 m. 46. 36 m.
47. 0.09 m. 48. 0.08 m. 49. 18 m. 50. 307 m.
51. 0.365 km. 52. 0.072 km. 53. 0.009 km.
54. 0.08 km. 55. 0.00027 km. 56. 0.0025 km.
57. 0.47 km. 58. 0.000075 km. 59. 100.
60. 100. 61. 1000. 62. 100. 63. 1000. 64. 100.
65. 3½ 66. 3½ 67. 3½ 68. 3½ 69. 3½.
70. 4½ 71. 4½ 72. 4½ 73. 4½ 74. 4½.
75. 3½ 76. 3½ 77. 3½ 78. 3½ 79. 3½.
80. 2½.
### Page 80
**EXERCISE 60**

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<td>3.</td>
<td>0.633</td>
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<td>4.</td>
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<td>£8 19s. 2½d</td>
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<td>26.</td>
<td>£2 9s. 9½d</td>
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<td>27.</td>
<td>£3 12s. 7½d</td>
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**EXERCISE 61**

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<td>4 tons 2 cwt. 2 qr. 17 lb</td>
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<td>1 ton 7 cwt. 1 qr. 23 lb</td>
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<td>43 ch. 15 yd. 0 ft</td>
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<td>34 gall. 2 qt. 1 pt</td>
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**EXERCISE 62**

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### Page 84
**EXERCISE 63**

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<td>1.20 in</td>
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<td>0.01 strips</td>
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<td>0.8 in</td>
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<td>25.</td>
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<td>35</td>
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<td>29-1 m.p.h</td>
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### Page 87
**EXERCISE 64**

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<thead>
<tr>
<th>S.</th>
<th>Ans.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>12.7 cm.</td>
</tr>
<tr>
<td>2.</td>
<td>39 in.</td>
</tr>
<tr>
<td>3.</td>
<td>1.09 yd</td>
</tr>
<tr>
<td>4.</td>
<td>0.5937 in.</td>
</tr>
<tr>
<td>5.</td>
<td>1.25 mi</td>
</tr>
<tr>
<td>6.</td>
<td>6.80 km</td>
</tr>
<tr>
<td>7.</td>
<td>3.878</td>
</tr>
<tr>
<td>8.</td>
<td>0.15 sq. in</td>
</tr>
<tr>
<td>9.</td>
<td>0.35 dm.</td>
</tr>
<tr>
<td>10.</td>
<td>1.5 sq. dm</td>
</tr>
<tr>
<td>11.</td>
<td>0.8 in.</td>
</tr>
<tr>
<td>12.</td>
<td>0.045 ft</td>
</tr>
<tr>
<td>13.</td>
<td>1500 mi</td>
</tr>
<tr>
<td>14.</td>
<td>2.5</td>
</tr>
<tr>
<td>15.</td>
<td>0.035 litre</td>
</tr>
<tr>
<td>16.</td>
<td>10 gm.</td>
</tr>
<tr>
<td>17.</td>
<td>1000</td>
</tr>
<tr>
<td>18.</td>
<td>154 lb</td>
</tr>
<tr>
<td>19.</td>
<td>15 kg</td>
</tr>
<tr>
<td>20.</td>
<td>7.34</td>
</tr>
<tr>
<td>21.</td>
<td>10.5 ft</td>
</tr>
<tr>
<td>22.</td>
<td>198 lb</td>
</tr>
<tr>
<td>23.</td>
<td>13.6 kg</td>
</tr>
</tbody>
</table>

### Page 89
**EXERCISE 65**

<table>
<thead>
<tr>
<th>S.</th>
<th>Ans.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>12.08</td>
</tr>
<tr>
<td>2.</td>
<td>0.015</td>
</tr>
<tr>
<td>3.</td>
<td>14.33 gm</td>
</tr>
<tr>
<td>4.</td>
<td>2.39 gm</td>
</tr>
<tr>
<td>5.</td>
<td>3.450 gm</td>
</tr>
<tr>
<td>6.</td>
<td>0.91 kg</td>
</tr>
<tr>
<td>7.</td>
<td>0.25 kg</td>
</tr>
<tr>
<td>8.</td>
<td>0.425 kg</td>
</tr>
<tr>
<td>9.</td>
<td>0.0854 kg</td>
</tr>
<tr>
<td>10.</td>
<td>4.155 kg</td>
</tr>
<tr>
<td>11.</td>
<td>0.085 m</td>
</tr>
<tr>
<td>12.</td>
<td>0.435 m</td>
</tr>
<tr>
<td>13.</td>
<td>39.7 ft.</td>
</tr>
<tr>
<td>14.</td>
<td>1.83 cm</td>
</tr>
<tr>
<td>15.</td>
<td>3.44 litres</td>
</tr>
<tr>
<td>16.</td>
<td>0.76375 kg</td>
</tr>
<tr>
<td>17.</td>
<td>0.9725 sq. m</td>
</tr>
<tr>
<td>18.</td>
<td>0.95 cu. dm</td>
</tr>
<tr>
<td>19.</td>
<td>7.1 kg</td>
</tr>
<tr>
<td>20.</td>
<td>650 c.c.</td>
</tr>
<tr>
<td>21.</td>
<td>11 litres</td>
</tr>
<tr>
<td>22.</td>
<td>0.46 sq. dm</td>
</tr>
<tr>
<td>23.</td>
<td>1.15 litres</td>
</tr>
<tr>
<td>24.</td>
<td>6 mm</td>
</tr>
<tr>
<td>25.</td>
<td>0.24 litres</td>
</tr>
<tr>
<td>26.</td>
<td>212.5 gm</td>
</tr>
<tr>
<td>27.</td>
<td>1.67 kg</td>
</tr>
</tbody>
</table>
### Page 90
**EXERCISE 66**

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 15 cm.</td>
<td>2. 8 ft.</td>
</tr>
<tr>
<td>3. 3 cm.</td>
<td>4. 21 sq. cm.</td>
</tr>
<tr>
<td>5. 5 mm.</td>
<td>6. 37 yr.</td>
</tr>
<tr>
<td>7. 17 kg.</td>
<td>8. 0-456 m.</td>
</tr>
<tr>
<td>9. 7-2 km. per hr.</td>
<td>10. 14 pieces</td>
</tr>
<tr>
<td>11. 18 cm.</td>
<td>12. 220 pt.</td>
</tr>
<tr>
<td>13. 1-274 m.</td>
<td>14. 7 fl. in.</td>
</tr>
<tr>
<td>16. 21-1 cm.</td>
<td>17. 12-8 cm.</td>
</tr>
<tr>
<td>18. 15-7 in.</td>
<td>19. 5-9 in.</td>
</tr>
<tr>
<td>20. 8-12 lb.</td>
<td>21. 50-7 lb.</td>
</tr>
</tbody>
</table>

### Page 92
**EXERCISE 67**

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 132 km.</td>
<td>2. 154 mg.</td>
</tr>
<tr>
<td>3. 350 grm.</td>
<td>4. 126 km. per hr.</td>
</tr>
<tr>
<td>5. 0.2161 kg.</td>
<td>6. 11.25 litres; 79 pts.</td>
</tr>
<tr>
<td>7. 2-35 dm.</td>
<td>8. 21000.</td>
</tr>
<tr>
<td>9. 1-5 kg.</td>
<td>10. 343 kg.</td>
</tr>
<tr>
<td>13. 6-5 dm.</td>
<td>14. 312 grm.</td>
</tr>
<tr>
<td>15. 1-05 litres.</td>
<td>16. 1 lb. 12 oz.</td>
</tr>
<tr>
<td>17. 107.31 grm.</td>
<td>18. 55 grm.</td>
</tr>
</tbody>
</table>

### Page 93
**Oral Work, Example 1**

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) 3, 11 in.</td>
<td>(ii) 4, 1, 6 gal.</td>
</tr>
<tr>
<td>(iii) 3, 1, 3 gal.</td>
<td>(iv) 1 p.m. to 2 p.m.</td>
</tr>
<tr>
<td>(v) Between 2 p.m. and 3 p.m.</td>
<td>(vi) 36, 2, 10, 0, 4, 16, 24 mi.</td>
</tr>
<tr>
<td>(vii) Number of gallons at 3:30 p.m.</td>
<td>(viii) Yes, yes, no.</td>
</tr>
</tbody>
</table>

### Page 94
**Oral Work, Example 2**

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) 3.</td>
<td>(ii) 3.</td>
</tr>
<tr>
<td>(iii) No.</td>
<td>(iv) No.</td>
</tr>
<tr>
<td>(v) 2nd, 3rd, 1st, 2nd.</td>
<td></td>
</tr>
</tbody>
</table>

### Page 95
**EXERCISE 68**

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Weight.</td>
<td>2. Time of day.</td>
</tr>
<tr>
<td>3. Age.</td>
<td>4. H. P.</td>
</tr>
<tr>
<td>5. Time of day.</td>
<td>6. Length.</td>
</tr>
<tr>
<td>11. 5 units to 1 in.; 5, 75.</td>
<td></td>
</tr>
<tr>
<td>12. 100 units to 1 in.; 100.</td>
<td>13. 2 units to 1 in.; 5, 25.</td>
</tr>
<tr>
<td>14. 10 units to 1 in.; 40, 100.</td>
<td>15. 50 units to 1 in.; 100, 260.</td>
</tr>
<tr>
<td>16. 0-2 units to 1 in.; 0, 1.</td>
<td>17. 50 units to 1 in.; 50, 300 yd.</td>
</tr>
<tr>
<td>18. 0-1 lb. to 1 in.; 2-7, 3-3 lb.</td>
<td>19. 0-05 m. to 1 in.; 0-2, 0-55 m.</td>
</tr>
<tr>
<td>20. £50,000 to 1 in.; £250,000, £800,000.</td>
<td></td>
</tr>
</tbody>
</table>

### Page 96
**EXERCISE 69**

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 18.1 ft.; 90 yr.</td>
<td>2. Thursday.</td>
</tr>
<tr>
<td>3. 102 lb.</td>
<td>4. July; January.</td>
</tr>
</tbody>
</table>

### Page 98
**Oral Work, Example 3**

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) 100, 45°; 101, 43°.</td>
<td>(ii) 3.20 p.m., 7 p.m.</td>
</tr>
<tr>
<td>(iii) 9.15 a.m., 1.25 p.m.</td>
<td></td>
</tr>
</tbody>
</table>

### Page 99
**Oral Work, Example 4**

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) 29-65, 29-76, 29-87, 29-33 in.</td>
<td>(ii) 9.36 a.m., 2.12 p.m.</td>
</tr>
<tr>
<td>(iii) 9.45 a.m., 2.12 p.m.</td>
<td>(iv) 9.36 a.m. to 2.12 p.m.</td>
</tr>
<tr>
<td>(v) 0.33 in.</td>
<td>(vi) 0.12 in.</td>
</tr>
<tr>
<td>(vii) The steeper the slope, the more rapid the fall.</td>
<td></td>
</tr>
</tbody>
</table>

### Page 99
**EXERCISE 70**

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 136, 336 yd.; 424 m.p.h.</td>
<td>2. £180, £267, £365; 32 yr.; 22-3 yr.</td>
</tr>
<tr>
<td>3. 30-5, 18, 10-7 yr.; 47, 56, 65 years old.</td>
<td>4. £17 5s., £21 10s.; age 43.</td>
</tr>
<tr>
<td>5. 14, 16-5 hr.; 41°, 58-5°.</td>
<td>6. £75, 2-42 sec.; 2-27 ft.; shorten by 4 in.</td>
</tr>
<tr>
<td>7. 3800 ft.; 18-2 (2), 44-8 (8) sec.; 41-2 (2) sec.</td>
<td>8. 56, 93 sec.; the same; 530, 925 yd.</td>
</tr>
<tr>
<td>9. Epidemic; half-term holiday.</td>
<td>10. Married at 23; severe illness at 29; son sent to school at 38 and to University at 43; death at 56.</td>
</tr>
<tr>
<td>11. Small sums spent on advertising are of little use; but it does not pay to spend more than £30,000.</td>
<td>12. Fare on meter starts at 8d. and jumps up 2d. at a time; a traffic block after 1½ miles.</td>
</tr>
<tr>
<td>13. A. pedestrians; B. pedal cyclists.</td>
<td></td>
</tr>
</tbody>
</table>

### Page 102
**Oral Work, Example 5**

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) 30, 60, 90 mi.</td>
<td>(ii) constant speed; 15 m.p.h.</td>
</tr>
<tr>
<td>(v) The steeper the graph, the greater the speed.</td>
<td></td>
</tr>
</tbody>
</table>

### Page 104
**EXERCISE 71**

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 12, 0, 3 m.p.h.; 6 m.p.h.; the steepest part.</td>
<td>2. 26, 34, 44 cm.; 12.5, 25, 60 gm.; 18 cm.; yes; extension is proportional to load; 2nd spring is more elastic.</td>
</tr>
<tr>
<td>3. 39 mi.; 12.10 p.m.; 2.09 p.m.; 15 m.p.h. (14-8).</td>
<td></td>
</tr>
</tbody>
</table>
4. 3.7 km; 3.9 mi.
5. 10, 0, 24, 4, 8, 0, 25 m.p.h.; 92 mi.; 1 p.m.; 24, 4, 0, 41 m.p.h.; 41 mi.; at 12.20 p.m.; 34.5 mi. out at 2.36 p.m.; 14 mi.; 6 mi.
6. 95° F.; 158° F.; 60° C.; 85° C.
7. 32, 67, 5 ft. per sec.; 13, 6, 51 ft. m.p.h.
8. 26; 67; 48.
9. 14, 5, 21, 34 mi.; 13, 5 mi.; 9, 35 a.m., 9, 54 a.m., 10, 13 a.m.
10. 2, 45 p.m.; 33 mi.
11. 3, 4 mi. from town, 2:6 hr. after cyclist started.
12. £270; £475.
13. 48 m.p.h.
14. 24 mi.
15. 55 mi.

Page 108
Oral Practice 1
1. 278. 2. 3194. 3. 3399. 4. 572. 5. 40 oz.
6. 2 ft. 5 in. 7. 2 tons 5 cwt. 8. 21/8. 9. 3s. 10. 8 oz.

Page 108
Oral Practice 2
1. 1955. 2. 4360. 3. 942. 4. 7 ft. 6 in.
5. 2300 yd. 6. 1 ch. 2 yd. 7. 1 cwt. 8. 125 cu. in.
9. 4 days. 10. 3°.

Page 108
Oral Practice 3
1. 1700. 2. 9708. 3. 9091. 4. 24 pts.
5. 6000. 6. 1 cwt. 62 lb. 7. 1 qr. 6 lb. 8. 3 ft.
9. 6s. 10. 21/3, 31/3, 51/3.

Page 109
Oral Practice 4
1. 334. 2. 43779. 3. 4. 4. 3 lb. 2 oz.
5. 3 ft. 4 in. 6. 4 mi. 3 fur. 7. 15 min. 8. 1s.
9. 21/3, 31/3. 10. 121 yd.

Page 109
Oral Practice 5
1. 1123. 2. 36342. 3. 8409. 4. 1 mi. 1240 yd.
5. 2420 sq. yd. 6. 5 gall. 1 qt. 7. 2 ft. 2 in. 8. 744s.
9. 21/3, 31/3, 51/3. 10. 6 hr.
### Page 112
 **Oral Practice 14**

<p>| | | | | | |</p>
<table>
<thead>
<tr>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>1 ft 1(\frac{1}{4}) in.</td>
<td>2.</td>
<td>1(\frac{1}{4})</td>
<td>3.</td>
<td>2; 1:2.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.</td>
<td>0:048.</td>
<td>5.</td>
<td>0:27.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>6.</td>
<td>0:65.</td>
<td>7.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>8.</td>
<td>2(\frac{1}{2}).</td>
<td>9.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>10.</td>
<td>1136 c.c.</td>
</tr>
</tbody>
</table>

### Page 112
 **Oral Practice 15**

<p>| | | | | | |</p>
<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>16s. 8d.</td>
<td>2.</td>
<td>1.</td>
<td>3.</td>
<td>1:715.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.</td>
<td>0:004.</td>
<td>5.</td>
<td>0:23.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>6.</td>
<td>5:045 kg.</td>
<td>7.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>8.</td>
<td>10:5 kg.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>10.</td>
<td>25 gall.</td>
</tr>
</tbody>
</table>

### Page 112
 **Oral Practice 16**

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>5s. 3d.</td>
<td>2.</td>
<td>$\frac{1}{10}$</td>
<td>3.</td>
<td>0:009.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.</td>
<td>0:000,027.</td>
<td>5.</td>
<td>2:02 m.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>6.</td>
<td>0:97.</td>
<td>7.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>8.</td>
<td>4; 2d.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>10.</td>
<td>63 sq. in.</td>
</tr>
</tbody>
</table>

### Page 113
 **Test 1**

<p>| | | | | | |</p>
<table>
<thead>
<tr>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>4(\frac{1}{8}); 1(\frac{1}{4}).</td>
<td>2.</td>
<td>0:2938; 1730.</td>
<td>3.</td>
<td>0:7875.</td>
</tr>
<tr>
<td></td>
<td>4.</td>
<td>14 cwt. 59 lb.</td>
<td>5.</td>
<td>$\frac{1}{2}$.</td>
<td>6.</td>
</tr>
</tbody>
</table>

### Page 113
 **Test 2**

<p>| | | | | | |</p>
<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>1(\frac{1}{2}); 6.</td>
<td>2.</td>
<td>54:7504; 0:0205.</td>
<td>3.</td>
<td>0:4375.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.</td>
<td>£3 12s. 2d.</td>
<td>5.</td>
<td>2(\frac{1}{2}).</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.</td>
<td>3; (\frac{1}{2}); (\frac{1}{2}).</td>
<td>2.</td>
<td>0:26103; 0:0217.</td>
<td>3.</td>
<td>0:61875.</td>
</tr>
<tr>
<td></td>
<td>4.</td>
<td>2 ft. 7 in.</td>
<td>5.</td>
<td>0:725; 0:0118.</td>
<td>6.</td>
</tr>
</tbody>
</table>

### Page 113
 **Test 3**

<p>| | | | | | |</p>
<table>
<thead>
<tr>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>2(\frac{1}{2}); 2(\frac{1}{2}).</td>
<td>2.</td>
<td>0:05152; 60:4.</td>
<td>3.</td>
<td>£0:6048.</td>
</tr>
<tr>
<td></td>
<td>4.</td>
<td>3 cwt. 31 lb. 5 oz.</td>
<td>5.</td>
<td>0:3133 Kg.</td>
<td>6.</td>
</tr>
</tbody>
</table>

### Page 114
 **Test 4**

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>1(\frac{1}{2}); $\frac{1}{2}$.</td>
<td>2.</td>
<td>0:02869; 20:4.</td>
<td>3.</td>
<td>$\frac{1}{8}$</td>
</tr>
<tr>
<td></td>
<td>4.</td>
<td>1:305.</td>
<td>5.</td>
<td>1705 c.c.</td>
<td>6.</td>
</tr>
</tbody>
</table>

### Page 114
 **Test 6**

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>24(\frac{3}{4}); $\frac{1}{2}$.</td>
<td>2.</td>
<td>5:51156; 0:000,145.</td>
<td>3.</td>
<td>$\frac{1}{3}$</td>
</tr>
<tr>
<td></td>
<td>4.</td>
<td>£8 17s. 8d.</td>
<td>5.</td>
<td>0:622.</td>
<td>6.</td>
</tr>
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</table>

### Page 114
 **Test 7**

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<tbody>
<tr>
<td>1.</td>
<td>20(\frac{3}{4}); $\frac{1}{2}$.</td>
<td>2.</td>
<td>0:0101906; 536.</td>
<td>3.</td>
<td>0:4856.</td>
</tr>
<tr>
<td></td>
<td>4.</td>
<td>£11 10s. 11d.</td>
<td>5.</td>
<td>132(\frac{1}{4}).</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>6.</td>
<td>2:40 oz.</td>
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### Page 115
 **Test 8**

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<tbody>
<tr>
<td>1.</td>
<td>$\frac{1}{2}$; $2\frac{1}{2}$.</td>
<td>2.</td>
<td>0:000,010,8; 1370.</td>
<td>3.</td>
<td>0:769.</td>
</tr>
<tr>
<td></td>
<td>4.</td>
<td>161; 45 cm.</td>
<td>5.</td>
<td>4(\frac{1}{2}).</td>
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<tr>
<td></td>
<td></td>
<td>6.</td>
<td>1:09 yd.</td>
<td></td>
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### Page 115
 **Paper 1**

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<tbody>
<tr>
<td>1.</td>
<td>309.</td>
<td>2.</td>
<td>1086 yr.</td>
<td>3.</td>
<td>1(\frac{1}{4})d.</td>
</tr>
<tr>
<td></td>
<td>4.</td>
<td>£4 18s. 3d.; £13,016 10s.</td>
<td>5.</td>
<td>870.</td>
<td>6.</td>
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### Page 115
 **Paper 2**

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<tbody>
<tr>
<td>1.</td>
<td>21227; 91054.</td>
<td>2.</td>
<td>23.</td>
<td>3.</td>
<td>87 days.</td>
</tr>
<tr>
<td></td>
<td>4.</td>
<td>£15 10s. 4d.</td>
<td>5.</td>
<td>11 ft.</td>
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### Page 116
 **Paper 3**

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<tbody>
<tr>
<td>1.</td>
<td>531.</td>
<td>2.</td>
<td>207.</td>
<td>3.</td>
<td>3s., 9s.</td>
</tr>
<tr>
<td></td>
<td>4.</td>
<td>Tuesday; Monday.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5.</td>
<td>165 revs.</td>
<td>6.</td>
<td>14 gall.</td>
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### Page 116
 **Paper 4**

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<tbody>
<tr>
<td>1.</td>
<td>5349.</td>
<td>2.</td>
<td>21,974,568; 181,608.</td>
<td>3.</td>
<td>£44 12s. 6d.</td>
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<tr>
<td></td>
<td>4.</td>
<td>778.</td>
<td>5.</td>
<td>135.</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>6.</td>
<td>278 boys.</td>
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### Page 117
 **Paper 5**

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<tbody>
<tr>
<td>1.</td>
<td>57, 93; 364, r. 100.</td>
<td>2.</td>
<td>5s. 6d., 7s. 9d.; £11 15s.</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>3.</td>
<td>360.</td>
<td>4.</td>
<td>45.</td>
<td></td>
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<tr>
<td></td>
<td>5.</td>
<td>30.</td>
<td>6.</td>
<td>972 yd.</td>
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### Page 117
 **Paper 6**

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<tbody>
<tr>
<td>1.</td>
<td>598.</td>
<td>2.</td>
<td>£117 17s. 10d.</td>
<td>3.</td>
<td>30 weeks.</td>
</tr>
<tr>
<td></td>
<td>4.</td>
<td>186 hurdles.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5.</td>
<td>22; 1s. 6d.</td>
<td>6.</td>
<td>88 yd.</td>
<td></td>
</tr>
</tbody>
</table>
Page 118

1. 1,547,699.  
   2. £5 14s. 10d.; £1 15s. 5d.  
   3. £1 8s. 8d.  
   4. 27 ft.  
   5. £18 15s.  
   6. 61; 130 sq. yd.

Page 118

1. 4003.  
   2. 2112.  
   3. £53 2s. 10d.  
   4. £322; £592 a year.  
   5. £1 5s. 8d.  
   6. 720.

Page 119

1. 2³, 3³, 5.7.  
   2. 960 yd.; 8 chairs.  
   3. 5s.; £186 13s. 4d.  
   4. 176 yd.  
   5. £1 14s. 7d.  
   6. 100 sq. in.; 3 cu. ft. 816 cu. in.

Page 119

1. 48.  
   2. 6 cwt. 107 lb. 11 oz.; 44. 3. 9s. 7d.  
   4. 13 gall.; 18s. 5d.  
   5. 1760 sq. ft.  
   6. 3456; 30 cwt.

Page 120

1. 3 mi. 2 ch. 12 yd.; 1 ton 14 cwt. 2 qr.  
   2. £197.  
   3. £27 10s.  
   4. 3s. 9d.; 4 hr.  
   5. 9 tons 15 cwt.  
   6. 116 sq. in.; 3 cu. ft. 384 cu. in.

Page 120

1. 36.  
   2. 12020 lb.; 5 mi. 21 ch. 18 yd.  
   3. £3 3s. 3d.  
   4. £12 16s. 8d.  
   5. £8; £5 12s.  
   6. 1576 cu. in.

Page 121

1. 2³, 3³, 7³; 6; 7.  
   2. 3 ch. 19 yd. 1 ft. 21 tons 17 cwt. 3 qr.  
   3. 60 mi.  
   4. 27 lb. 13 oz.  
   5. 82 sq. in.  
   6. 3 tons 10 cwt.

Page 121

1. 2³, 3³, 7³; 336.  
   2. 23; 29 ch. 17 yd. 1 ft. 2 in.  
   3. 70 lb. per min.  
   4. 15 days.  
   5. 84 sq. cm.  
   6. 16 in.; 140 sq. in.

Page 121

1. 84.  
   2. 54 sq. yd.; 16 sq. in.; 11s. 6d.  
   3. 18 days.  
   4. £10 8s. 4d.; £3 15s. 5d.  
   5. 149 sq. yd.  
   6. 8640.

Page 122

1. 594.  
   2. 3653 days; £41 1s. 8d.  
   3. £920 rails.  
   4. 80 sq. in.; 80 ac.  
   5. 210 sq. yd.; 2190 sq. ft.  
   6. 1740 cu. in.

Page 123

1. £708,930.  
   2. 3⁵⁄₈; 7⁵⁄₈.  
   3. 3³⁴⁄₈; 0·13892.  
   4. 30 m.p.h.  
   5. 43·2 cm.  
   6. 480 lb.; 0·214 ton.

Page 123

1. 3⁵⁄₈; 17s. 6d.  
   2. 0·1072; 0·00605.  
   3. £4 17s. 9d.; 0·709.  
   4. 9 fr. 45 c.  
   5. 200.  
   6. 4³⁄₈ sq. in.; 36 lb.

Page 124

1. £20 2s.; £995.  
   2. 7⁷⁄₈; 7⁷⁄₈.  
   3. 2·683 m.; 10·857726.  
   4. 6³⁄₈ gall.; 160 chairs.  
   5. A £275, B £245, C £230.  
   6. £338 8s. 4d.

Page 124

1. 2³, 3³, 5.7·11.  
   2. 8⁷⁄₈; 8⁷⁄₈.  
   3. 6·203; 0·0056.  
   4. 50 min.  
   5. £3 12s. 3d.; 2s. per hr.  
   6. 31·5 sq. cm.; 7·56 cu. dm.

Page 125

1. 18.  
   2. 2³⁄₈; 1³⁄₈.  
   3. 2 ft. 2 in.; 0·00003, 2.  
   4. 39·4 in.  
   5. £5 0s. 3d.  
   6. 64 sq. in.; 68³⁄₈ sq. in.

Page 125

1. 5⁵⁄₈; 2⁵⁄₈.  
   2. £3 14s. 7d.; 30·28.  
   3. 232·875 fr.  
   4. 3³⁄₄ m.p.h. faster.  
   5. 3⁹⁄₁₀d. per lb.  
   6. 15 sq. in.; 25⁵⁄₈ oz.
ANSWERS

Page 126  Paper 23

1. $\frac{5}{9}$; 19. 2. 0:000,15; £0·692. 3. 1 $\frac{1}{2}$ m.p.h. 4. £1962 a year. 5. 93 cu. in. 6. 55 yd. 71 in.; 55 yd. 4 in.

Page 126  Paper 24

1. 14,989,564 bunches; 510,000,000 lb. 2. 124 $\frac{1}{2}$; 55 pieces; 1 $\frac{1}{2}$ in. 3. 12·75; 0·528. 4. £8 5s. 5. £60 15s. 6. 49·47 sq. cm.; 445 gm.

Page 129  EXERCISE 72

1. 3d. 2. 4s. 3. 2s. 4. 4 in. 5. 4 cwt. 6. 6 in. 7. 1 $\frac{1}{2}$d. 8. 12 min. 9. 14 lb. 10. 1 pt. 11. 5 $\frac{1}{4}$ yd. 12. 220 yd. 13. 2 s. 6d. 14. 2 $\frac{1}{3}$d. 15. 6s. 8d. 16. 1s. 3d. 17. $\frac{1}{2}$ 18. $\frac{1}{2}$ 19. $\frac{3}{4}$ 20. $\frac{1}{2}$. 21. $\frac{1}{2}$ 22. $\frac{1}{2}$ 23. $\frac{2}{3}$ 24. $\frac{1}{3}$. 25. $\frac{1}{2}$

Page 129  EXERCISE 73

1. 8d. 2. 12s. 3. 13s. 4d. 4. 7s. 6d. 5. 8s. 6. 7s. 6d. 7. 2s. 8. 6s. 8d. 9. 12 oz. 10. 8 cwt. 11. 2 ft. 3 in. 12. 1100 yd. 13. 42 min. 14. 2 gall. 1 qt. 15. 8s. 4d. 16. 8s. 9d. 17. 16s. 10d. 18. 9s. 10d. 19. £3 10s. 20. £2 9s. 8d. 21. 4 yd. 1 ft. 3 in. 22. 4 tons 6 cwt. 1 qr. 23. £1 17s. 6d. 24. £2 9s. 25. 4d.

Page 131  EXERCISE 75

1. 19s. 6d. 2. 11s. 3d. 3. £1 5s. 6d. 4. £2 18s. 9d. 5. £7 1s. 8d. 6. £4 0s. 1d. 7. £9 15s. 8. £11 7s. 9. £115 4s. 10. £112 14s. 11. £340 3s. 4d. 12. £311 15s. 13. £398 15s. 4d. 14. £197 6s. 8d. 15. £625 6s. 3d. 16. £350 12s. 6d. 17. 19 lb. 11 oz. 18. 33 qr. 7 lb. 19. 18 qr. 18 lb. 20. 87 lb. 14 oz. 21. 92 tons 16 cwt. 22. 262 tons 4 cwt. 23. 49 yd. 1 ft. 9 in. 24. 213 yd. 2 ft. 4 in. 25. 125 hr. 40 min. 26. 141 cwt. 42 lb.

Page 133  EXERCISE 76

1. £134 8s. 2. £123 15s. 3. £312 16s. 6d. 4. £85 8s. 2d. 5. £150 13s. 6. £114 3s. 8d. 7. £254 7s. 3d. 8. £139 15s. 0$\frac{1}{2}$d. 9. £1599 9s. 7d. 10. £2573 12s. 11d. 11. 22 tons 6 cwt. 14 lb. 12. 221 tons. 13. 534 tons 7 cwt. 2 qr. 14. 146 tons 8 cwt. 15. 43 ch. 21 yd. 2 ft. 9 in. 16. 568 ch. 2 yd. 2 ft. 17. £553 3s. 9d. 18. £9114 11s. 8d. 19. £13 4s. 4$d$. 20. £1 19s. 10$d$. 21. £25 0s. 6d. 22. £34 2lb. 9 oz. 23. 127 cwt. 2 qr. 20 lb. 24. £129 15s.

PART II
Page 184

EXERCISE 98

1. 8; 1/4 cu. in.
2. 60; 7/4 cu. in.; 7/4.
3. 1000; 0.001 cu. dm.
4. 2520; 0.001, 2.52 cu. dm.
5. 105; 1/4 cu. in.; 13/4.
6. 1728 cu. in.; 1/4 ft.
7. 1000 c.c.; 0.001 litres.
8. 81 cu. ft.
9. 0.1 cu. dm.
10. 0.01 c.c.
11. 3/4 cu. ft.
12. 10 cu. in.
13. 63 cu. in.
14. 100 c.c.
15. 60 cu. dm.

Page 186

EXERCISE 99

1. 49 3/4 cu. ft.
2. 540 c.c.
3. 19.5 cu. in.
4. 7 cu. ft.
5. 1632 cu. ft.
6. 9 cu. ft.
7. 10,080 cu. ft.
8. 39,312; 4180; 15 cu. ft. 1400 cu. in.
9. 12,800.
10. 480 litres; 1309; 31013 cu. dm.
11. 2:16 litres.
12. 45 lb.
13. 60 lb.
14. 300 gm.
15. 504 kg.
16. 23 in.
17. 1 1/4 in.
18. 5 in.
19. 42.9 cm.
20. 6 in.
21. 2 ft.
22. 50 gal.
23. 16,500 gal.
24. 18,750 gal.
25. 1 ft.
26. 9/16 tons.
27. 30 kg.
28. 275 litres.
29. 104 kg.
30. 18 cm.
31. 16 dm.
32. 4 ft.
33. 14 in.
34. £114.
35. 12 loads.
36. 4' 8" in.

Page 189

EXERCISE 100

1. 216 cu. in.
2. 12 1/4 cu. in.
3. 348 c.c.
4. 17 cu. in.
5. 1065 cu. in.
6. 1675 c.c.
7. 105 cu. in.
8. 600 c.c.
9. 760 cu. in.
10. 503 c.c.
11. 6 lb.
12. 16 tons.
13. 118 gall.
14. 18 1/2 cu. ft.; 94 lb.
15. 53 tons.
16. 16s. 2d.

Page 191

EXERCISE 101

1. 81 cu. in.
2. 1 1/4 cu. dm.
3. 16 ft. 8 in.
4. 3 1/2 m.
5. 34 sq. ft.
6. 0.145 sq. cm.
7. 6 5/8 cm.
8. 40 sq. cm.
9. 11 7/8 kg.
10. 99 cu. in.; 30 lb. (29-7).
11. 7250 c.c.; 57 kg. (36:55).
12. 26 3/4 in.
13. 0:8 in.
14. 2130 tons.
15. 550 lb.
16. 35 sq. in.
17. 937 1/2 gal.
18. 11 sq. in. (10).
19. 2 1/8 hr.
20. 312 1/4 gal.

Page 192

EXERCISE 102

1. 5 ft. 3 in.
2. 3 1/4 yd.
3. 8 lb. 7 oz.
4. 2092 1/2 sq. ft.
5. 225. 2d.
6. 16 in.
7. 375. 6d.
8. 5 min.
9. 4 in.
10. 498 lb.
11. 315 litres.
12. 2275, 14,400 sq. yd.
13. 532 sq. in.
14. 350 cu. ft.
15. 45 3/4 litres.
16. 49 lb.
17. 0.8 sq. mm.
18. 3 1/4 lb.

Page 196

EXERCISE 103

1. 88 in.
2. 17 6/16 cm.
3. 39.6 m.
4. 18 8/9 in.
5. 28.3 in.
6. 62.8 in.
7. 47.1 cm.
8. 78.5 yd.
9. 4.40 yd.
10. 1.75 in.
11. 14 cm.
12. 3.5 yd.
13. 15.9 yd.
14. 1.02 dm.
15. 1.27 in.
16. 50.2 in.
17. 70 yd.
18. 9 in.
19. 114 yd.
20. 23.6 (6).
21. 720.
22. 12.5 m.p.h.
23. 96.
24. 91.7 ft. per sec.
25. 117.
26. 314 yd.; yes.
27. 503 yd.
28. 9 ft. 10 in.
29. 25 1/2 in.
30. 1.75 cm.
31. 8.96 in.
32. 39 in.
33. 4 17 in. (4 1/4).
34. 7.00 in.

Page 199

EXERCISE 104

1. 154 sq. cm.
2. 1390 sq. in.
3. 95.0 sq. cm.
4. 154 yd.
5. 13 sq. in.
6. 693 sq. in.
7. 4 1/4 sq. in.
8. 314. 628 sq. yd.
9. 12,100 sq. yd. (2 1/2 ac.);
10. 15,400 sq. yd. (3 1/8 ac.).
11. 22 sq. in.
12. 3 3/4 sq. in.
13. 770 sq. ft.
14. 27 1/2 lb.
15. 5 in.
16. 201 sq. in.
17. 23.5 sq. cm.
18. 112.6 sq. yd.

Page 201

EXERCISE 105

1. 3 1/2 in.; 55 sq. in.;
2. 22, 6 cm.; 132 sq. cm.
3. 220 sq. in.
4. 188 sq. cm.
5. 70.4 sq. cm.
6. 56.5 sq. in.
7. 88 sq. in.
8. 660 sq. cm.
9. 11 sq. ft.
10. 220 sq. cm.
11. 56 sq. in.
12. 217 sq. cm.
13. 3 1/2 in.
14. 5 1/2 in.
15. 3-5 mm.
16. 16 in.
17. 9 cm.
18. 204 sq. cm.
19. 314 lb.
20. 1100 sq. ft.
21. 490 sq. in.
22. 126 sq. ft.
23. 895 sq. yd.
24. 27s. 6d.
### EXERCISE 106

**Page 204**

<table>
<thead>
<tr>
<th>Problem</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>2. 69-3 c.c.</td>
<td>15.4 cu. dm.</td>
</tr>
<tr>
<td>4. 9-24 cu. in.</td>
<td>8. 7 ccm.</td>
</tr>
<tr>
<td>6. 69-3 dm.</td>
<td>12. 3-11 ft.</td>
</tr>
<tr>
<td>7. 4 in.</td>
<td>13. 198 cu. in.</td>
</tr>
<tr>
<td>4. 9-24 cu. in.</td>
<td>10. 9-41 cu. ft.</td>
</tr>
<tr>
<td>13. 132 c.c.</td>
<td>20. 32.2 cu. in.</td>
</tr>
<tr>
<td>17. 18 in.</td>
<td>22. 3560 cu. in. (1538)</td>
</tr>
<tr>
<td>24. 15,500 gall.</td>
<td>25. 1-10 ft. per sec.</td>
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</table>

### EXERCISE 107

**Page 206**

<table>
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<tr>
<th>Problem</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>1. 64 sq. ft.</td>
<td>8. 51 ac.</td>
</tr>
<tr>
<td>2. 1-57 in.</td>
<td>4. 576.</td>
</tr>
<tr>
<td>5. 3-8(6) sq. in.</td>
<td>7. 8 lb.</td>
</tr>
<tr>
<td>6. 3-5 ft.</td>
<td>8. 81-5 cu. in.</td>
</tr>
<tr>
<td>7. 13 in.</td>
<td>12. 26 ft.</td>
</tr>
<tr>
<td>10. 43-6 oz.</td>
<td>16. 614 sq. yd.</td>
</tr>
<tr>
<td>12. 129'</td>
<td>19. 315 min.</td>
</tr>
<tr>
<td>14. 315 min.</td>
<td>20. 7-84% 24 in.</td>
</tr>
<tr>
<td>18. 1350 gall.</td>
<td>21. 460(5) lb.</td>
</tr>
<tr>
<td>22. 28-6'</td>
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### EXERCISE 108

**Page 208**

<table>
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<tr>
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<tbody>
<tr>
<td>1. £3, £103.</td>
<td>3. £12, £112.</td>
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<tr>
<td>4. £20, £120.</td>
<td>6. £7, £107.</td>
</tr>
<tr>
<td>5. £4, £104.</td>
<td>8. £1, £10.</td>
</tr>
<tr>
<td>10. £11, £321.</td>
<td>11. £24, £424.</td>
</tr>
<tr>
<td>13. £260, £266 4s.; £6 4s.</td>
<td>15. £380, £382; £2.</td>
</tr>
<tr>
<td>18. £4,000, £4,060 16s.; £6 16s.</td>
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### EXERCISE 109

**Page 212**

<table>
<thead>
<tr>
<th>Problem</th>
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<tbody>
<tr>
<td>1. £63, £413.</td>
<td>3. £18 8s., £202 8s.</td>
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<tr>
<td>4. £177 9s., £1022 9s.</td>
<td>6. £4 10s., £67.</td>
</tr>
<tr>
<td>5. £45, £420.</td>
<td>8. £8 16s., £200 16s.</td>
</tr>
<tr>
<td>10. £20 8s.</td>
<td>11. £27 14s. 5d.</td>
</tr>
<tr>
<td>12. £24 19s. 2d.</td>
<td>13. £20 4s. 5d.</td>
</tr>
<tr>
<td>14. £5 15s. 2d.</td>
<td>15. £17 4s. 7d.</td>
</tr>
<tr>
<td>16. £11 8s. 6d.</td>
<td>17. £5 17s. 10d.</td>
</tr>
<tr>
<td>18. £10 1s. 10d.</td>
<td></td>
</tr>
</tbody>
</table>

### EXERCISE 110

**Page 213**

<table>
<thead>
<tr>
<th>Problem</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 3%</td>
<td>3. 3%</td>
</tr>
<tr>
<td>2. 3%</td>
<td>6. 3%</td>
</tr>
<tr>
<td>3. 6%</td>
<td>9. £400.</td>
</tr>
<tr>
<td>4. 4%</td>
<td>10. £700.</td>
</tr>
<tr>
<td>5. 4 yr.</td>
<td></td>
</tr>
<tr>
<td>6. 3 yr.</td>
<td></td>
</tr>
<tr>
<td>7. 6 yr.</td>
<td></td>
</tr>
<tr>
<td>8. 4 yr.</td>
<td></td>
</tr>
<tr>
<td>11. £600.</td>
<td>13. 100 x 1 x</td>
</tr>
<tr>
<td>12. £270.</td>
<td>15.</td>
</tr>
<tr>
<td>13. y%</td>
<td>14. 100 x PT</td>
</tr>
<tr>
<td>16. 100 x 1</td>
<td>17. 2 - 100 x RT</td>
</tr>
<tr>
<td>18. £</td>
<td></td>
</tr>
</tbody>
</table>

### EXERCISE 111

**Page 214**

<table>
<thead>
<tr>
<th>Question</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. £138; 5%</td>
<td>2. £56; 34%</td>
</tr>
<tr>
<td>4. £1158; 33%</td>
<td>6. £720; £768.</td>
</tr>
<tr>
<td>7. £40 10s.</td>
<td>8. 3s. 9d.; £10 3s. 9d.</td>
</tr>
<tr>
<td>9. £1805 14s.; 48%</td>
<td>10. 2s. 1s.; 28%</td>
</tr>
<tr>
<td>11. £2500 6s. 3d.; 14%</td>
<td>12. £6825; £7166 5s.</td>
</tr>
<tr>
<td>13. £75 2s. 6d.; £560 12s. 6d.</td>
<td>14. £73 10s. 3d.; 28%</td>
</tr>
<tr>
<td>15. £47 10s. 6d.; 6%</td>
<td>16. £44 16s.; 4%</td>
</tr>
<tr>
<td>17. 7% 18. 2 yr.</td>
<td>19. £14; 20. 60%</td>
</tr>
<tr>
<td>19. £1 2s. 6d.</td>
<td>22. £4 1s.</td>
</tr>
<tr>
<td>23. £88; 4%</td>
<td>24. £1 18s. 4d.</td>
</tr>
<tr>
<td>25. 120 days.</td>
<td>26. £169.</td>
</tr>
<tr>
<td>27. £275.</td>
<td></td>
</tr>
</tbody>
</table>

### EXERCISE 112

**Page 217**

<table>
<thead>
<tr>
<th>Question</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. £600.</td>
<td>2. £750.</td>
</tr>
<tr>
<td>5. £960.</td>
<td>6. £560.</td>
</tr>
<tr>
<td>8. £200; £4.</td>
<td>11. £198 6s. 5d.; £1 19s. 7d.</td>
</tr>
<tr>
<td>12. £72 2s. 4d.; £2 17s. 8d.</td>
<td>13. £338 11s. 4d.; £1 8s. 6d.</td>
</tr>
<tr>
<td>14. £70; £4 16s. 3d.</td>
<td>15. £42 1s. 5d.; £13 8s. 7d.</td>
</tr>
<tr>
<td>16. 46%</td>
<td>17. £115.</td>
</tr>
<tr>
<td>18. 66%</td>
<td>19. 4s. 7d.; 31%</td>
</tr>
</tbody>
</table>

### EXERCISE 113

**Page 218**

<table>
<thead>
<tr>
<th>Question</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 5% 16%</td>
<td>2. 2s.; 7s. 6d.</td>
</tr>
<tr>
<td>4. £168.</td>
<td>5. £15 7s. 1d.; £15 1s. 10d.</td>
</tr>
<tr>
<td>7. 4%</td>
<td>8. 16% 3% 3s. 4d. in the £.</td>
</tr>
<tr>
<td>9. 15s.</td>
<td>10. £. 16s. 6d. 12. £10 16s.</td>
</tr>
<tr>
<td>11. £4. 16s. 6d.</td>
<td>13. £115.</td>
</tr>
<tr>
<td>12. £4. 7d.; 31%</td>
<td></td>
</tr>
</tbody>
</table>

### EXERCISE 114

**Page 220**

<table>
<thead>
<tr>
<th>Question</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. £99 12s. 4d.</td>
<td>2. £899 16s. 2d.</td>
</tr>
<tr>
<td>4. £748 3s. 0d.</td>
<td>5. £1796 11s. 0d.</td>
</tr>
<tr>
<td>7. £1 3s. 10d.</td>
<td>8. £18 15s. 4d.</td>
</tr>
<tr>
<td>10. 7s. 0d.</td>
<td>11. 4%</td>
</tr>
<tr>
<td>13. 14s. 2d.</td>
<td>14. 19s. 10d.</td>
</tr>
<tr>
<td>15. £1 13s. 8d.</td>
<td>17. £4 16s. 4d.</td>
</tr>
<tr>
<td>16. £1 3s. 8d.</td>
<td></td>
</tr>
</tbody>
</table>

### EXERCISE 115

**Page 222**

<table>
<thead>
<tr>
<th>Question</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. £22 15s.</td>
<td>2. £15. 3%</td>
</tr>
<tr>
<td>5. 10% 33% 21%</td>
<td>6. £74 4s. 4d.</td>
</tr>
<tr>
<td>8. £8 8s.</td>
<td>9. 1 1 yr. 10. £1 17s. 9d.</td>
</tr>
<tr>
<td>11. £2 16s.</td>
<td>12. 4%</td>
</tr>
<tr>
<td>13. £135.</td>
<td>14. 24%</td>
</tr>
<tr>
<td>15. 9 months.</td>
<td>16. £2 3s. 3d.</td>
</tr>
<tr>
<td>17. £966; 5%</td>
<td>18. £1 10s. less.</td>
</tr>
<tr>
<td>19. £875.</td>
<td>21. £4 11s.</td>
</tr>
<tr>
<td>20. 6%</td>
<td>22. 32a. per ton.</td>
</tr>
<tr>
<td>23. £692.</td>
<td>24. £30 17s. 8d.</td>
</tr>
</tbody>
</table>
ANSWERS

Page 225
EXERCISE 116
1. 3. 29. 5. 2. 6. 8. 9. 11. 12. 13. 14. 15.
2. 293. 3. 5.253. 4. 10-91. 5. 2.622. 6. 3.926. 7. 2.58. 8. 5.58. 
9. 42,770,000; 44,790,000; 2,020,000. 10. £237,900,000; £255,300,000; £175,500,000. 
11. 20,600,000 letters; 155,000 passengers; 6,260,000 miles.
12. £8,930,000; £5,030,000; £5,890,000. 13. 0.3333. 14. 0.1667. 15. 0.2500.
16. 0.7181. 17. 0.1333. 18. 0.142. 
Page 227
EXERCISE 117
1. 1. 2. 79. 3. 81,000,000. 4. 9.37. 5. 2,970,000,000,000. 
6. 0.91. 7. 4.68. 8. 8.64. 9. 4700. 
10. 0.93. 11. 0.457. 12. 98.18. 13. 554.3. 
14. 28,430. 15. 0.003270. 16. £48,720,000. 17. 97,400.
18. 14.69. 19. 413. 20. 182. 
Page 229
EXERCISE 118
1. 1. 2. 0.0063. 3. 7.300. 4. 5.95. 5. 1.280.
6. 0.65. 7. 1.61. 8. 0.2633. 9. 666,000. 10. 0.01193. 
11. 7000. 12. 0.137. 13. 0.597. 14. 3.24. 15. 56.70%. 
16. 19. 17. 0.17. 
Page 230
EXERCISE 119
1. 0.05 cm.; 0.01 ; 1%. 2. 0.5 gm.; 0.002; 0.2%. 
3. 6.12; 0.004; 0.4%. 4. 0.1 sec.; 0.01; 1%. 
5. 6 oz.; 0.004; 0.4%. 6. 5 yd.; 0.002; 0.2%. 
7. 20 lb.; 0.01; 1%. 8. 2%; 9. 0.1%. 10. 0.04%. 
11. 1%. 12. 1%. 13. 7.2; 7.8 cm. 14. 0.48 f. 
15. 200 yd. 
Page 232
EXERCISE 120
1. 1. 2. 1600; 1658 ± 2. 3. 2756; 2756 ± 0.25. 
4. 590; 587.3 ± 1.5. 5. 8; 8 ± 2; 8.522 ± 0.001. 
6. 5; 271; 2.7083 ± 0.00055. 7. 8; 90; 85 ± 0.55. 
8. 900; 913 ± 2.5. 9. 13; 13.3 ± 0.002. 
10. 16.35; 16.05 cm.; 16.2 ± 0.15 cm. 11. 2; 124; 123.64 ± 0.025. 
12. 5; 5.047; 5.0726 ± 0.0011. 13. 6; 400; 403 ± 1.
14. 8; 90; 85 ± 0.55. 15. 10; 900; 913 ± 2.5. 
16. 12; 13; 13.249 ± 0.0115. 
Page 235
EXERCISE 121
1. 1. 2. 20; 20.25 ± 4.5. 2. 0.2; 0.156 ± 0.0045. 
3. 9; 8.88 ± 0.31. 4. 0.57; 0.5708 ± 0.0030. 
5. 0.4; 0.4 ± 0.0122. 6. 3; 2.6 ± 0.12. 
7. 0.25; 0.253 ± 0.001. 8. 30 sq. cm.; 26.9 ± 0.54 sq. cm. 
9. 0.32; 0.32 ± 0.0015. 10. 48,000 sq. ft.; 48,000 ± 300 sq. ft. 
11. 29/2 in.; 29/21 ± 0.011 in. 12. 0.8 sec.; 0.84 ± 0.007 sec. 
13. 4 ft.; 4.77 ± 0.013 in. 14. 4 ft.; 4.72 ± 0.024 cm. 
15. 510,000; 306,700 ± 1300. 16. 7 g.; 7 ± 0.054 g. 
17. 30 × 10²; 2,970,000 ± 18,500. 18. Total between 1013.92 and 1013.6. 
19. (i) 300 m.p.h.; 304.3 ± 1.3. (ii) 304 m.p.h.; 304.3 ± 0.14. 
Page 236
EXERCISE 122
1. 0.88 yd. 2. 3.146; 0.1753 ± 0.00055. 3. 0.645635. 
4. 0.68 mi. 5. 1182. 6. 153 ft. 8 in. 7. 10 gm. 
8. 0.084 in. per hr. 9. 3.28 ft. 10. 1.76 ft. 11. 17.2. 
12. 4 figures. 13. 19.22 gm. 14. 72.3%. 15. 67.9%. 
16. 6080 ft. 17. £1,000,000. 18. 0.01%. 19. 28.8 sec. 
20. 66.4%; 48.4%. 21. 454 gm. 22. 350 m.p.h. 
23. 64 lb. per cu. ft. 24. 4.14d. per £. 
Page 239
EXERCISE 123
1. 9. 2. 760. 3. 3.45. 4. 32/9. 5. 45. 2d. 
6. 2 tons 18 cwt. 2 qr. 7. 2s. per lb. 8. 8d. per lb. 
9. 3s. 10½d. per lb. 10. 3s. 9d.; 3s. 9d. 11. 13. 
12. 1s. 9½d.; £2 8s. 9½d. 13. 6 hr.; 10½ m.p.h. 
14. 34 m.p.h. 15. 12. 16. 58. 17. £8964. 
18. 865. 19. £1300. 20. 2s. 10½d., 2s. 3d., per lb. 
21. 3s. 2s. per ton. 22. 4 ft. 7 in. 23. 
24. 36 m.p.h.; 70 years. 25. 51°. 
26. 70 m.p.h.; 50 years. 27. 40 m.p.h.; 75 years. 
28. 55 m.p.h.; 70 years. 29. 42 m.p.h.; 75 years.
<table>
<thead>
<tr>
<th>Page 248</th>
<th>Oral Practice 17</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>3.</td>
</tr>
<tr>
<td>2.</td>
<td>4.</td>
</tr>
<tr>
<td>3.</td>
<td>5.</td>
</tr>
<tr>
<td>4.</td>
<td>6.</td>
</tr>
<tr>
<td>5.</td>
<td>7.</td>
</tr>
<tr>
<td>6.</td>
<td>8.</td>
</tr>
<tr>
<td>7.</td>
<td>9.</td>
</tr>
<tr>
<td>8.</td>
<td>10.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Page 248</th>
<th>Oral Practice 18</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>3.</td>
</tr>
<tr>
<td>2.</td>
<td>4.</td>
</tr>
<tr>
<td>3.</td>
<td>5.</td>
</tr>
<tr>
<td>4.</td>
<td>6.</td>
</tr>
<tr>
<td>5.</td>
<td>7.</td>
</tr>
<tr>
<td>6.</td>
<td>8.</td>
</tr>
<tr>
<td>7.</td>
<td>9.</td>
</tr>
<tr>
<td>8.</td>
<td>10.</td>
</tr>
</tbody>
</table>
### ANSWERS

**Page 251**

<table>
<thead>
<tr>
<th>Test 12</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 3½</td>
</tr>
<tr>
<td>2. 0-011,121,201; 6304.</td>
</tr>
<tr>
<td>3. £14 11s. 10d.</td>
</tr>
<tr>
<td>4. 12 yd. 1 ft. 5 in.</td>
</tr>
<tr>
<td>5. 3:8; 7½%.</td>
</tr>
<tr>
<td>6. £5 1s. 8d.</td>
</tr>
</tbody>
</table>

**Page 251**

<table>
<thead>
<tr>
<th>Test 18</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. £0-9198.</td>
</tr>
<tr>
<td>2. 0-253,504; 0-016.</td>
</tr>
<tr>
<td>3. ½.</td>
</tr>
<tr>
<td>4. £25 9s. 2d.</td>
</tr>
<tr>
<td>5. 75%; £101,300.</td>
</tr>
<tr>
<td>6. 17s. 6d.</td>
</tr>
</tbody>
</table>

**Page 251**

<table>
<thead>
<tr>
<th>Test 14</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 65 yd. 1 ft. 11 in.</td>
</tr>
<tr>
<td>2. 3½: 1-01.</td>
</tr>
<tr>
<td>3. £8 16s. 5¾d.</td>
</tr>
<tr>
<td>4. 19:33.</td>
</tr>
<tr>
<td>5. 18 yd. 2 ft. 6 in.</td>
</tr>
<tr>
<td>6. 6-9%.</td>
</tr>
</tbody>
</table>

**Page 252**

<table>
<thead>
<tr>
<th>Test 15</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ½h</td>
</tr>
<tr>
<td>2. 0-064,373,082,3; 0-000,006,44.</td>
</tr>
<tr>
<td>3. £587 10s.</td>
</tr>
<tr>
<td>4. 112 6s. 1d.</td>
</tr>
<tr>
<td>5. £15 7s. 6d.</td>
</tr>
<tr>
<td>6. £7 4s.</td>
</tr>
</tbody>
</table>

**Page 252**

<table>
<thead>
<tr>
<th>Test 16</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 0-36.</td>
</tr>
<tr>
<td>2. 1½; 8 cwt. 3 qr. 21 lb.</td>
</tr>
<tr>
<td>3. 4-2788 fur.</td>
</tr>
<tr>
<td>4. 30,008 6s. 5d.</td>
</tr>
<tr>
<td>5. 57-8%.</td>
</tr>
<tr>
<td>6. £4,418,000.</td>
</tr>
</tbody>
</table>

**Page 252**

<table>
<thead>
<tr>
<th>Test 17</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 23; 0-37.</td>
</tr>
<tr>
<td>2. £604 16s. 10d.</td>
</tr>
<tr>
<td>3. 26½%; 28:29.</td>
</tr>
<tr>
<td>4. 3 hr. 15 min. 7 sec.</td>
</tr>
<tr>
<td>5. 36 lb.</td>
</tr>
<tr>
<td>6. 3 years.</td>
</tr>
</tbody>
</table>

**Page 252**

<table>
<thead>
<tr>
<th>Test 18</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ½h; 120.</td>
</tr>
<tr>
<td>2. £1 18s. 10d.</td>
</tr>
<tr>
<td>3. 10 tons 14 cwt. 2 qr.; £67 10s.</td>
</tr>
<tr>
<td>4. £74 6s. 3d.</td>
</tr>
<tr>
<td>5. 87 sq. in.</td>
</tr>
<tr>
<td>6. £9 2s. 5d.</td>
</tr>
</tbody>
</table>

**Page 253**

<table>
<thead>
<tr>
<th>Test 19</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 33-9; 758.</td>
</tr>
<tr>
<td>2. 0-116,25; 8s. 2d.</td>
</tr>
<tr>
<td>3. 1s. 10d. per lb.</td>
</tr>
<tr>
<td>4. £36 6s. 9d.</td>
</tr>
<tr>
<td>5. £1 14s. 5d.</td>
</tr>
<tr>
<td>6. £4 13s.</td>
</tr>
</tbody>
</table>

**Page 253**

<table>
<thead>
<tr>
<th>Test 20</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 1-039,73; 0-18.</td>
</tr>
<tr>
<td>2. 4th, 2nd, 3rd, 1st.</td>
</tr>
<tr>
<td>3. 6s.</td>
</tr>
<tr>
<td>4. 560; £251 2s.</td>
</tr>
<tr>
<td>5. £9 3s. 9d.</td>
</tr>
<tr>
<td>6. 4½%.</td>
</tr>
</tbody>
</table>

**Page 253**

<table>
<thead>
<tr>
<th>Test 21</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 0-352; 242 yd.</td>
</tr>
<tr>
<td>2. 5:12; 12s. 9¾d.</td>
</tr>
<tr>
<td>3. £507 7s. 6d.</td>
</tr>
<tr>
<td>4. 3s. 4d., 5s., 11s. 8d.</td>
</tr>
<tr>
<td>5. 31 oz.</td>
</tr>
</tbody>
</table>

**Page 254**

<table>
<thead>
<tr>
<th>Test 22</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ½h; 0-16.</td>
</tr>
<tr>
<td>2. 75%; 9 lb. 10 oz.</td>
</tr>
<tr>
<td>3. £6 6s. 1d.</td>
</tr>
<tr>
<td>4. £48 2s. 6d.</td>
</tr>
<tr>
<td>5. 30s., 32s., 48s.</td>
</tr>
<tr>
<td>6. 7-9 mm.</td>
</tr>
</tbody>
</table>

**Page 254**

<table>
<thead>
<tr>
<th>Test 23</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 32 ½: 0-017,75.</td>
</tr>
<tr>
<td>2. 3:5; 8s. 3d.</td>
</tr>
<tr>
<td>3. 28; 0-05 yd.</td>
</tr>
<tr>
<td>4. 46,293.</td>
</tr>
<tr>
<td>5. 7¾ tons.</td>
</tr>
<tr>
<td>6. 73 sq. yd.</td>
</tr>
</tbody>
</table>

**Page 255**

<table>
<thead>
<tr>
<th>Test 24</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 9; 80.</td>
</tr>
<tr>
<td>2. 0-065; £4 3s. 10d.</td>
</tr>
<tr>
<td>3. £8.</td>
</tr>
<tr>
<td>4. 25½ m.p.h.</td>
</tr>
<tr>
<td>5. 467 lb. (466½).</td>
</tr>
<tr>
<td>6. 2-6 in.</td>
</tr>
</tbody>
</table>

**Page 255**

<table>
<thead>
<tr>
<th>Test 25</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 2s. 7½d.; 3:10.</td>
</tr>
<tr>
<td>2. £36 1s. 6d.</td>
</tr>
<tr>
<td>3. £4 18s. 6d.</td>
</tr>
<tr>
<td>4. 45½%.</td>
</tr>
<tr>
<td>5. £281.</td>
</tr>
<tr>
<td>6. £18 7s. 10½d.</td>
</tr>
</tbody>
</table>

**Page 256**

<table>
<thead>
<tr>
<th>Test 26</th>
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<tbody>
<tr>
<td>1. 32 ½: 82-4%.</td>
</tr>
<tr>
<td>2. £78.</td>
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<tr>
<td>3. £6 0s. 9d.</td>
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<tr>
<td>4. 1½%.</td>
</tr>
<tr>
<td>5. £12.</td>
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<tr>
<td>6. 104 sq. ft.</td>
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**Page 256**

<table>
<thead>
<tr>
<th>Test 27</th>
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<tbody>
<tr>
<td>1. 5:42; £1020, £600.</td>
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<tr>
<td>2. 0 1542.</td>
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<tr>
<td>3. £317 1s. 10½d.</td>
</tr>
<tr>
<td>4. 54 hr.</td>
</tr>
<tr>
<td>5. 182-7 sq. cm.</td>
</tr>
<tr>
<td>6. £267 15s. 1d.</td>
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**Page 256**

<table>
<thead>
<tr>
<th>Test 28</th>
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</thead>
<tbody>
<tr>
<td>1. 18s.</td>
</tr>
<tr>
<td>2. 0-000,75.</td>
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<tr>
<td>3. £25 12s. 8d.</td>
</tr>
<tr>
<td>4. £3670 (3666½).</td>
</tr>
<tr>
<td>5. 7 months.</td>
</tr>
<tr>
<td>6. 3-8 gall.</td>
</tr>
</tbody>
</table>

**Page 257**

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>1. 5-860; 0-4233; 1-16.</td>
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<tr>
<td>2. 375 ac.</td>
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<td>3. 3s. 9d.</td>
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<tr>
<td>4. 480 lb.</td>
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<td>5. £8 19s. 8d.</td>
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<tr>
<td>6. 16-1 sq. in.</td>
</tr>
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</table>
Page 257  Test 30
1. 28% .  2. 0:32175.  3. 11-7 ac.  4. 22s. 6d., 37s. 6d., 62s. 6d., 97s. 6d.  6. 2s. 71/4d.  6. £4000.

Page 258  Test 31
1. 7 yd. 1 ft.  2. 1s. 4d.; 28%.  3. 15%.  4. 221/2, 3, 41/2 cwt.  5. £24 8s. 3d.  6. 4 1 oz.

Page 258  Test 32
1. 71 m.p.h.  2. £5856 11s. 10d.  3. £520 18s. 11d.  4. 151/2%.  5. 72-8%.  6. 3 47 in.

Page 259  Paper 25
1. 38% ; 0:033.3.  2. £1 2s. 6d.; £484 8s.  3. 60%; £3 7s. 6d.  4. 1744 sq. ft.  5. £23.  6. £13 10s.

Page 259  Paper 26
1. 28.3, 5, 11.17; 1 yd. 1 ft. 11 in.  2. 17; 2s. 1d.; £291 7s. 6d.  3. 371/2%; £2 12s. 6d.  4. 2 49 ac.  5. £16 13s. 4d.  6. 12%.

Page 259  Paper 27
1. 5 mi. 914 yd.; 165.  2. 12 ft. 0 in.; £7 17s. 6d.  3. 3: 8; 27 6%.  4. 17%.  5. 9 oz.  6. £51 6s. 8d.

Page 260  Paper 28
1. 2 1/8; £0.8948.  2. £7 19s. 4d.; £4 10s. 8d.  3. £3 6s. 8d.; 261.  4. 3s. 9d. in the £.  5. £4 9 sec.; 13 m.p.h.

Page 261  Paper 29
1. 2 1/4; 34 ch. 5 yd. 2 ft.  2. £49 2s. 10½d.; £2 9s. 2d.  3. 325.; 2-8%.  4. 4½ min.  5. 494 ac.  6. 10%; 3½d. per lb.

Page 261  Paper 30
1. 495; 0:004.45.  2. £75 7s. 4d.; 4 tons 6 cwt.  3. 223%; £28.  4. 131/2 sec.  5. 1404 kg.  6. 52½%.

Page 262  Paper 31
1. 0:655; 0:000,016.  2. 10 1d.; £9 1s. 5d.  3. 14s.; 20%.  4. £1 0s. 8d.  5. £5 5s., £2 12s., £1 1s. 8d.  6. £33 6s. 8d.; 5 1/2%.

Page 262  Paper 32
1. 11 lb. 13 oz.; 39 ares.  2. 9s. 1d.; £10 5s. 1d.  3. 20s. 8d.; 12½%.  4. 16 25 cm.; 56 gm.  5. 123 Men.  6. £900.

Page 263  Paper 33
1. 2½, 3½, 5, 7, 13.23; 2093.  2. 13½; £13 6s.  3. £3 11s. 3d.  4. £930; 32½%.  5. 460 c.c.  6. 15-7 in.; 19-6 sq. in.

Page 264  Paper 34
1. 1 dozen.  2. 2nd, 4th, 5th, 3rd, 1st.  3. 2240 ac.  4. 4%.  5. £1610.  6. 56-6 cu. in.; 75-4 sq. in.

Page 264  Paper 35
1. 3 hr. 58½ min.  2. 67½%; 18½%.  3. 1s. 4½d.  4. Second kind; 5%.  5. £60 15s.  6. 27 in.

Page 265  Paper 36
1. £2800; 9s. 4d.  2. 28 cwt. 49 lb.  3. 54%; 15s. 10d.  4. 5 women.  5. £1 3s. 9d.  6. 628 lb.

Page 265  Paper 37
1. 17s. 7½d., r. 3s. 3½d.; £11 13s. 4d.  2. 50%; 2s. per lb.  3. 2-16d. per hr.  4. 1260. 280.  5. 449 gall.  6. 3-39 kg.

Page 266  Paper 38
1. 240.; 36 m.p.h.  2. 300 ft.  3. 38s. 6d.  4. 2.30 p.m., 15 min. from Q.  5. 9 min.  6. 127 m. ; 1-27 Ha.

Page 266  Paper 39
1. 37½ hr.; 77 cm.  2. £99,496 5s.  3. £4 2s. 3d.  4. 4-9 lb. per sq. ft.; 27-6 m.p.h.  5. 90 cu. in.  6. 176 sq. cm.
**ANSWERS**

**Page 267**

1. £8 18s. 6d.; 16 bills, £1. 2. 9416 yd. 3. 4 : 25; 3 : 16; 183 8%. 4. 25%. 5. £13 10s. 6. 42,800 cm. ft.; 2839 sq. ft.

**Page 268**

1. 3 : 2; 6s. 6d., 6s. 3d., 11s. 3d. 2. 24 m.p.h. 3. £281,539 3s. 4d.; £3 9s. 11d. 4. £7 11s. 3d. 5. £17 6s. 10d. 6. 5 : 3.

**Page 268**

1. 8%; 5 : 9. 2. 6d. 3. £37 2s. 1d.; 18 3 8%. 4. 373 revs. (373 1/2). 5. 41%. 6. 108.

**Page 269**

1. 228; 18s. 10d. 2. 11:26(4) km. 3. 384; 56-6%. 4. 130 gm. (129-6). 5. £43 8s. 11d. 6. 18 3 8 ft.

**Page 269**

1. £163 17s.; £3 13s. 4d. 2. £24 17s. 1d. 3. 50,400,000 gall. 4. £39 14s. 4d. 5. 589 gall. 6. 10%.

**Page 270**

1. 2.833, 2.828, 2.827. 2. 26%: 0 1%. 3. 924 yd., £69 6s. 4. 2 : 3. 5. £225. 6. £120.

**Page 271**

1. 8%; 2 8%. 2. £89 15s. 6d. 3. 58%. 4. 6%. 5. 678 gm. 6. 23 days.

**Page 271**

1. 41 3/8%; 43.2 cm. 2. 263. 3. 1.06 p.m. 4. (i); (iii). 5. 2.8 ft. 6. £96 a year.

**Page 272**

1. 0:407; £2 11s. 8d. 2. £4520 16s. 8d. 3. 71. 4. 5 1/2 tons. 5. £3 10s. 6. 10 1/2 days; £15, £10, £5.

**ANSWERS TO PART III**

**Page 273**

Oral Work

(i) 11-6; 46-2; 74-0; 84-6. (ii) 5-83; 7-21; 8-60; 9-38. (iii) 6-78; 7-62; 9-59; 3-46.

**Page 275**

EXERCISE 129

1. 23. 2. 29. 3. 34. 4. 38. 5. 47. 6. 57. 7. 68. 8. 66. 9. 99. 10. 123. 11. 243. 12. 371.

**Page 276**

EXERCISE 130

1. 0-5. 2. 0-8. 3. 0-2. 4. 0-8. 5. 0-8. 6. 0-3. 7. 0-0. 8. 0-6. 9. 0-2. 10. 1-1. 11. 1. 12. 0-5.

**Page 277**

EXERCISE 131

1. 2-6. 2. 3-9. 3. 0-52. 4. 0-28. 5. 0-058. 6. 0-152. 7. 1-14. 8. 0-119. 9. 39-4. 10. 0-334.

**Page 277**

EXERCISE 132

1. 1-41. 2. 4-47. 3. 3-16. 4. 0-548. 5. 2-72. 6. 0-869. 7. 0-628. 8. 0-199. 9. 2-21. 10. 31-0. 11. 0-308. 12. 0-904.

**Page 278**


**PART III**
### Page 278

**EXERCISE 133**

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**Page 280**

**EXERCISE 134**

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**Page 281**

**EXERCISE 135**

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**Page 282**

**EXERCISE 136**

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**Page 284**

**EXERCISE 137**

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Page 290
EXERCISE 141
1. 1-6. 2. 5-0. 3. 2-6 (3). 4. 6-9 (2). 5. 10-0-90, 10-0-88. 6. 10-0-87. 7. 10-0-78. 8. 10-0-96. 9. 10-0-18. 10. 0-45; 0-50; 0-95; 8-9 (5). 11. 7-3 (5); 7-3. 12. 0-93; 0-58; 0-35; 2-2 (4). 13. 2-5 (5); 1-2 (4). 14. 0-90; 0-45; 2-6 (2). 15. 0-78; 0-39; 2-4 (5). 16. 0-95; 0-32; 2-0 (9).

Page 293
EXERCISE 142
The indices are as follows:

Page 294
EXERCISE 143
1. 4-2. 2. 5-9. 3. 6. 4. 3-49. 5. 3-56. 6. 5-08. 7. 8-14. 8. 9-36. 9. 5-03. 10. 3-75. 11. 6-04. 12. 8-02. 13. 2-52 (3). 14. 3-35 (6). 15. 5-72 (9). 16. 7-31 (5). 17. 3-04 (4). 18. 5-05 (8). 19. 6-02 (7). 20. 8-03 (9). 21. 3-35 (4). 22. 4-57 (4). 23. 1-88 (5). 24. 1-54 (2). 25. 1-58 (8). 26. 4-07 (8). 27. 5-17 (3). 28. 5-78 (2). 29. 7-00 (6). 30. 8-00 (5). 31. 9-33 (4). 32. 1-07 (1). 33. 1-07 (5). 34. 1-00 (6). 35. 1-00 (7). 36. 9-00 (8).

Page 295
EXERCISE 144

Page 296
EXERCISE 145
1. 2. 2. 3. 3. 4. 1. 5. 0. 6. 4. 7. 5. 8. 4. 9. 7. 10. 2. 11. 4. 12. 3. 13. 5. 14. 4. 15. 6. 16. 1. 17. 18. 4. 19. 5.

Page 298
EXERCISE 147

Page 299
EXERCISE 148
1. 3-42. 2. 13-860. 3. 329-800. 4. 2-791. 5. 25-9 (7). 6. 43-0 (3). 7. 11-2 (0). 8. 12-3 (9). 9. 2-86 (8). 10. 108 (0).

Page 300
EXERCISE 149
1. 7-28 (5). 2. 2300 c.c. 3. 20-1 m. 4. 485. 5. 4-42. 6. 22-7 kg. 7. 35-3%. 8. 1-36 kg. 9. 7-42 (5) grn. 10. 1-04 ac. 11. 13-2%. 12. 39-2%. 13. 54-4; 1-78. 14. 4-51; 7-00. 15. 1-690; 27-6. 16. 8-63. 17. 118 yr. 18. 22-0 sq. in. 19. 26-3 m.p.h. 20. 247 ac.
Page 302

EXERCISE 150

1. 2.6839. 2. 3.8808. 3. 1.004. 4. 9552. 5. 4.771. 6. 2.8476. 7. 2.0418. 8. 2.4472. 9. 0.391. 10. 0.0046. 11. 0.0000934. 12. 0.000334. 13. 0.00203. 14. 0.9977. 15. 0.00148. 16. 0.0000275. 

Page 303

EXERCISE 151

1. 3.7. 2. 2.8. 3. 5.4. 4. 1.5. 5. 3.5. 6. 0.6. 7. 7.4. 8. 5.5. 9. 1.68. 10. 5.2. 11. 2.6. 12. 3.3. 13. 4.7. 14. 3.5. 15. 2.5. 16. 3.2. 17. 2.7. 18. 3.7. 19. 1.48. 20. 1.16. 21. 1.68. 22. 1.5. 23. 1.4. 24. 0.00148. 

Page 304

EXERCISE 152


Page 305

EXERCISE 153


Page 306

EXERCISE 154

1. 6.64. 2. 6.29. 3. 1.69. 4. 0.131. 5. 7.27. 6. 13.4 7. 2. 8. 3. 9. 1. 10. 2. 11. 1. 12. 1.09. 13. 1.87. 14. 45. 15. 143. 16. 8. 17. 2.29. 18. 13.0 years. 19. 14.2 years. 20. 11.9 years. 

Page 307

EXERCISE 155

1. 1.58 (5). 2. 1.65%. 3. 40-3 m.p.h. 4. 0.943. 5. 0.564. 6. 0.135. 7. 13.2. 8. 0.005,49 (5). 9. 0.005,07. 10. 0.567 litres. 11. 30.5 cm. 12. 0.0208; 6. 12. 13. 1510 tons. 14. 1.14 (5). 15. 0.397. 16. 1.21. 17. 2.57. 18. 13.5 days. 19. 5.30 ft. 20. 67,450. 

Page 308

EXERCISE 156

1. 40 sq. cm. 2. 12 sq. in. 3. 3.65 sq. in. 4. 689 sq. cm. 5. 3.87 in. 6. 4.37 cm. 7. 13.5 sq. in.; 4.12 in. 8. 22.5 sq. cm.; 7.89 (5) cm. 9. 17.5 sq. in. 10. 26.8 sq. cm. 11. 8.36 sq. in. 12. 64.9. 13. 26.4 sq. cm. 14. 8.91 cm. 15. 12 sq. in. 16. 19.0 sq. cm. 17. 6.64 sq. in. 18. 1.17 (5) sq. cm. 19. 0.16 (5) sq. in. 20. 1.11 sq. ft. 21. 4.97 cm. 22. 387 (5) sq. ft. 23. 20.1 sq. in. 24. 1.50 ac. 25. 3.9 sq. in.; 1.65 sq. in.; 1.94 in. 26. 14.7 sq. cm.; 5.88 cm. 27. 12.2 sq. in.; 5.30 in. 28. 5.17 ac.; 242 yd. 29. 2.94 ac.; 7.96 ch. 30. 16.7 sq. in. 

Page 309
<table>
<thead>
<tr>
<th>Page 313</th>
<th>ANSWERS</th>
<th>EXERCISE 157</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 218 c.c.</td>
<td>2. 304 cu. in.</td>
<td>3. 1060 cu. ft.</td>
</tr>
<tr>
<td>4. 662 (5) cu. ft.</td>
<td>5. 142 in.</td>
<td>6. 23,100 cu. ft.</td>
</tr>
<tr>
<td>7. 63,800 cu. ft.</td>
<td>8. 180 c.c.; 264 sq. cm.</td>
<td>9. 9-12 (5) cu. ft.</td>
</tr>
<tr>
<td>10. 97,500 gall.</td>
<td>11. 13,100 gall.</td>
<td>12. 1380 sq. cm.; 18-4 kg.</td>
</tr>
<tr>
<td>13. 1,750,000 gall.</td>
<td>14. 10,800 cu. ft.</td>
<td>15. 168 cu. in.</td>
</tr>
<tr>
<td>16. 16-5 sq. cm.</td>
<td>17. 22-5 cu. ft.</td>
<td>18. 2 ft. 8 in.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Page 316</th>
<th>EXERCISE 158</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 15 cu. in.</td>
<td>2. 40 c.c.</td>
</tr>
<tr>
<td>5. 18 sq. cm.</td>
<td>6. 192 c.c.</td>
</tr>
<tr>
<td>9. 58-8 cu. in.</td>
<td>10. 5 in.; 0-63 in.</td>
</tr>
<tr>
<td>12. 96 sq. cm.</td>
<td>13. 79-6 sq. in.</td>
</tr>
<tr>
<td>15. 9 in.</td>
<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Page 318</th>
<th>EXERCISE 159</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 85-5 (5) in.; 593 sq. in.</td>
<td>2. 159 yd.; 796 sq. yd.</td>
</tr>
<tr>
<td>3. 39-3 (5) yd.; 247 yd.</td>
<td>4. 611.</td>
</tr>
<tr>
<td>5. 27-8 sq. cm.</td>
<td>6. 31-8 cu. in.</td>
</tr>
<tr>
<td>7. 13-1 kg.</td>
<td>8. 970 sq. ft.</td>
</tr>
<tr>
<td>9. 2-02 ft.</td>
<td>10. 664 sq. in.</td>
</tr>
<tr>
<td>11. 13-86 sq. in.</td>
<td>12. 19-8 in.</td>
</tr>
<tr>
<td>15. 10-41 in.</td>
<td>16. 20-8 in.</td>
</tr>
<tr>
<td>17. 144 (5) sq. in.</td>
<td>18. 144 (5) sq. in.</td>
</tr>
<tr>
<td>19. 1-85 pt.</td>
<td>20. 13-0 hr.</td>
</tr>
<tr>
<td>21. 2-21 in.</td>
<td>22. 3-31 cwt.</td>
</tr>
<tr>
<td>23. 177-5 sq. yd.</td>
<td>24. 11-1 in.</td>
</tr>
<tr>
<td>25. 0-003,12 in.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Page 320</th>
<th>EXERCISE 160</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 20 cu. in.</td>
<td>2. 75-4 c.c.</td>
</tr>
<tr>
<td>3. 50-3 cu. in.</td>
<td>4. 8-49 cu. in.</td>
</tr>
<tr>
<td>5. 207 cu. in.</td>
<td>6. 151 sq. cm.</td>
</tr>
<tr>
<td>7. 204 sq. cm.</td>
<td>8. 52-5 sq. in.</td>
</tr>
<tr>
<td>9. 198-5 sq. in.</td>
<td>10. 185 sq. in.</td>
</tr>
<tr>
<td>11. 240 (5) in.</td>
<td>12. 12 sq. cm.</td>
</tr>
<tr>
<td>13. 8 in.</td>
<td>14. 530 cu. ft.; 278 sq. ft.</td>
</tr>
<tr>
<td>15. 5-46 in.</td>
<td>16. 8190 (8190)</td>
</tr>
<tr>
<td>17. 299 sq. ft.</td>
<td>18. 3-58 in.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Page 322</th>
<th>EXERCISE 161</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 76-2 cu. in.; 86-9 sq. in.</td>
<td>2. 202 cu. in.; 166 (5) sq. in.</td>
</tr>
<tr>
<td>3. 10-1 cm.</td>
<td>4. 0-885 in.</td>
</tr>
<tr>
<td>5. 60-5 c.c.</td>
<td>6. 64-5 sq. in.</td>
</tr>
<tr>
<td>7. 15 in.</td>
<td>8. 13-5.</td>
</tr>
<tr>
<td>9. 1-59 cm.</td>
<td>10. 102 sq. in.</td>
</tr>
<tr>
<td>11. 20-45 sq. in.</td>
<td>12. 373 gm.</td>
</tr>
<tr>
<td>13. 7-205 in.</td>
<td>14. 0-490 in.</td>
</tr>
<tr>
<td>15. 1570 c.c.; 676 sq. cm.</td>
<td>16. 5-24 cm.</td>
</tr>
<tr>
<td>17. 20-50 cm.; 13-8 cm.</td>
<td>18. 2-50 cm.; 13-8 cm.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Page 325</th>
<th>ANSWERS</th>
<th>EXERCISE 162</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 4 : 9</td>
<td>2. 25 : 4</td>
<td>3. 25 : 36</td>
</tr>
<tr>
<td>4. 8 : 27</td>
<td>5. 125 : 64</td>
<td>6. 25 : 16</td>
</tr>
<tr>
<td>7. 9 : 25</td>
<td>8. 67-5 lb.</td>
<td>9. 1½ gal.</td>
</tr>
<tr>
<td>10. 12-44 tons</td>
<td>11. 4 : 5</td>
<td>12. 5 : 3</td>
</tr>
<tr>
<td>13. 25 : 9</td>
<td>14. 27 days</td>
<td>15. 1-12 : 1</td>
</tr>
<tr>
<td>16. 1-14 : 1</td>
<td>17. 1-14 : 1</td>
<td>18. 1-16 : 0-289 sq. in.</td>
</tr>
<tr>
<td>19. 10% ; 33-1%</td>
<td>20. 1-6 ; 0-6 sq. in.</td>
<td></td>
</tr>
<tr>
<td>21. 288 pt.</td>
<td>22. 120 sq. in.</td>
<td>23. 22-3 min.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Page 327</th>
<th>EXERCISE 163</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 25s. 9d.</td>
<td>2. 10s. 6d.</td>
</tr>
<tr>
<td>3. 180 sq. in.</td>
<td>4. 153 sq. ft. (152½)</td>
</tr>
<tr>
<td>5. £7 7s. 6d.</td>
<td>6. £16</td>
</tr>
<tr>
<td>7. 110 in.</td>
<td>8. £16</td>
</tr>
<tr>
<td>11. 113 sq. dm.</td>
<td>12. 54-6 sq. in.</td>
</tr>
<tr>
<td>13. 1446 tons</td>
<td>14. 117 cu. in.</td>
</tr>
<tr>
<td>15. 286 in.; 45 sq. in.</td>
<td>16. 21 in.</td>
</tr>
<tr>
<td>17. 12½ cu. ft.</td>
<td>18. 1564 tons</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Page 331</th>
<th>EXERCISE 164</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 67 (5) lb.</td>
<td>2. 13 kg.</td>
</tr>
<tr>
<td>3. 23-1 kg.</td>
<td>4. 8-33</td>
</tr>
<tr>
<td>5. 52 c.c.</td>
<td>6. 34 cu. in.</td>
</tr>
<tr>
<td>7. 13 kg.</td>
<td>8. 0-73</td>
</tr>
<tr>
<td>9. 2-3</td>
<td>10. 13-2 cm.</td>
</tr>
<tr>
<td>11. 130 mm.</td>
<td>12. 43 cm.</td>
</tr>
<tr>
<td>13. 1200 kg.</td>
<td>14. 17-9 mm.</td>
</tr>
<tr>
<td>15. 46-5 (5) lb.</td>
<td>16. 20s.</td>
</tr>
<tr>
<td>17. 11-7</td>
<td>18. 0-67</td>
</tr>
<tr>
<td>21. 3-83 c.c.</td>
<td>22. 0-078</td>
</tr>
<tr>
<td>23. 0-63 (5)</td>
<td>24. 67 gm.</td>
</tr>
<tr>
<td>25. 13-14; 13</td>
<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Page 333</th>
<th>EXERCISE 165</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 36 gm.</td>
<td>2. 44 gm.</td>
</tr>
<tr>
<td>3. 39 c.c.</td>
<td>4. 5-8 oz.</td>
</tr>
<tr>
<td>5. 9-6 oz.</td>
<td>6. 27 cu. ft.</td>
</tr>
<tr>
<td>7. 270 gm. (274)</td>
<td>8. 250 gm.</td>
</tr>
<tr>
<td>9. 240 gm. (244)</td>
<td>10. 27; 110 gm. (111)</td>
</tr>
<tr>
<td>11. 120 gm. (117); 120 gm. (123)</td>
<td>12. 14 gm. (13-8)</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Page 337</th>
<th>EXERCISE 166</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. £19 11s. 8d.</td>
<td>2. £22 10s. 8d.</td>
</tr>
<tr>
<td>3. £47 7s. 1d.</td>
<td>4. £19 11s. 8d.</td>
</tr>
<tr>
<td>5. 665 7s. 6d.</td>
<td>6. £22 15s. 9d.</td>
</tr>
<tr>
<td>7. £22 15s. 9d.</td>
<td>8. £22 15s. 9d.</td>
</tr>
<tr>
<td>9. £12 15s. 9d.</td>
<td>10. £12 15s. 9d.</td>
</tr>
<tr>
<td>11. £74 8s. 2d.</td>
<td>12. £71 2s. 8d.</td>
</tr>
<tr>
<td>13. £135 1s. 8d.</td>
<td>14. £64 15s. 9d.</td>
</tr>
<tr>
<td>15. £219 2s. 8d.</td>
<td>16. £49 8s. 7d.</td>
</tr>
<tr>
<td>17. £299 8s. 7d.</td>
<td>18. £505 13s. 0d.</td>
</tr>
</tbody>
</table>
ANSWERS

Page 340

Exercise 167

1. 1; 04; 1; 03; 1; 025; 1; 5; 1; 0375.
2. 0.95; 0.96; 0.968; 0.9375; 0.9; 0.8725.
3. 4, 12, 23, 25, 13, 16, 4, 23.

Page 343

Exercise 168

1. £162; 16; 126; 6; 1677.
2. £89; 89; £97.93.
3. £1424; 1424; 23.
4. £546; 546; 02 (9).
5. £2036; 2036; 1 (9).
6. £1,000; 1,000; 025; 1 (9).
7. £130; 8, 696; 9, 293 (5).
8. £243 (7).
9. £97 (1).
10. £323 (2).
11. £2336.
12. £59 (5).
13. £949 (5).
14. £91; 1000; £1,025 (9).
15. 6; 01; 1 yr.
16. 4; 5; 17, 4; 3; 18, 6; 01 yr.
17. 4; 11 (5); 45 (2) yr.
18. £5; 5; 4.
19. £3; 233.
20. 0.922; 26, 1; 025; 29, 300.

Page 346

Exercise 169

1. £390.
2. £3,645.
3. £5760.
4. 20; 90.
5. 12%.
6. 13; gain.
7. 44s.
8. £40 5s.
9. 2 ft; 8 in.
10. 160; 11.
11. £18; 12.
12. 12; 9.
13. 4l; 1.
14. 12; 23; 17.
15. 50; 9.
16. 55; 7.
17. £250.
18. £5.
19. 26; 2.
20. 26; 2.
21. 26; 2.
22. 26; 2.
23. 26; 2.
24. 26; 2.
25. 26; 2.
26. 26; 2.
27. 26; 2.

Page 349

Exercise 170

1. 5%; 2. 2s per lb.
3. 46; 2s. 10d per lb.
4. 1s.
5. 19%.
6. 14s. 6d.
7. 50 gm.
8. 1.7.
9. 3; 4.
10. 3; 1.
11. 5; 8.
12. 37%.
13. 96%; 40%.
14. 1; 19.
15. 9; 4.
16. 40%.
17. £1920; £690.
18. £7; 14s.
19. 51%.
20. 86 lb; 8643 lb.

Page 353

Exercise 171

1. £400; 24.
2. £40; 2.
3. 400 shares; £8.
4. 200 shares; £40.
5. £24.
6. £400.
7. 4s.
8. 10s.
9. 4%.
10. 6%.

Page 354

Exercise 172

1. £81; 4 5.
2. £151 10s.; £6.
3. £101 5s.; £6.
4. £112 10s.; £12 10s.
5. £127 10s.; £6 15s.
6. 40; £3 4s.
7. £160; £4 16s.
8. £600; £10 10s.
9. £600; £10 10s.
10. £115; £13 10s.
11. £15.
12. £13 10s.
13. £183 15s.
14. £540.
15. £480 10s.
16. 6; 17; 5; 19.
17. 10; 6; 19.
18. 6; 20; 7; 10.
19. 7; 10.
20. 7; 10.
21. 17; 2; 19.
22. 17; 2; 19.
23. 4; 7; 10.
24. 4; 7; 10.
25. 4; 7; 10.
26. 4; 7; 10.
27. 4; 7; 10.
28. 4; 7; 10.

Page 357

Exercise 173

1. 180.
2. 200.
3. 14s. 8d.
4. 2s. 9d.
5. £4 5s.
6. £120.
7. 480; £36.
8. £13 10s. gain.
9. 720; 2 more.
10. £1000; £12 less.
11. 2560; 8 less.
12. 13; 8% p.a.
13. £13 15s.; 11%.
14. 8; 4.
15. 7; 3; 4.
16. £139 7s. 6d.; £138 2s. 6d.
17. £99 10s.; £98 10s.
18. £151 17s. 6d.; £140 12s. 6d.
19. £415 10s.; £403 10s.
20. 2s. 6d.
21. 10s.

Page 362

Exercise 174

1. £240; £12.
2. £240; £12.
3. £200 stock; £14.
4. £50 stock; £15.
5. £450.
6. £324.
7. 80.
8. 110.
9. £6 per cent.
10. £5 per cent.

Page 363

Exercise 175

1. £336; £12.
2. £180; £7 10s.
3. £97; £28 17s. 6d.
4. £599 4s.; £33 12s.
5. £116 6s.; £4 7s. 3d.
6. £500 stock; £38 8s.
7. £666 13s. 4d. stock; £16.
8. £1250 stock; £70.
9. £320 4s. 6d. stock; £7 13s. 8d.
10. £454 1s. 1d. stock; £9 1s. 7d.
11. £675.
12. £522.
13. £342.
14. £282 13s. 9d.
15. £9 6s. 8d. per cent.
16. £10 per cent.
17. £3 2s. 6d. per cent.
18. £4 1s. 1d. per cent.
19. 125.
### Page 365

**EXERCISE 176**

| 1. | £312 stock.       | 2. | £584 stock.       | 3. | £56 5s.          |
| 4. | £34 15s. 9d.     | 5. | £570 more.        | 6. | £30 loss.        |
| 7. | £366 15s. 4d. more. | 8. | £10 more.        | 9. | £8 18s. 7d. more.|
| 10. | £5 less.    | 11. | £208.          | 12. | £2668.         |
| 13. | £135; £3400 stock. | 14. | £2 more.       | 15. | £4725.         |
| 16. | £275 stock.    | 17. | £800 stock.      | 18. | 94s.          |
| 20. | 100s.       | 21. | 76s.            | 22. | 110s.         |
| 24. | £1295.      |

### Page 369

**EXERCISE 177**

| 1. | 33 7 ft. per sec. | 2. | 11 6 m.p.h.     | 3. | 20 7 m.p.h.     |
| 4. | 20 1 m. per sec. | 5. | 34 7 cu. in.    | 6. | 4 54 litres.   |
| 7. | 84 60 gm. per cc. | 8. | 0 42 0 oz. per cu. in. | 9. | 5s. 3d.       |
| 10. | 13 5 fr.   | 11. | 4s. 4d.         | 12. | 3s. 3d.      |
| 13. | 73 9 fr. | 14. | 73 9 litre.    | 15. | 1 47 fr.    |
| 16. | 4 54 litres. | 17. | 0 76 2 kg.   | 18. | £128.        |
| 19. | 10 0 lb.   | 20. | 19 8 lb. per sq. in. | 21. | 2 59 kg. per sq. cm. |
| 22. | Direct £146 more. | 23. | Direct 16s. more. |

### Page 372

**EXERCISE 178**

| 1. | 7 1/8 m.p.h.; 8 1/2 m.p.h. | 2. | 4 1/2 hr.; 9 1/2 hr. | 3. | 48 6 m.p.h.       |
| 4. | 18 sec.            | 5. | 77 yd.; 12 sec.  | 6. | 20 m.p.h.        |
| 7. | 88 yd.; 30 m.p.h.  | 8. | 22 1/2 sec.    | 9. | 15 m.p.h.        |
| 10. | 15 min. | 11. | 7 1/2 mi.     | 12. | 12 sec.         |
| 13. | 44 yd.           | 14. | 61 6 sec.     | 15. | 3 35 p.m.   |

### Page 374

**EXERCISE 179**

| 1. | 10 1/2 min., 43 1/2 min. past 5; 21 1/2 min., 54 3/4 min. past 7. | 2. | 16 1/2 min. past 3. | 3. | 16 1/2 min. past 9. |
| 4. | 21 1/2 min., 43 1/2 min. past 6. | 5. | 48 1/2 min. to 6. | 6. | 111 1/2 min to 9. |
| 7. | 48 1/2 min. to 7. |

### Page 385

**EXERCISE 180**

| 1. | 65%.       | 2. | 210 out or 150 not out. |
| 4. | 46-5625, 45-2025 sq. cm.; 1 fig.; 0-68 sq. cm. | 6. | 459 lb.       |
| 8. | 23 5 (5) mi. | 7. | 38 3/4%; 25%. |
| 10. | 19s. 11d.  | 11. | 25 ft.; 92,000 cu. ft. |
| 13. | 1-83 in. per min. | 14. | 242 tons; £176. |
| 16. | 8s. a year. | 17. | 50%.    |
| 19. | 82 sq. cm. | 20. | 15s 1/2 hr.; 15 1/2 hr. |
| 25. | 30 1/2% | 26. | £4 5s. 7d.; £1 17s. 9d. |
| 28. | 79 m.p.h., 59 m.p.h. | 29. | 4 4 ft. per sec. |
| 31. | 64% | 32. | £1320 7s. 6d. |
| 34. | 2-11 kg. per sq. cm. | 35. | 1-41 in.; 1-89 cu. in.; 10-9 sq. in. |
| 36. | £70. | 37. | 12.35 p.m.; 16-7 mi. (16%), from X. |
| 39. | 22 3/4 m.p.h.; 20 1/2 m.p.h. | 40. | 30 min. |
| 41. | £19 10s. | 42. | 250 cu. in.; 70 cu. in. |
| 44. | 21/2% | 45. | £2300. |
| 47. | 4 1/2% p.a. | 48. | 63 m.p.h.; 36 1/2 m.p.h. |
| 50. | 12,350 ft.; 24 0/4. | 51. | 15 1/2% |

**Test 381**

| 1. | £12,876. | 2. | 23 3/4%. | 3. | 0 308; 3 33. |
| 4. | 0 293 (s); 0 579. | 5. | 2 78 yd. | 6. | 13 7 in. |

**Test 381**

| 1. | 1. | 2. | 14 6. | 3. | 4 07; 0 348. |
| 4. | 0 009,98; 0 853 (s). | 5. | 730 tons. | 6. | 1 93 in. |

**Test 382**

| 1. | 10 08; 0 756. | 2. | 15 9%. | 3. | 3 667; 0 270. |
| 4. | 68 0 yd. | 5. | £94 9s. 9d. | 6. | 5 57 in.; 66 8 cu. in. |

**Test 382**

<p>| 1. | 0 304; 0 0243. | 2. | 1 09 yd. | 3. | 2 23 5 s. 4d. |
| 4. | 0 231; 0 696. | 5. | 6 7 in.; 8 55 (s) sq. in. | 6. | 1 55 in. |</p>
<table>
<thead>
<tr>
<th>Page 393</th>
<th>Paper 57</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 276 gm.; 33%</td>
<td>2. 0.81; 19.8</td>
</tr>
<tr>
<td>4. 1.05 in.</td>
<td>5. 0.04</td>
</tr>
<tr>
<td>6.</td>
<td>£25 16s. 11d.</td>
</tr>
<tr>
<td>3.</td>
<td>£31 15s. 6d. at 90.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Page 394</th>
<th>Paper 58</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 12% 0.01</td>
<td>2. 1.03; 18</td>
</tr>
<tr>
<td>4. 127 cu. in.; 170 sq. in.</td>
<td>5. £13 4s.</td>
</tr>
<tr>
<td>3. 7 yr. (6-90); 12 yr.</td>
<td>6. £288.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Page 395</th>
<th>Paper 59</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 4:5; 0.611</td>
<td>2. 15 sec.</td>
</tr>
<tr>
<td>4. 8.84</td>
<td>5. 160 lb. per cu. ft.</td>
</tr>
<tr>
<td>3. 0.56 in.</td>
<td>6. £8 16s. 1d.</td>
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<thead>
<tr>
<th>Page 396</th>
<th>Paper 60</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 121 cu. in.; 1.26</td>
<td>2. 1.55; 0.356 (5)</td>
</tr>
<tr>
<td>4. 658 tons; £3 1s. 11d.</td>
<td>5. 74.5%; 25-5%</td>
</tr>
<tr>
<td>3. 8s.</td>
<td>6. £8000 stock.</td>
</tr>
</tbody>
</table>

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<tr>
<th>Page 397</th>
<th>Paper 61</th>
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</thead>
<tbody>
<tr>
<td>1. 52%; 363.</td>
<td>2. 70.5 cm.</td>
</tr>
<tr>
<td>4. 4.8; 4.3; 1.8</td>
<td>5. £100; 4s. 3d.</td>
</tr>
<tr>
<td>3. £27 12s. 3d.</td>
<td>6. 4.9%; 15% less.</td>
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<tr>
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<th>Paper 62</th>
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<tbody>
<tr>
<td>1. 9.49 kg. per sq. cm.; 0.160</td>
<td>2. 25 days</td>
</tr>
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<td>4. £345</td>
<td>5. 17; 16</td>
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<tr>
<td>3. 395 cm.</td>
<td>6. 5.%; £1488 4s. 2d.</td>
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</table>

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<tr>
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<th>Paper 63</th>
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<tbody>
<tr>
<td>1. 67s.; 76 (4) min.</td>
<td>2. £710s. loss</td>
</tr>
<tr>
<td>4. £23 5s. 2s. 8%</td>
<td>5. 65.1 gm.</td>
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<tr>
<td>3. 140 c.c.</td>
<td>6. At 114%</td>
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<tr>
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<th>Paper 64</th>
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<tbody>
<tr>
<td>1. 13 lb. 15 oz.; £4 3s. 6d.</td>
<td>2. £5 8s.</td>
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<td>4. 1.65 in.</td>
<td>5. 16,500 cu. ft.</td>
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<td>3. 33%</td>
<td>6. £19 8s. 11d. more.</td>
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### ANSWERS TO APPENDIX, PART I

#### EXERCISE 3 (a)

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<tbody>
<tr>
<td>1. 2³, 3².</td>
<td>2. 2.19.</td>
<td>3. Prime.</td>
<td>4. 2³², 5.</td>
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<tr>
<td>5. 2² ³.</td>
<td>6. Prime.</td>
<td>7. 3.5.7.</td>
<td>8. 4.7.</td>
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<td>9. Prime.</td>
<td>10. 7.17.</td>
<td>11. 2³², 7.</td>
<td>12. 3².5.</td>
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<tr>
<td>13. 2³, 7².</td>
<td>14. 3.7.11.</td>
<td>15. 3.7.13.</td>
<td>16. 7³.</td>
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<td>17. 2³², 7.</td>
<td>18. 2³².3³.7.</td>
<td>19. 3.11.19.</td>
<td>20. 3².</td>
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<tr>
<td>21. 3², 5.7.11.</td>
<td>22. 2³².3³.71.11.</td>
<td>23. 5.11³.13.</td>
<td>24. 2³².7.11².</td>
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#### EXERCISE 5 (a)

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<td>2. 7⁵.</td>
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<td>4. 2¹⁰.</td>
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<td>5. 3⁶.</td>
<td>6. 7².</td>
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<td>8. d⁴.</td>
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<td>9. 7⁶.</td>
<td>10. 3⁶.</td>
<td>11. a³⁶.</td>
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<td>14. 1.</td>
<td>15. 2³², 7³.</td>
<td>16. a³⁵.</td>
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<td>17. a³², 8.</td>
<td>18. 2⁴.7.</td>
<td>19. 2³², 5³.</td>
<td>20. 2³³.</td>
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<tr>
<td>21. 2³², 5³.</td>
<td>22. 2³², 3³.7².</td>
<td>23. 3³, 5².</td>
<td>24. 2³², 5³.</td>
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#### EXERCISE 6 (a)

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<td>1. 2³, 3³, 7³.</td>
<td>2. 2³, 3³, 5³².</td>
<td>3. 3³; 2², 3³, 5³.</td>
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<td>4. 2³²; 2³², 3³, 7³.</td>
<td>5. 3³.</td>
<td>6. 2³².</td>
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<tr>
<td>7. 18.</td>
<td>8. 18.</td>
<td>9. 32.</td>
<td>10. 45.</td>
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<tr>
<td>15. 15³.</td>
<td>16. 154.</td>
<td>17. 245.</td>
<td>18. 286.</td>
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<td>19. 40³.</td>
<td>20. 18.</td>
<td>21. 33.</td>
<td>22. 42.</td>
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<td>27. 10. 90.</td>
<td>28. 12. 504. 882. 126.</td>
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<td>29. 1, 4, 9, 6, 5, 9, 4, 1, 0; 3, 7, 8.</td>
<td>30. 126 yd.</td>
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<td>31. 2³.</td>
<td>32. 60 in.</td>
<td>33. 3³, 3³², 11.</td>
<td>24. 3³, 3³², 11.</td>
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<tr>
<td>35. 2³², 5³, 23.</td>
<td>36. 2³², 3³², 7³.</td>
<td>37. 2³, 3³.</td>
<td>38. 2³², 3³.</td>
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<tr>
<td>39. 2³², 5³.</td>
<td>40. 3³², 7.11.</td>
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<tr>
<td>1. 28.</td>
<td>2. 42.</td>
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<td>7. 33.</td>
<td>8. 11.</td>
<td>9. 175.</td>
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<td>19. 143.</td>
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<tbody>
<tr>
<td>1. 2^3 \cdot 3.7</td>
<td>2. 2^3 \cdot 3^2 \cdot 5</td>
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<td>4. 2^4 \cdot 3^3 \cdot 13</td>
<td>5. 2^2 \cdot 3 \cdot 5 \cdot 7</td>
</tr>
<tr>
<td>7. 2^3 \cdot 3 \cdot 11 \cdot 19</td>
<td>8. 2^4 \cdot 3^2 \cdot 5 \cdot 7</td>
</tr>
<tr>
<td>10. 2 \cdot 3^2 \cdot 7^2</td>
<td>11. 2^2 \cdot 3 \cdot 7^2</td>
</tr>
<tr>
<td>13. 3 \cdot 5^2 \cdot 7 \cdot 11</td>
<td>14. 2 \cdot 3 \cdot 7^2 \cdot 11 \cdot 13</td>
</tr>
<tr>
<td>16. 2 \cdot 3^2 \cdot 5 \cdot 7</td>
<td>17. 2 \cdot 3 \cdot 5^2 \cdot 7</td>
</tr>
<tr>
<td>19. 2^3 \cdot 3^2 \cdot 5 \cdot 7 \cdot 11</td>
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<tbody>
<tr>
<td>1. 20.</td>
<td>2. 35 ft.</td>
</tr>
<tr>
<td>7. 20 days.</td>
<td>8. 1 ft. 6 in.</td>
</tr>
<tr>
<td>13. 25, 35, 115, 161, 175, 575, 805.</td>
<td>14. 41; 26.</td>
</tr>
</tbody>
</table>

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<tr>
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<th>EXERCISE 13 (a)</th>
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</thead>
<tbody>
<tr>
<td>1. 98 sq. in.</td>
<td>2. 64 sq. ft.</td>
</tr>
<tr>
<td>5. 450 sq. in.</td>
<td>6. 70 sq. ft.</td>
</tr>
<tr>
<td>9. 120 sq. in.</td>
<td>10. 432 sq. in.</td>
</tr>
<tr>
<td>12. 5 ft.; 58 ft.</td>
<td>13. 72.</td>
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</tbody>
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<th>Page 417</th>
<th>EXERCISE 14 (a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 24 ac.</td>
<td>2. 10 ac.</td>
</tr>
<tr>
<td>5. 80 ac.</td>
<td>6. 42 sq. in.</td>
</tr>
<tr>
<td>9. 254 sq. ft.</td>
<td>10. 107 sq. ft.</td>
</tr>
<tr>
<td>13. 440 sq. ft.</td>
<td>14. 99 sq. ft.</td>
</tr>
<tr>
<td>17. 1380 sq. ft.</td>
<td>18. 22 sq. ft.; 228 sq. in.</td>
</tr>
</tbody>
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<tr>
<th>Page 418</th>
<th>ANSWERS TO APPENDIX</th>
<th>EXERCISE 15 (a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 210.</td>
<td>2. 648.</td>
<td>3. 64.</td>
</tr>
<tr>
<td>6. 960 cu. in.</td>
<td>7. 36 cu. ft.</td>
<td>8. 96 cu. in.</td>
</tr>
<tr>
<td>10. 1440 cu. ft.</td>
<td>11. 864.</td>
<td>12. 6 ft.; 45 sq. ft.</td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th>Page 419</th>
<th>EXERCISE 16 (a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 1180 cu. in.</td>
<td>2. 374 cu. in.</td>
</tr>
<tr>
<td>5. 744 sq. ft.; 5952 cu. ft.</td>
<td>6. 432 cu. ft.</td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th>Page 420</th>
<th>EXERCISE 18 (a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 24s.</td>
<td>2. 5s. 4d.</td>
</tr>
<tr>
<td>6. £1.</td>
<td>7. 15.</td>
</tr>
<tr>
<td>15. 12.</td>
<td>16. 15 pesetas.</td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th>Page 421</th>
<th>EXERCISE 21 (a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. £30.</td>
<td>2. 2 hr. 30 min.</td>
</tr>
<tr>
<td>5. 15s. 9d.</td>
<td>6. 24 min.</td>
</tr>
<tr>
<td>10. No answer.</td>
<td>11. 5 sq. in.</td>
</tr>
<tr>
<td>14. 9 days.</td>
<td>15. 26, 14 lb.</td>
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<td>1. 3.</td>
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<tr>
<td>6. 3.</td>
<td>7. 1.</td>
</tr>
<tr>
<td>15. 3.</td>
<td>16. 3.</td>
</tr>
<tr>
<td>20. 1.</td>
<td>21. 1\frac{1}{6}</td>
</tr>
<tr>
<td>25. 2.</td>
<td>26. 2.</td>
</tr>
<tr>
<td>30. 10.</td>
<td>31. 5.</td>
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<th>EXERCISE 24 (a)</th>
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<tbody>
<tr>
<td>1. 7.</td>
<td>2. 1.</td>
</tr>
<tr>
<td>7. 4\frac{1}{2}.</td>
<td>8. 4\frac{1}{2}.</td>
</tr>
<tr>
<td>13. 1\frac{1}{2}.</td>
<td>14. 3.</td>
</tr>
<tr>
<td>19. 1\frac{1}{2}.</td>
<td>20. 1\frac{1}{2}.</td>
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### EXERCISE 29 (a)

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### EXERCISE 39 (a)

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Page 441

D 5

1. 2 cwt. 3 qr. 15 lb. 1 oz.
2. 37 tons 3 cwt.
3. 5 tons 15 cwt. 9 lb.
4. 18 mi. 2 ch.
5. 9 mi. 7 fur. 37 p.
6. 17 pk. 1 gall. 1 qt.
7. 1 qr. 11 lb.
8. 11 cwt. 62 lb.
9. 2 tons 8 cwt. 2 qr. 18 lb. 12 oz.
10. 4 fur. 6 ch.
11. 1 fur. 3 ch. 7 yd.
12. 5 fur. 8 ch. 2 yd. 9 in.
13. 2 ac. 6 sq. ch. 403 sq. yd. 2 sq. ft.
14. 4 sq. in.
15. 1 qr. 7 bush. 3 gall. 1 qt. 1 pt.

Page 442

D 6

1. £1 9s. 2. £1 6s. 3d.
2. £2 4s. 6d.
3. £2 19s. 10½d.
4. £4 17s. 2d.
5. £7 8s. 4d.
6. £40 4s. 8d.
7. £54 8s. 8d.
8. £1567 16s. 10½d.
9. £833 19s. 2d.
10. £17,532 17s. 2d.
11. £26,091 16s. 10½d.
12. £19,579 4s. 4d.
13. 9 tons 8 cwt. 2 qr. 8 lb.
14. 124 tons 13 cwt. 23 lb.
15. 241 mi. 6 fur. 2 ch. 20 yd.
16. 1230 mi. 7 fur. 5 ch.
17. 278 ac. 2 r. 30 a.
18. £14 3s. 7d.
19. £22 13s. 5¾d.
20. £23 13s. 10¾d.
21. £7 1¼s.
22. 5 yd. 2 ft. 7 in.
23. 161 yd. 1 ft. 8 in.
24. 7 tons 3 cwt. 20 lb.
25. 5 tons 5 cwt. 1 qr. 4 lb.
26. £34 18s. 33d.
27. 3 mi. 1 fur. 7 ch. 14 yd.; 71 yd.
28. 1 qr. 1 lb. 8 oz.; 60 oz.
29. 3 gall. 1 pt.; 43 pt.
30. 16 ac. 2033 sq. yd.; 40 sq. yd.
31. 70. 60. 24. 61. 78.

Page 443

D 7

1. 9.
2. 6½.
3. 4½.
4. 2½.
5. 1½.
6. 1½.
7. 7½.
8. 1½.
9. 1½.
10. 1½.
11. 9½.
12. 2½.
13. 3½.
14. 4½.
15. 3½.
16. 2½.
17. 2½.
18. 4½.
19. 3½.
20. 2½.
21. 1½.
22. 3½.
23. 2½.
24. 1½.
25. 4½.
26. 4½.
27. 12½.
28. 12½.
29. 3½.
30. 3½.
31. 2½.
32. 10.
33. 2½.
34. 1½.
35. 21½.
36. 7½.
37. 5½.
38. ½.
39. 12½.
40. 1.

Page 444

D 8

1. 1½.
2. 4.
3. 1½.
4. 1½.
5. 1½.
6. 3½.
7. 2.
8. 7½.
9. 20.
10. 9.
11. 1½.
12. 1½.
13. 3.
14. 1.
15. 5.
16. 5.
17. 1½.
18. 4½.
19. 3.
20. ½.
21. 21.
22. 18.
23. 24.
24. 8½.

Page 445

D 9

1. 9.
2. 3½.
3. 1½.
4. 3½.
5. 3½.
6. 1½.
7. 8½.
8. 5½.
9. 2½.
10. 1½.
11. 2½.
12. 9½.
13. 1½.
14. 1½.
15. 3½.
16. 5.
17. 1½.
18. 1½.
19. 2¼.
20. 2½.
21. 4½.
22. 1¾.
23. ¾.
24. 4.
25. 3½.
26. 3½.
27. 1½.
28. 3½.
29. 14½.
30. 1.
31. 3½.
32. 12½.
33. 66½.
34. 12½.
35. 18VISION.
36. 1.
37. 8½.
38. 2½.
39. 1½.
40. 4½.
41. 24.
42. ½.
43. 8½.
44. 0.
45. ½.

Page 446

D 10

1. 3½.
2. 1.
3. 1½.
4. 0½.
5. 0½.
6. 11½.
7. 3½.
8. 1½.
9. 0½.
10. 7½.
11. 0½.
12. 0½.
13. 67½.
14. 70½.
15. 44½.
16. 50½.
17. 47½.
18. 48½.
19. 50½.
20. 50½.
21. 10½.
22. 10½.
23. 17½.
24. 57½.
25. 82½.
26. 5½.
27. 918½.
28. 2½.
29. 23½.
30. 38½.
D 11

1. 65-1, 65-1, 759-5, 0-7595.
2. 65-1, 65-1, 759-5, 0-7595.
3. 0-47, 0-0047, 1-034, 1-128.
5. 0-1843, 0-6596, 3-589.
6. 10-2.
7. 249898.
8. 0-001, 111.
9. 43043.
10. 194.
11. 20, 54.702.
12. 24.
13. 0-001711.
15. 0-001, 555.2.
16. 0-001, 555.2.
17. 33.
18. 11-4.
19. 32.
20. 33.
21. £3 14s. 2d.
22. £3 14s. 2d.
23. £7 7s. 11d.
24. 7 cwt. 1 qr. 23 lb.
25. 7 cwt. 2 qr. 13 lb. 15 oz.
26. 4 ac. 5 sq. ch. 294 sq. yd.
27. 6 km. 30 m. 162 mm.
28. 1016 kg.
29. 28.3 cu. dm.
30. 1609 m.

D 12

1. 0-24, 36, 0-0016, 2-88.
2. 0-036, 2160, 90, 120.
3. 0-0024, 0-64, 0-00008, 0-12.
4. 0-0024, 0-64, 0-00008, 0-12.
5. 220.
7. 14-3.
8. 0-00004, 22.
9. 220.
10. 30, 000.
11. 126.
12. 13.
13. 21030.
14. 21030.
15. 126.
16. 0-3007.
17. 250.
18. 0-032.
19. 250.
20. 0-032.
21. 250.
22. 250.
23. 34-3168.
24. 34-3168.
25. 0-0526.
26. 34-0991.
27. 3-13.
28. 3-13.
29. 33.
30. 33.
31. 0-0991.
32. 25-7.
33. 0-0361.
34. 25-7.
35. 0-0080.
36. 0-0080.
37. 9-18.
38. 0-918.
39. 46.
40. 0-368.
41. 46.
42. 47.
43. 47.
44. 47.
45. 47.
46. 47.
47. 47.
48. 47.
49. 47.
50. 47.
Page 451
1. 1 ton 7 cwt. 3 qr. 3 lb.; 1 mi. 27 ch. 14 yd. 1 ft.
2. 15; 12s. 7d.
3. £11 13s. 4d.
4. 75 min.
5. 42 sq. in.
6. 4 ft.

Page 452
1. 25 ch. 4 yd. 2 ft. 8 in.; 2 tons 12 cwt. 3 qr.
2. 18; 114, 6d.
3. £14.
4. 22 tons 1 cwt. 108 lb.
5. 3 in.
6. 30 sq. in.

Page 452
1. 2.78; 11; 154; 484.
2. 12 cwt. 2 qr. 9 lb.; 12 mi. 6 fur. 177 yd.
3. 960; 75 tons.
4. £9 12s. 6d.
5. 1134.
6. 84 sq. ft.; 15 in.

Page 453
1. 21¹; 38; 27. 34.
2. 15 ch. 15 yd. 2 ft. 6 in.; 18 tons 32 lb.
3. £156 10s.
4. 16 gall.; 17s., 22s. 8d.
5. 720 ac.; £486.
6. 8100 cu. ft.

Page 454
1. 12; 101.
2. 12 mi. 7 fur. 3 ch. 10 yd.
3. £3 17s. 2d.
4. 1 lb. 5 oz.
5. 171 sq. in.
6. 7 ft.; 8 ft.; 14 ft.

Page 454
1. £10 10s.
2. 12 cwt.; 5 tons 15 cwt.
3. 221 men.
4. £27 10s. loss.
5. 1100 yd.
6. 4 lb. 12 oz.

Page 455
1. 28; 8.
2. 0.0305.
3. £9 14s. 7½d.; 231 cm.
4. £7.

Page 455
1. 158,427.
2. ½; ½.
3. 0.03125; 0.113,283,6.
4. £1 19s. 7d.
5. £6 10s. 1½d.
6. 10s.
## ANSWERS TO APPENDIX, PART II

### EXERCISE 73 (a)

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<td>9d.</td>
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<td>2. 14s.</td>
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<td>6s. 8d.</td>
<td>6. 4s. 6d.</td>
<td>3. 12s. 6d.</td>
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<td>9.</td>
<td>7½ in.</td>
<td>10. 528 yd.</td>
<td>7. £2 8s.</td>
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<td>9s. 10d.</td>
<td>14. 9s. 6d.</td>
<td>8. 11s. 8d.</td>
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<td>17.</td>
<td>3 yd.</td>
<td>18. 1 ton 17 cwt. 2 qr.</td>
<td>12. 3 cwt. 3 qr.</td>
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<td>20.</td>
<td>£18; £7 8s. 5d.</td>
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<td>16. £3 2s. 3½d.</td>
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<td>19. 21,000 yd.</td>
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### EXERCISE 76 (a)

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<td>£49 0s. 6d.</td>
<td>2. £174 17s.</td>
<td>3. £191 0s. 8d.</td>
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<td>4.</td>
<td>£339 16s. 6d.</td>
<td>5. £401 6s. 8d.</td>
<td>6. £89 13s. 6d.</td>
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<td>7.</td>
<td>£722 2s. 1d.</td>
<td>8. £1959 3s. 7d.</td>
<td>9. 51 tons 16 cwt. 3 qr.</td>
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<td>10.</td>
<td>459 tons 14 cwt. 62 lb.</td>
<td>11. 94 ch. 17 yd. 1 ft. 6 in.</td>
<td>13. £5 3s. 1½d.</td>
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<td>12.</td>
<td>569 ch. 11 yd. 6 in.</td>
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<td>14. £298 1s. 8d.</td>
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<td>15.</td>
<td>£4 10s. 1d.</td>
<td>16. £6 13s. 1½d.</td>
<td>17. £370 8s. 4d.</td>
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<td>18.</td>
<td>1190 yd.</td>
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### EXERCISE 77 (a)

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<td>£6 3s. 6d.</td>
<td>2. £8 11s. 10½d.</td>
<td>3. £28 12s. 11d.</td>
</tr>
<tr>
<td>4.</td>
<td>£110 1s. 8d.</td>
<td>5. £22 5s. 11½d.</td>
<td>6. £21 15s. 1d.</td>
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<tr>
<td>7.</td>
<td>£3 7s. 7d.</td>
<td>8. £28 18s. 1d.</td>
<td>9. £14 18s. 4d.</td>
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<td>10.</td>
<td>£205 16s.</td>
<td>11. £10 3s. 4d.</td>
<td>12. £143 9s. 9d.</td>
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<td>£25 19s. 5d.</td>
<td>14. £137 15s. 4d.</td>
<td>15. £179 16s. 5d.</td>
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<td>£1142 3s. 4d.</td>
<td>17. £299 14s. 11d.</td>
<td>18. £4112 3s. 1d.</td>
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### EXERCISE 78 (a)

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<td>1.</td>
<td>£20 11s. 3d.</td>
<td>2. £1 19s. 5d.</td>
<td>3. 6 yd. 4 in.</td>
</tr>
<tr>
<td>4.</td>
<td>£32 18s. 9d.</td>
<td>5. 383 grn.</td>
<td>6. £30 12s.</td>
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<tr>
<td>7.</td>
<td>£1 8s. 1½d.</td>
<td>8. £1 12s. 1d.</td>
<td>9. 2s. 0d.</td>
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<tr>
<td>10.</td>
<td>£1107 13s. 8d.</td>
<td>11. £22 18s. 4d.; £12 14s. 4½d.</td>
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<tr>
<td>12.</td>
<td>£8 8s.</td>
<td>13. 123.8 mi.</td>
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<td>15.</td>
<td>£336 2s. 6d.</td>
<td>16. £2105 0s. 6d.</td>
<td>14. £6 13s. 4d.</td>
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**PART II**
**Exercise 80 (a)**

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<td>5. 27:8. 9:4.</td>
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<td>6. 7:5.</td>
<td>7. 10:13. 8. 2:3.</td>
</tr>
<tr>
<td>9. 4:5.</td>
<td>10. 8:5.</td>
</tr>
<tr>
<td>11. £397s. 6d.</td>
<td>12. £5,108,541 13s. 4d.</td>
</tr>
<tr>
<td>13. 14s. 3d.</td>
<td>14. £591s. 3d.</td>
</tr>
<tr>
<td>15. £7.</td>
<td>16. £3200. 17. £5315s.</td>
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<td>18. 8s. 3d. in the £.</td>
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**Exercise 81 (a)**

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<tr>
<td>1. 1:3:1.</td>
<td>2. 2:4:1. 3. 0:44:1.</td>
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<td>4. 0:5.</td>
<td>5. 0.47:1. 7. 1:46:1.</td>
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<td>8. 0:50:1.</td>
<td>9. 0:11:1. 10. 10.560.</td>
</tr>
<tr>
<td>11. 1100 ac.</td>
<td>12. 28² in.</td>
</tr>
<tr>
<td>13. 6400 yd.</td>
<td>14. 1100 ac. 15. 97.7 sq. in.</td>
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<td>16. 12.54 mi.</td>
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**Exercise 82 (a)**

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<tbody>
<tr>
<td>1. 5s. 3d.</td>
<td>2. 6 ft. 3. £2 18s. 4d. 4. 1 ton 4 cwt. 3 qr.</td>
</tr>
<tr>
<td>5. 10 in.</td>
<td>6. 6 guineas a week. 7. 3:5. 8. 8:5.</td>
</tr>
<tr>
<td>9. 4:3.</td>
<td>10. 8. 11. 12. 13. 14. 10s. 6d.</td>
</tr>
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<td>20. 15:16 decrease.</td>
<td>21. £5.</td>
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**Exercise 83 (a)**

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<tr>
<td>1. 4s. 6d.</td>
<td>2. £2 8s. 3. 12 days.</td>
</tr>
<tr>
<td>4. 18 days.</td>
<td>5. 19s. 3d. 6. 2½ hr. 7. 15s. 9d.</td>
</tr>
<tr>
<td>8. £2 17s. 6d.</td>
<td>9. 25 min. 10. 384 mi. 11. 2s. 8d.</td>
</tr>
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<td>12. £2 14s.</td>
<td>13. 14 days. 14. 1:2 in. 15. £400.</td>
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**Exercise 84 (a)**

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<tbody>
<tr>
<td>1. 3 bush.</td>
<td>2. 8 days. 3. 28 ac.</td>
</tr>
<tr>
<td>4. 49.</td>
<td>5. ¾d. 6. £37 6s. 8d. 7. 52 weeks.</td>
</tr>
<tr>
<td>8. 18.</td>
<td>9. £7PR 100² 10. 100x PR years. 11. 15 hours.</td>
</tr>
<tr>
<td>12. 15½ weeks.</td>
<td></td>
</tr>
</tbody>
</table>

**Exercise 85 (a)**

<table>
<thead>
<tr>
<th>Page 468</th>
<th>EXERCISE 85 (a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 12s., 6s.; 10s., 8s.; 4s. 6d., 13s. 6d.</td>
<td>2. 8 ft. 4 in., 6 ft. 8 in.; 10 ft. 6 in., 4 ft. 6 in.; 13 ft. 4 in., 1 ft. 8 in.</td>
</tr>
<tr>
<td>3. £12, £8, £20. 4. 36s., 18s., 16s. 5. 4s., 6d., 8s., 6d. 8d.</td>
<td>6. 3:2:10. 7. 5:30:8. 8. 1:2, 3:6, 6 cm.</td>
</tr>
<tr>
<td>15. 15:10:6. 16. 7:2:5. 17. £2 2s., £1 8s., 14s.</td>
<td>18. 60, 24, 36 lb. 19. Table £4 16s., chair £2 8s., rug 16s.</td>
</tr>
<tr>
<td>20. A £2 6s. 8d., B £1 13s. 4d.</td>
<td>21. B £12 10s., C £8, D £4 10s.</td>
</tr>
</tbody>
</table>

**Exercise 90 (a)**

<table>
<thead>
<tr>
<th>Page 473</th>
<th>EXERCISE 90 (a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 477.</td>
<td>2. 14. 3. 80. 4. £4 3s. 4d.</td>
</tr>
<tr>
<td>5. 4 tons 16 cwt.</td>
<td>6. 225. 7. 375. 8. £8.</td>
</tr>
<tr>
<td>13. £13 15s.</td>
<td>14. 14,500. 15. £3750 a year. 16. £62 10s.</td>
</tr>
<tr>
<td>17. 4%.</td>
<td>18. 4%. 19. 750. 20. 290 sq. in.; 14½%.</td>
</tr>
<tr>
<td>21. 7s. 4d.</td>
<td>22. £275.</td>
</tr>
</tbody>
</table>

**Exercise 92 (a)**

<table>
<thead>
<tr>
<th>Page 474</th>
<th>EXERCISE 92 (a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. £27.</td>
<td>2. 1s. 11d. 3. 6s. 2d. 4. £2 13s. 4d.</td>
</tr>
<tr>
<td>5. 37½% l.</td>
<td>6. 20%. g. 7. 11½%, g. 8. 21½%, g.</td>
</tr>
<tr>
<td>9. 33½%</td>
<td>10. 10%, l. 11. 18½%, g. 12. 16½%, l.</td>
</tr>
<tr>
<td>13. £40.</td>
<td>14. £72. 15. 25s.</td>
</tr>
<tr>
<td>16. 8s. 2d.</td>
<td>17. 42%. 18. 16½%. 19. £1 11s. 6d. 20. £25.</td>
</tr>
<tr>
<td>21. £275.</td>
<td>22. 1s. 4d. 23. 31½%. 24. 43½%.</td>
</tr>
<tr>
<td>25. £120.</td>
<td>26. 33%. 27. 60%. 28. 33%.</td>
</tr>
<tr>
<td>29. 16s. 8d.</td>
<td>30. 37½%.</td>
</tr>
</tbody>
</table>
ANSWERS TO APPENDIX

EXERCISE 93 (a)

1. 7\%.
2. 12\%.
3. 58\%.
4. 57\%.
5. 4\%.
6. £188 10s.
7. 30\%.
8. £3 6s. 2d.
9. 60\%.
10. 9\%.
11. £520.
12. £4 10s.
13. 2s. 11d.
14. 15,160.
15. 37\%.
16. 16\%.
17. 6s.
18. 61.
19. £2 14s.
20. £20 14s.
21. £4 4s.
22. £32,000.
23. £7500.
24. 6\%.
25. 3\%.
26. £240, £360, £450.
27. 35\%.

EXERCISE 94 (a)

1. 100; 0-01 sq. dm.
2. 294; 0-01, 2-94 sq. in.
3. 17, 13 mm.; 221; 221 sq. mm., 2-21 sq. cm.
4. 66; 8\% sq. in.
5. 100 sq. dm.; 0-01 sq. m.
6. 10 sq. ch., 4840 sq. yd.
7. 12; 1\% sq. ft.
8. 2 sq. cm.
9. 0-45 sq. m.
10. 9 sq. ch.
11. 1 sq. cm.
12. 10 sq. dm.
13. 10 ac.

EXERCISE 95 (a)

1. 8 yd. 2 ft.; 22 sq. ft.
2. 36 ft. 6 in.; 82 sq. ft.
3. 48 ft. 8 in.; 138 sq. ft.
4. 42 dm. 8 cm.; 47-04 sq. cm.
5. 2-2 sq. m.
6. 3 yd.; 3\% sq. ft.
7. 14 ft.
8. 87 m.
9. 3 yd.; 3\% sq. ft.
10. 8 ft. 8 in.
11. 61-2 ac.
12. 20 ac.
13. 11 yd.
14. 1089 sq. yd.
15. 18 sq. 6d.
16. 20 sq. 6d.
17. 20 sq. in.
18. 20 sq. in.
19. 28 sq. in.
20. 28 sq. in.
21. 28 sq. in.
22. 75 sq. in.
23. 30 sq. dm.
24. 32 ft.
25. 32 ft.
26. 1 doc.
27. 299 sq. cm.

EXERCISE 96 (a)

1. 1720 sq. ft.
2. 3016 sq. ft.
3. 682 sq. m.
4. 197 sq. ft.
5. 210, 243\% sq. yd.
6. 650 sq. in.
7. 518 sq. in.
8. 1008 black, 1296 white.
9. 14, 3466 sq. yd.
10. 148, 100 sq. yd.

EXERCISE 97 (a)

1. 169 sq. yd.
2. 58 sq. in.
3. 149\% sq. ft.
4. £38 10s.
5. £19 14s. 6d.

ANSWERS TO APPENDIX

EXERCISE 98 (a)

1. 64; \(\frac{1}{2}\) cu. in.
2. 240; \(\frac{1}{2}\) \(\frac{3}{4}\) cu. in.; \(\frac{3}{4}\).
3. 1920; \(\frac{1}{2}\) 922 c.c.; \(\frac{1}{2}\)
4. \(\frac{1}{2}\) in., \(\frac{1}{2}\) \(\frac{3}{4}\) cu. in.; \(\frac{3}{4}\) 3600; \(\frac{1}{2}\) \(\frac{3}{4}\) cu. in.; \(\frac{3}{4}\).
5. 27 cu. ft.; \(\frac{1}{2}\) cu. yd.
6. 0-000,001 cu. m.; 0-000,001 cu. dm.
7. \(\frac{1}{2}\) cu. yd.
8. 0-025 cu. m.
9. 0-001 cu. m.
10. 0-5 litre.
11. 3 cu. in.
12. 5 cu. in.
13. 3 c.c.
14. 72 c.c.

EXERCISE 99 (a)

1. 105 cu. in.
2. 1980 cu. yd.
3. 384.
4. 15 cu. ft.
5. 4500 litres.
6. 840 litres.
7. 1920; 128; 100 cu. in.
8. 800.
9. 9 6 ft. 6 in.
10. 9\%.
11. 2100 lb.
12. 480 lb.
13. 2 in.
14. 1\% in. per min.
15. 312\% gall.
16. 9\%.
17. 4 ft.
18. 4-5 dm.
19. £4 15s. 8d.
20. 200 sheets.
21. 0-09 in.
22. 4-48 ft.

EXERCISE 100 (a)

1. 70 cu. in.
2. 740 cu. in.
3. 2-15 cu. dm.
4. 122-5 cu. in.
5. 246 c.c.
6. 88 cu. in.
7. 150 c.c.
8. 231\% lb.
9. 6160 cu. ft.
10. 74 lb.
11. 3936 cu. ft.; £37 7s. 6d.

EXERCISE 101 (a)

1. 1\% cu. ft.
2. 540 c.c.
3. 7-2 in.
4. 2-5 dm.
5. 2 sq. ft.
6. 75 sq. cm.
7. 17\% in.
8. 7 lb. 13 oz.
9. 530 cu. in.; 130 lb. (127-2).
10. 6 in.
11. 1\%.
12. 3 in.
13. 663,000 tons.
14. 0-045 sq. cm.
15. 3900 gal.

EXERCISE 102 (a)

1. 770 yd.
2. 147 sq. in.
3. 2s. 8d.
4. 17,500 lb.
5. 56\% cu. in.
6. 15 dm.
7. 8 in.
8. 1\% c = \(\frac{32}{4}\) abd.
9. 9-945 in.
10. 265 tons.
11. 3 ft. 5\% in.
**ANSWERS TO APPENDIX**

**EXERCISE 103 (a)**

<table>
<thead>
<tr>
<th>Page 488</th>
<th>EXERCISE 103 (a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 22 in.</td>
<td>2. 26-4 km.</td>
</tr>
<tr>
<td>5. 66 in.</td>
<td>6. 18-8 yd.</td>
</tr>
<tr>
<td>8. 280 yd.</td>
<td>10. 24,900 mi.</td>
</tr>
<tr>
<td>13. 63(6) m.</td>
<td>14. 21-4 in.</td>
</tr>
<tr>
<td>17. 180</td>
<td>18. 15-1 cm.</td>
</tr>
<tr>
<td>4. 31-4 cm.</td>
<td>8. 1-59 in.</td>
</tr>
<tr>
<td>12. 262</td>
<td>18. 750 m.p.h.</td>
</tr>
<tr>
<td>20. 2011 yd.</td>
<td></td>
</tr>
</tbody>
</table>

**EXERCISE 104 (a)**

<table>
<thead>
<tr>
<th>Page 489</th>
<th>EXERCISE 104 (a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 38-5 sq. in.</td>
<td>2. 24-6 sq. cm.</td>
</tr>
<tr>
<td>5. 28-3 sq. yd.</td>
<td>6. 177 sq. in.</td>
</tr>
<tr>
<td>9. 3-4 km.</td>
<td>10. 17-1 sq. ft.</td>
</tr>
<tr>
<td>12. 24; 6-2 sq. in.</td>
<td>13. 136 sq. yd.</td>
</tr>
<tr>
<td>15. 1-57 sq. in.</td>
<td>16. 424-3 sq. in.</td>
</tr>
<tr>
<td>19. 32-7 sq. in.</td>
<td>18. 5 in.</td>
</tr>
<tr>
<td>4. 73-5 sq. cm.</td>
<td>8. 21 yd.</td>
</tr>
<tr>
<td>11. 264 m</td>
<td>14. 36 cm.</td>
</tr>
<tr>
<td>17. 56-10s</td>
<td>18. 616 sq. yd.</td>
</tr>
<tr>
<td>22. 5 in.</td>
<td></td>
</tr>
</tbody>
</table>

**EXERCISE 105 (a)**

<table>
<thead>
<tr>
<th>Page 490</th>
<th>EXERCISE 105 (a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 264 sq. cm.</td>
<td>2. 75-4 sq. in.</td>
</tr>
<tr>
<td>5. 352 sq. cm.</td>
<td>6. 16-0 sq. cm.</td>
</tr>
<tr>
<td>9. 176 sq. cm.</td>
<td>10. 21-2 sq. ft.</td>
</tr>
<tr>
<td>12. 192 sq. in.</td>
<td>14. 1830 sq. ft.</td>
</tr>
<tr>
<td>17. 484 sq. cm.</td>
<td>18. 62-8 sq. in.</td>
</tr>
<tr>
<td>3. 66 sq. ft.</td>
<td>7. 39-3 sq. in.</td>
</tr>
<tr>
<td>11. 3-5 in.</td>
<td>15. 60-5 sq. in.</td>
</tr>
<tr>
<td>19. 127 cu. in.</td>
<td>4. 18-8 sq. dm.</td>
</tr>
<tr>
<td>8. 36-7 sq. ft.</td>
<td>12. 45400 cu. ft.</td>
</tr>
</tbody>
</table>

**EXERCISE 106 (a)**

<table>
<thead>
<tr>
<th>Page 491</th>
<th>EXERCISE 106 (a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 275 cu. in.</td>
<td>2. 792 c.c.</td>
</tr>
<tr>
<td>5. 7 cm.</td>
<td>6. 3-18 in.</td>
</tr>
<tr>
<td>9. 87-9 gall.</td>
<td>10. 631</td>
</tr>
<tr>
<td>13. 396 cu. in.</td>
<td>14. 297,726 cu. in.</td>
</tr>
<tr>
<td>17. 11-8 in.</td>
<td>18. 3-84 ft. per sec.</td>
</tr>
<tr>
<td>3. 785 c.c.</td>
<td>8. 4 cm.</td>
</tr>
<tr>
<td>11. 5 in.</td>
<td>12. 33 c.c.</td>
</tr>
<tr>
<td>15. 84-8 lb.</td>
<td>16. 45,400 cu. ft.</td>
</tr>
<tr>
<td>19. 199 gal.</td>
<td></td>
</tr>
</tbody>
</table>

**EXERCISE 109 (a)**

<table>
<thead>
<tr>
<th>Page 493</th>
<th>EXERCISE 109 (a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. £31 4s.; £291 4s.</td>
<td>2. £99 15s.; £669 15s.</td>
</tr>
<tr>
<td>3. £85 14s. 6d.; £720 14s. 6d.</td>
<td>4. £33 15s.; £190.</td>
</tr>
<tr>
<td>5. £12 12s.; £120 12s.</td>
<td>8. £3; £264.</td>
</tr>
<tr>
<td>7. £23 15s. 2d.</td>
<td>8. £43 13s. 7d.</td>
</tr>
<tr>
<td>10. £99 8s. 7d.</td>
<td>11. £43 4s. 9d.</td>
</tr>
<tr>
<td>13. £52 14s. 11d.</td>
<td>14. £15 3s. 2d.</td>
</tr>
<tr>
<td>16. £120 5s. 0d.</td>
<td>17. £83 11s. 5d.</td>
</tr>
<tr>
<td>19. £12 £8 4d</td>
<td>20. £24 17s. 1d.</td>
</tr>
<tr>
<td>9. £36 3s. 10d.</td>
<td>10. £43 5s. 0d.</td>
</tr>
<tr>
<td>12. £15 6s. 7d.</td>
<td>15. £28 17s. 9d.</td>
</tr>
<tr>
<td>18. £13 14s. 0d.</td>
<td>19. £24 17s. 1d.</td>
</tr>
</tbody>
</table>

**EXERCISE 122 (a)**

<table>
<thead>
<tr>
<th>Page 498</th>
<th>EXERCISE 122 (a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 0-367,88</td>
<td>2. 0-583 cm.</td>
</tr>
<tr>
<td>3. 39-4</td>
<td>4. 129 m.</td>
</tr>
<tr>
<td>5. 36-3 gal.</td>
<td>6. 2-57.</td>
</tr>
<tr>
<td>9. 0-13%</td>
<td>10. 2-205 lb.</td>
</tr>
<tr>
<td>12. 18-3%</td>
<td>13. 10-7%</td>
</tr>
<tr>
<td>15. 1-196 sq. yd.</td>
<td>16. 46.</td>
</tr>
<tr>
<td>17. 43 lb.</td>
<td>18. 14-4 m.p.h.</td>
</tr>
<tr>
<td>20. 11%.</td>
<td>19. 365-2425 days.</td>
</tr>
</tbody>
</table>

**EXERCISE 111 (a)**

<table>
<thead>
<tr>
<th>Page 493</th>
<th>EXERCISE 111 (a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. £364; 73%</td>
<td>2. £30; 1 1/8 yr.</td>
</tr>
<tr>
<td>4. £4 11s.; 48%</td>
<td>5. £463 10s.; 8 mo.</td>
</tr>
<tr>
<td>7. £810; 41%</td>
<td>8. £51; £53 11s.</td>
</tr>
<tr>
<td>10. £850; £869 2s. 6d.</td>
<td>11. 83%</td>
</tr>
<tr>
<td>13. 1%</td>
<td>14. 10 2s. 6d.</td>
</tr>
<tr>
<td>16. 14 1/2 yr.; £5782 10s.</td>
<td>18. 8 mo.</td>
</tr>
</tbody>
</table>

**EXERCISE 115 (a)**

<table>
<thead>
<tr>
<th>Page 494</th>
<th>EXERCISE 115 (a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. £552 12s.</td>
<td>2. £36.</td>
</tr>
<tr>
<td>5. 3%</td>
<td>6. £72.</td>
</tr>
<tr>
<td>9. £1200</td>
<td>10. 9s. 4d.</td>
</tr>
<tr>
<td>17. 4 yr.</td>
<td>18. £40 13s. 4d.; 26 1/2%</td>
</tr>
</tbody>
</table>

**EXERCISE 117 (a)**

<table>
<thead>
<tr>
<th>Page 496</th>
<th>EXERCISE 117 (a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 52</td>
<td>2. 8-4.</td>
</tr>
<tr>
<td>5. 9,760,000</td>
<td>6. 0-59.</td>
</tr>
<tr>
<td>9. 0-549</td>
<td>10. 74-1.</td>
</tr>
<tr>
<td>13. 745-7</td>
<td>14. 0-064.</td>
</tr>
<tr>
<td>16. 928</td>
<td></td>
</tr>
</tbody>
</table>

**EXERCISE 118 (a)**

<table>
<thead>
<tr>
<th>Page 496</th>
<th>EXERCISE 118 (a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 2-0</td>
<td>2. 61-1.</td>
</tr>
<tr>
<td>5. 1310</td>
<td>6. 1-14.</td>
</tr>
<tr>
<td>8. 4-502.</td>
<td></td>
</tr>
<tr>
<td>9. 0-006,679</td>
<td>10. 3-279.</td>
</tr>
<tr>
<td>12. 18. 3-018.</td>
<td></td>
</tr>
</tbody>
</table>

**EXERCISE 119 (a)**

<table>
<thead>
<tr>
<th>Page 497</th>
<th>EXERCISE 119 (a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 3 1/4 in.; 0-02; 2%</td>
<td>2. 1 1/4 ds.; 0-005; 0-5%.</td>
</tr>
<tr>
<td>3. 50 sq. yd.; 0-002; 0-2%</td>
<td>4. 5 min.; 0-005; 0-5%.</td>
</tr>
<tr>
<td>5. 2%</td>
<td>6. 0-4%.</td>
</tr>
<tr>
<td>9. 0-2%</td>
<td>10. 0-07%.</td>
</tr>
<tr>
<td>12. 0-2%</td>
<td>15. 35 9. 8-31.</td>
</tr>
<tr>
<td>18. 17-92 oz.</td>
<td>14. 15s. 2 1/4d.</td>
</tr>
</tbody>
</table>
ANSWERS TO APPENDIX

Page 499

**EXERCISE 123 (a)**
1. 43. 2. 6776. 3. £2 18s. 11½d. 4. 4 yd. 4 in.
5. 1s. 8d. 6. 9s. 3½d. 7. 74.
8. 1s. 11d.; 1s. 11d.
9. 1-2 in. 10. 2s. 2d. 11. 16½ m.p.h.
12. 25-4 m.p.h.
13. 18s. per lb.; 22s. 6d. per lb.
14. 2s. 8d. per lb.
15. 9s. 16. 22½ m.p.h.
17. £4 16s. 6d.
18. \( b + \frac{m(b-c)}{n} \)
19. \( \frac{px + ry}{x + y} \) %.
20. 2-4 hr.

Page 501

**EXERCISE 128 (a)**
1. 20 days. 2. 45 days.
3. 4 hours. 4. 12 m.p.h.
5. 16 gall. 6. 5: 3.
7. 24 mi. 8. 72 days.
9. £13 6s. 8d., £10 13s. 4d., £10.
10. A 105 days, B 140 days.
11. 66 days.

**DRILL EXERCISES D 13-20**

Page 502

**D 13**
1. £253. 2. £77. 3. £232. 4. £993 15s.
5. £2540 6s. 6. £337 1s. 6d. 7. £469 14s.
8. £4897 1s. 3d. 9. £529 11s. 0½d. 10. £223 11s. 3d.
11. £193 19s. 11d. 12. £1925 6s. 6½d. 13. £1906 8s. 9d.
14. £1603 0s. 5½d. 15. £696 1s. 11d. 16. £1268 10s.
17. £885 4s. 6d. 18. £482 15s. 8d. 19. £698 5s. 7½d.
20. £2770 9s. 11d. 21. £145 2s. 0½d. 22. £16,718 3s. 10½d.
23. £1075 8s. 8d. 24. £5783 10s. 7½d. 25. £136 15s. 4d.
26. £71 2s. 2d.

Page 502

**D 14**
1. £204 13s. 1½d. 2. £74 17s. 3d. 3. £34 8s. 6d.
4. £2 15s. 4½d. 5. £50 11s. 2½d. 6. £21 18s. 4½d.
7. £3 7s. 0½d. 8. £21 19s. 1½d. 9. £9 7s. 10½d.
10. £5118 2s. 5d. 11. £111 10s. 12. £951 19s. 10½d.
13. £7 16s. 7d. 14. £101 14s. 10d. 15. £127 3s. 5d.
16. £444 17s. 9d. 17. £35 2s. 5d. 18. £307 11s. 5d.
19. £432 17s. 11d. 20. £648 0s. 3d.

Page 503

**D 15**
1. 70, 0-35; 60, 0-075; 50, 0-625; 40, 1-375; 30, 3-5; 20, 8-5; 10, 105.
1. 12; 10; 8; 5; 3½; 1½; 10; 105.

Page 504

**D 16**
1. Gain 25%. 2. Loss 20%.
3. Gain 5%. 4. Gain 22%.
5. Loss 21½%.
6. Gain 3½%.
7. Loss 22½%.
8. Gain 33½%.
9. Loss 25%.
10. £1 2s. 6d.
11. £2 18s. 4d.
12. £23 16s.
13. £9 18s.
14. £9 18s.
15. £9 18s.
16. £23 16s.
17. £23 16s.
18. £23 16s.
19. £23 16s.
20. £23 16s.
21. £23 16s.
22. £23 16s.
23. £23 16s.
24. £23 16s.
25. £23 16s.
26. £23 16s.
27. £23 16s.
28. £23 16s.
29. £23 16s.
30. £23 16s.
ANSWERS TO APPENDIX

Page 506

D 19

1. 7.7.  2. 0.16.  3. 50.  4. 51.84.  5. 30.  6. 30.
7. \(\frac{2}{8}\).  8. \(\frac{3}{8}\).  9. \(\frac{5}{8}\).  10. \(\frac{2}{5}\).  11. \(\frac{2}{11}\).  12. \(\frac{1}{7}\).
13. 6.  14. \(7\frac{1}{8}\).  15. \(\frac{4}{11}\).  16. 10.  17. \(2\frac{1}{4}\).  18. \(\frac{3}{8}\).
19. \(\frac{7}{11}\).  20. 8.  21. \(\frac{1}{11}\).  22. \(\frac{3}{8}\).  23. \(\frac{1}{8}\).  24. 5.

Page 507

D 20

1. 7.70.  2. 30.  3. 0.001.  4. 86.400.  5. 300.
6. 54.0.  7. 0.0429.  8. 23%.  9. 2.36; 3-11.  10. 4.4%
11. 26 cm, 26.4 cm.  12. 2.74 sec., 0.732 sec.  13. 251,600.
14. 730.  15. 6.93 cm.  16. 1-649.  17. 18.  18. 166.8 ± 0.6 yd.
19. 0.662.  20. 0.000,379.  21. 4.04.  22. 0.081.
23. 33%.  24. 22.8%.  25. 7.52%.  26. 6.07%.
27. 9.01%.  28. 3.48%.  29. 17.  30. 3.33%.
31. 2.1%.  32. 0.040%.  33. 0.1%.  34. 0.58%.
35. 227,000.  36. 299,000.  37. 385,000.  38. 770,000.

REVISION PAPERS A 25-48

Page 509

Paper A 25
1. 8 yd. 1 ft. 10 in.; 51,200 nails.  2. 5\(\frac{2}{3}\); £710 12s.
3. 45%; 90 yd.  4. £6 13s. 6d.  5. £8 9s. 9d.  6. 51%.

Page 510

Paper A 26
1. 2\(\frac{1}{2}\); £3 12s. 2\(\frac{1}{2}\).  2. 26\(\frac{3}{4}\); £948 9s. 4d.  3. 5; 18; 57%.
4. £480, £1600 a year.  5. 176\(\frac{3}{4}\) sq. ft.  6. £78 8s., £47 12s.

Page 510

Paper A 27
1. 5s. 11d.; £0-3677.  2. £15 16s. 3d.; £21 17s. 6d.
3. 40%; 3:11.  4. 29 times.  5. £15.  6. 16\(\frac{2}{3}\); 23\(\frac{1}{2}\), 30 hrs.

Page 511

Paper A 28
1. 0.0778; £2 13s. 10d.  2. 800 pesetas; £97 16s. 10d.
3. 70%; £51.  4. 16s. 4\(\frac{1}{2}\)d.  5. 16%.  6. 2.85 mi.; 5.7 min.; after 6 min.

Page 511

Paper A 29
1. 882; the first.  2. £2 7s. 7\(\frac{1}{4}\)d.; £38 19s. 3\(\frac{1}{4}\)d.
3. £2 14s.; 37\(\frac{1}{2}\)%  4. 3\(\frac{1}{2}\) oz.  5. 6.23 gall.; 62.3 lb.
6. 100, 75, 80 ac.

Page 512

Paper A 30
1. 31 lb. 7 oz.; 80 ac.  2. £92 2s. 9d.; 37 payments; 10s. 3d.
3. 9:10; £13 15s.; 4. £25 5s. 9d.  5. \(\frac{5}{4}\) ft. per sec.
6. Approximately 29-75, 29-97 in.; 9.36 a.m., 2.10 p.m., 2.33 p.m.;
10.35 a.m. to 11.25 a.m. and 12.20 p.m. to 1.55 p.m.

Page 513

Paper A 31
1. 35; \(\frac{1}{4}\) ft. per sec.  2. £22 1s. 3d.; 20.
3. 7:13; \(\frac{3}{4}\).  4. £4 5s. 2d.  5. £12 10s.; 20%.
6. £420, £360, £288, £192.

Page 514

Paper A 32
1. March 4; 0:30.  2. £3 8s. 3d.; 37\(\frac{1}{2}\) sec.  3. 75%; 22s. 6d.
4. 50%.  5. \(\frac{2}{3}\) oz.  6. 544 mi.

Page 515

Paper A 33
1. 9s. 7d.; £5 6s. 8d.  2. 0.67 in.  3. 19,400 tons.
4. £400.  5. 28\(\frac{1}{2}\); 50%.  6. 240.

Page 515

Paper A 34
1. 2\(\frac{2}{3}\), 7, 11, 21; 1078.  2. 12\(\frac{1}{4}\) m.p.h.  3. 31s. 6d.; 16\(\frac{2}{3}\).
4. 416.  5. 65 lb.  6. 9-55 in.; 30-006,1 in.

Page 515

Paper A 35
1. 2 cwt. 99 lb.; 3\(\frac{1}{2}\).  2. 21 days.  3. 15, 20, 30, 40, 45.
4. 10% loss.  5. 9450 litres; 11-1 cm.  6. 1-1 c.c.; 15-0 gm.

Page 516

Paper A 36
1. 11 lb. 14 oz.; 126 articles, 6d.  2. 35 hr.  3. 8\(\frac{1}{4}\).
4. 10.42 a.m.; 7 mi. from B.  5. 616 sq. in.  6. 1270 sq. yd.

Page 516

Paper A 37
1. 4 mi. 170 yd.; £495 9s. 4d.  2. 12, 3\(\frac{1}{2}\) in.; 13, \(\frac{3}{8}\) in.
3. 525 lb. per cu. ft.  4. 62\(\frac{1}{4}\).  5. 6-4 ft.
6. 25-4 ft.; 44-1 sq. ft.

Page 517

Paper A 38
1. 4s. 1d.; \(\frac{2}{8}\).  2. 15 days; £4 2s. 6d.
3. £112, £144, £208, £256.  4. £103 2s. 6d.  5. \(\frac{3}{8}\) in.
6. 40 cu. in.
ANSWERS TO APPENDIX

Page 518
Paper A 39
1. 1460 yd.; 0.034 oz.
2. £83 6s. 8d., £86 13s. 4d.; £125, £120.
3. 14.25%.
4. £2, £1 12s., £1 8s.; man £20, each woman £16, each boy £14.
5. 18 cm. 6. 25 cu. in.

Page 518
Paper A 40
1. 0.868; £6 7s. 4d.
2. 1st kind; 2nd kind.
3. £5 16s.; £7 15s. 6d.
4. 21.6%. 5. 9-6 in. per hr. 6. 5 lb.

Page 519
Paper A 41
1. 2828,431; 6 figures.
2. 3/25.
3. 4s. 9d.
4. 12 3/2 sq. ft.
5. 3 3/4%.
6. 5-28, 5-94 km. per hr.

Page 520
Paper A 42
1. 320; 6 tons 5 cwt.
2. 1%.
3. £299,016 13s. 4d.; £4 4s.
4. £1 8s. 9d.
5. 35s.
6. 10s. 9d.

Page 520
Paper A 43
1. 88.
2. 7.2 m.p.h.
3. £178 10s., £180 16s. 8d.
4. 500 c.c.
5. 40% p.a.
6. 6 days; £7 4s., £6, £4 16s.

Page 521
Paper A 44
1. 83.
2. 23.5425, 22.5425 sq. cm.; 2%.
3. 9600 mi.
4. 2 2/3 yr.
5. 2 ft. 3 in.
6. 13.7%.

Page 521
Paper A 45
1. 30%; 109 yd. 1 ft. 1 in.
2. £2 11s.
3. £5 18s. 1d.
4. £3192.

Page 522
Paper A 46
1. 7; 0.05%.
2. 1s. 3d. per lb.
3. 7.3%.
4. £1 16s.
5. 44-5 cm.; 126-54 sq. cm.
6. 22 1/2%.

Page 522
Paper A 47
1. 149; 1/2 lb.
2. 6s. 4d.
3. £625.
4. £5 18s.
5. £4160.

Page 523
Paper A 48
1. £12 11s.
2. 126,607.
3. 176 ft.; 24 m.p.h.
4. 4%.
5. 1-8 m.p.h.
6. 33 3/8%.

Page 525
EXERCISE 129 (a)
1. 19. 2. 33. 3. 46. 4. 59. 5. 23. 6. 93.
18. 1207.

Page 525
EXERCISE 131 (a)
1. 2-8. 2. 5-4. 3. 0-64. 4. 0-24. 5. 0-072.
6. 0-155. 7. 17-4. 8. 3-04. 9. 4-01. 10. 64-3.

Page 525
EXERCISE 132 (a)
1. 1-24. 2. 7-07. 3. 0-837. 4. 19-2. 5. 0-0949.
16. 0-791. 17. 3-40. 18. 0-120. 19. 0-85. 20. 5-44.
21. 0-254. 22. 7/36. 23. 7/8. 24. 27/38. 25. 7/36.
26. 0-354. 27. 0-603. 28. 1-29. 29. 3-16. 30. 1-41.
31. 7-07.

Page 525
EXERCISE 136 (a)
1. 1-87 in. 2. 184 yd. 3. 4-66 yd. 4. 62-5. 5. 57-3.
6. 62-5. 7. 9-39. 8. 8 ft. 8 in. 9. 170 yd. 10. 3-79 cm.
11. 3-98 in. 12. 2-89 in.; 50 sq. in. 13. 6-74(5) in.

PART III
### Page 527

**EXERCISE 138 (a)**

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 10-2(5).</td>
<td>2. 0-428.</td>
</tr>
<tr>
<td>5. 0-408.</td>
<td>6. 0-018(2).</td>
</tr>
<tr>
<td></td>
<td>15. 2-232%.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>4. 0-001(10).</td>
<td>8. 0-354.</td>
</tr>
<tr>
<td>12. 0-81(2).</td>
<td>16. 127 and 129.</td>
</tr>
</tbody>
</table>

### Page 527

**EXERCISE 144 (a)**

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 8-51(9).</td>
<td>2. 8-58(5).</td>
</tr>
<tr>
<td>5. 7-66(1).</td>
<td>6. 8-88(8).</td>
</tr>
<tr>
<td>9. 1-03(3).</td>
<td>10. 1-36(0).</td>
</tr>
<tr>
<td>13. 8-24(0).</td>
<td>14. 2-05(5).</td>
</tr>
<tr>
<td></td>
<td>15. 1-77(4).</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>4. 3-95(9).</td>
<td>8. 2-45(3).</td>
</tr>
</tbody>
</table>

### Page 528

**EXERCISE 147 (a)**

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 1137.</td>
<td>2. 2183.</td>
</tr>
<tr>
<td>5. 70-5(0).</td>
<td>6. 479(9).</td>
</tr>
<tr>
<td>9. 20,850,000.</td>
<td>10. 8-90(0).</td>
</tr>
<tr>
<td>21. 100(4).</td>
<td>22. 7-94(0).</td>
</tr>
<tr>
<td>25. 36,200,000.</td>
<td>26. 99-3(7).</td>
</tr>
<tr>
<td>29. 4-66(5).</td>
<td>30. 3-27(1).</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>4. 259(5).</td>
<td>8. 223(9).</td>
</tr>
<tr>
<td>28. 312(2).</td>
<td></td>
</tr>
</tbody>
</table>

### Page 528

**EXERCISE 148 (a)**

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 150(5).</td>
<td>2. 112,200.</td>
</tr>
<tr>
<td>6. 33-5(8).</td>
<td>7. 3-94(5).</td>
</tr>
<tr>
<td>11. 337(1).</td>
<td>12. 2-87(1).</td>
</tr>
<tr>
<td>16. 13-1(8).</td>
<td>17. 5-16(8).</td>
</tr>
<tr>
<td>21. 1-73(7).</td>
<td>22. 4-86(0).</td>
</tr>
<tr>
<td></td>
<td>23. 16-8(4).</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>5. 18-3(7).</td>
<td>10. 11-4(7).</td>
</tr>
<tr>
<td>15. 1-41(6).</td>
<td>20. 6-97(9).</td>
</tr>
</tbody>
</table>

### Page 529

**EXERCISE 149 (a)**

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 2-54 cm.</td>
<td>2. 21-8 m.p.h.</td>
</tr>
<tr>
<td>5. 11-9%.</td>
<td>6. 30-4%.</td>
</tr>
<tr>
<td>9. 119; 2-88.</td>
<td>10. 3-78; 23-8.</td>
</tr>
<tr>
<td>13. 2-97 kg.</td>
<td>14. 47-0 sq. in.</td>
</tr>
<tr>
<td>17. 11-2 oz.</td>
<td>18. 2-62.</td>
</tr>
<tr>
<td></td>
<td>19. 3150 ft. per sec.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>4. 138 gm.</td>
<td>8. 114.</td>
</tr>
<tr>
<td>12. 187 fr.</td>
<td>16. 2-54 cm.</td>
</tr>
<tr>
<td>20. 18-5 cm.</td>
<td></td>
</tr>
</tbody>
</table>

### Page 530

**EXERCISE 152 (a)**

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 0-618(8).</td>
<td>2. 0-019(2).</td>
</tr>
<tr>
<td>5. 0-021(3)(0).</td>
<td>6. 48-8(7).</td>
</tr>
<tr>
<td>9. 0-008,197(7).</td>
<td>10. 0-958(6).</td>
</tr>
<tr>
<td>13. 0-683(9).</td>
<td>14. 0-000,088,3(4).</td>
</tr>
<tr>
<td>17. 0-000,005,000.</td>
<td>20. 0-319(1).</td>
</tr>
<tr>
<td>23. 0-394(5).</td>
<td>24. 0-901(3).</td>
</tr>
<tr>
<td>27. 0-148(0).</td>
<td>28. 0-140(7).</td>
</tr>
</tbody>
</table>

### Page 531

**EXERCISE 153 (a)**

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 0-018(4).</td>
<td>2. 0-000,050,2(3).</td>
</tr>
<tr>
<td>5. 0-001,254.</td>
<td>6. 27-370.</td>
</tr>
<tr>
<td>13. 0-283(9).</td>
<td>14. 0-038,3(1).</td>
</tr>
<tr>
<td>17. 0-051(0).</td>
<td>18. 1-13(0).</td>
</tr>
<tr>
<td>25. 4-46(1).</td>
<td>26. 15-2(3).</td>
</tr>
</tbody>
</table>

### Page 531

**EXERCISE 154 (a)**

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 27-0%.</td>
<td>2. 53-5%.</td>
</tr>
<tr>
<td>6. 501 sq. yd.</td>
<td>7. 0-764 cu. m.</td>
</tr>
<tr>
<td>14. 0-261.</td>
<td>15. 0-318.</td>
</tr>
<tr>
<td>18. 69-8.</td>
<td>20. 0-898.</td>
</tr>
</tbody>
</table>

### Page 533

**EXERCISE 156 (a)**

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 32-4 sq. in.</td>
<td>2. 88-6 sq. cm.</td>
</tr>
<tr>
<td>5. 9-44 cm.</td>
<td>6. 15-6 in.</td>
</tr>
<tr>
<td>9. 96 sq. in.</td>
<td>10. 17-9 sq. cm.</td>
</tr>
</tbody>
</table>

### Page 533

**EXERCISE 158 (a)**

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 40 cu. in.</td>
<td>2. 112 c.c.</td>
</tr>
<tr>
<td>5. 1152 cu. in.</td>
<td>6. 48 c.c.</td>
</tr>
<tr>
<td>8. 864 sq. in.</td>
<td>9. 179 sq. cm.</td>
</tr>
<tr>
<td>12. 665 cu. ft.</td>
<td></td>
</tr>
</tbody>
</table>
ANSWERS TO APPENDIX

Page 534  EXERCISE 159 (a)
1. 49:2 dm.; 193 sq. dm.
2. 4:77(5) in.; 71:6 sq. in.
3. 564 m.; 3540 m.
4. 10:9 kg.
5. 3:16 kg.
6. 5:83 cm.
7. 3:63 ft.
8. 1:65 in.
9. 31:4 in. per sec.
10. 20:4 oz.
11. 5:16 hr.
12. 45:1 in.
13. 4:12 in.
14. 44:1 in.
15. 1:63 in.

Page 535  EXERCISE 160 (a)
1. 91:6 cu. in.
2. 199 c.c.
3. 210 c.c.
4. 42:5(5) sq. in.
5. 167 sq. cm.
6. 67:9 sq. cm.
7. 3:67 cm.
8. 153 cu. in.
9. 302 cu. ft.
10. 188:5 sq. ft.
11. 5:66 in.

Page 536  EXERCISE 161 (a)
1. 56:4(5) cu. in.; 71:2 sq. in.
2. 2:29 in.
3. 3:39 in.
4. 94:0 cu. dm.
5. 375,000.
6. 12.
7. 287 c.c.
8. 4:13 in.
9. 7:65 lb.
10. 1470 c.c.; 653 sq. cm.
11. 71:5 cu. in.

Page 537  EXERCISE 163 (a)
1. £457.
2. 620 yd.; 3:42 ac.
3. 3:3 sq. ft.
4. 25 sq. in.; 4:47 in.
5. 69,400,000 tons.
6. 31,900 cu. yd.
7. 9750 sq. yd.
8. 2:23 oz.
9. 8:19 in.
10. 181 lb.
11. 3:9 sq. in.
12. 1:77 in.
14. 3 yd.
15. £277 4s.; £33 16s.
16. 168 lb. per cu. ft.
17. 0:985 in.
18. 2:24 sq. in.; 6:90 in.

Page 539  EXERCISE 164 (a)
1. 790 gm.
2. 6:9.
3. 0:75.
4. 8:8 kg.
5. 190 cu. in.
6. 86 kg.
7. 2:7.
8. 15 lb.
9. 1:6 kg.
10. 73 gm.
11. 0:66 in.
12. 0:25 mm.
13. 7:0.
14. 26 oz.
15. 0:67.
16. 17.
17. 1:0(3).

Page 540  EXERCISE 165 (a)
1. 10 oz.
2. 49 cu. in.
3. 3.
4. 120 gm. (119 gm.).
5. 25 gm.
6. 280 gm.
7. 7:1.
8. 1:2 mm.; 11 gm.
9. 35 mm.
10. 75 cm.
11. 5:83 in.
12. 7:1.
13. 10 ft.
14. 12 kg.
15. 10 sq. ft.
16. 10 sq. cm.
17. 10 cu. ft.
18. 10 cu. cm.
19. 10 cm.
20. 10 sq. cm.

Page 541  EXERCISE 166 (a)
1. £25 11s. 7d.
2. £96 8s. 2d.
3. £36 6s. 9d.
4. £27 7s. 9d.
5. £38 14s. 8d.
6. £31 2s. 9d.
7. £47 12s. 4d.
8. £25 14s. 3d.
9. £53 15s. 2d.
10. £34 19s. 9d.
11. £78 9s. 1d.
12. £50 10s. 6d.
13. £4109 1s. 1d.
14. £182 4s. 3d.
15. £1383 2s. 2d.
16. £899 12s. 2d.
17. £113 17s. 7d.
18. £2 7s. 2d.
19. £115 15s. 3d.
20. £161 7s. (£161 6s. 10d.).

Page 542  EXERCISE 168 (a)
1. £1642; £1642 2(8).
2. £1242; £1241 7(7).
3. £1354; £1353 5(0).
4. £5478; £5477 6(3).
5. £427.
7. £1583.
8. £1193.
10. £678 5(5).
11. 3.
12. 3:7.
13. 3:7.
14. 3:7.
15. £1; (18s.).
16. £750.
17. 1:50 ft.
18. 0:912.

Page 543  EXERCISE 169 (a)
1. 28s.; 50%.
2. 33:1%.
3. £39.
4. 5% gain.
5. 6%.
6. 33%.
7. 35s. 7d.
8. 9:6 st.
9. £41 13s. 4d.; 65:6%.
10. £125.
11. £50.
12. 20s. 10d.
13. 26%.
14. 12:4% increase.
15. 144% increase.
16. 6:6% increase.
17. 132:4%, 172:8% increase.
18. 22% decrease.

Page 544  EXERCISE 170 (a)
1. 28%.
2. 32:8%.
3. 2s. 4d. per lb.
4. 5%.
5. 23:13.
7. 3:4.
8. 93:5%.
9. 46 lb.
10. £8 per cwt.
11. £14,987.
12. 75.
13. 95.
14. £415.
15. 180%.

Page 546  EXERCISE 172 (a)
1. £120; £6 8s.
2. £112; £8 8s.
3. £280; £19 4s.
4. £587 10s.; £45.
5. 250; £3 15s.
6. 200; £14.
7. 600; £60.
8. £1650.
9. £170.
10. £148 10s.
11. 50%.
12. 75%.
13. 93%.
14. 4%.
15. 7s. 6d.
16. 8s. 6d.
17. 32%.
18. 78%.
19. At 5.
20. At 3s. 6d.
ANSWERS TO APPENDIX

Page 547

EXERCISE 173 (a)

1. 1.440. 2. 224. 3. 8s. 3d. 4. £10 8s. 4d. 5. £500. 6. Nothing.
7. 400; £1 more.
8. 9000; £5 8s. less.
9. £11 15s. more.
10. 5 3/8% p.a.
11. £40. 12. 1,250,000; £64,532.
13. £119; £117.
14. £111 5s.; £108 15s.
15. £132 15s.; £127 2s. 6d.
16. £682 10s.; £637 10s.
17. £70.

Page 549

EXERCISE 175 (a)

1. £518; £35.
2. £992; £48 12s.
3. £395 7s. 8d.; £18 2s. 9d.
4. £500 stock; £60.
5. £301 14s. 6d. stock; £15 1s. 9d.
6. £35 1s. 9d. stock; £24 1s. 7d.
7. £850 10s.
8. £781 4s.
9. £469 0s. 2d.
10. £6 13s. 4d. per cent.
11. £6 13s. 9d. per cent.
12. £4 4s. 5d. per cent.
13. £6 13s. 9d. per cent.
16. 112.
17. £1700; £4 11s. 9d. per cent.
18. £173 16s. 8d.; £7 17s. 6d.
19. 6% stock. 20. £98 2s. 21. 112 1/8.
22. £1631 5s.; £1618 15s.
23. £664; £657 12s.
24. £977 10s.; £969.
25. £591; £939.
26. £409 10s.; £404 2s.
27. £336 17s. 6d.; £293 2s. 6d.
29. 70 3/4.
30. 115.

Page 550

EXERCISE 176 (a)

1. £600 stock. 2. £7200 stock. 3. £168710s. 4. £50.
5. £40 more.
6. £7 10s. more.
7. £7 9s. 8d. more.
8. £433 16s.
9. £35 more.
10. 78.
11. £17 7s. 6d. loss.
13. 73 1/8.
14. £82 10s. gain.
15. 6%.
16. £132.

Page 552

EXERCISE 178 (a)

1. 22 1/2 min. 2. 10 1/8 m.p.h. 3. 12-6 sec.
4. 4 m.p.h., same way.
5. 49-8 m.p.h.; 21-1 sec.
6. 3 1/8 m.p.h.
7. 8-6 sec.
8. 9 p.m., Monday.

ANSWERS TO APPENDIX

Page 553

EXERCISE 180 (a)

1. 25%.
2. 62-4 m.p.h.; 65-3 m.p.h.
3. 2-5 kg.
4. 889 gall.
5. 20%.
6. 1040 m.p.h.
7. £2700.
8. 1%.
9. 1760.
10. £4 6s. 6d.
11. 8 gm. (7-82 ± 0.07).
12. 3 ft. per sec.
13. 110 gall.
14. \( e^{-\frac{(u-v)}{t}} \)
15. 1s. 2d. per yd.
16. 16-5 sq. in.
17. 1-89 in.
18. £1200.
19. 9 1/3 in.
20. 132 gm.
21. 12.02 p.m.; 8-1 mi. from X (8 1/3).
22. £157.
23. 200 cu. ft.
24. 8-33, 4-64 lb.
25. 27 3/8%.
26. 8 mi.

Page 556

DRILL EXERCISES D 21-26

Page 556

D 21

Page 557

D 22

Note.—Answers to Exercises D 22, D 23 are given correct to 4 figures.
### Answers to Appendix

**Page 557**

<table>
<thead>
<tr>
<th>D 23</th>
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<tbody>
<tr>
<td>1</td>
<td>2. 634.</td>
</tr>
<tr>
<td>2</td>
<td>2. 450.</td>
</tr>
<tr>
<td>3</td>
<td>25. 75.</td>
</tr>
<tr>
<td>4</td>
<td>3. 861.</td>
</tr>
<tr>
<td>5</td>
<td>0. 8854.</td>
</tr>
<tr>
<td>6</td>
<td>0. 2156.</td>
</tr>
<tr>
<td>7</td>
<td>0. 9364.</td>
</tr>
<tr>
<td>8</td>
<td>0. 4241.</td>
</tr>
<tr>
<td>9</td>
<td>0. 7892.</td>
</tr>
<tr>
<td>10</td>
<td>0. 6826.</td>
</tr>
<tr>
<td>11</td>
<td>0. 8806.</td>
</tr>
<tr>
<td>12</td>
<td>0. 3960.</td>
</tr>
<tr>
<td>13</td>
<td>0. 5454.</td>
</tr>
<tr>
<td>14</td>
<td>0. 2913.</td>
</tr>
<tr>
<td>15</td>
<td>0. 8106.</td>
</tr>
<tr>
<td>16</td>
<td>0. 01965.</td>
</tr>
<tr>
<td>17</td>
<td>0. 6925.</td>
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<td>18</td>
<td>2. 736.</td>
</tr>
<tr>
<td>19</td>
<td>0. 6215.</td>
</tr>
<tr>
<td>20</td>
<td>2. 422.</td>
</tr>
<tr>
<td>21</td>
<td>0. 4806.</td>
</tr>
<tr>
<td>22</td>
<td>1. 035.</td>
</tr>
<tr>
<td>23</td>
<td>0. 7612.</td>
</tr>
<tr>
<td>24</td>
<td>0. 1585.</td>
</tr>
<tr>
<td>25</td>
<td>569. 9.</td>
</tr>
<tr>
<td>26</td>
<td>0. 000003, 218.</td>
</tr>
<tr>
<td>27</td>
<td>0. 03939.</td>
</tr>
<tr>
<td>28</td>
<td>0. 827.</td>
</tr>
<tr>
<td>29</td>
<td>1. 832.</td>
</tr>
<tr>
<td>30</td>
<td>0. 7644.</td>
</tr>
<tr>
<td>32</td>
<td>4. 051.</td>
</tr>
<tr>
<td>33</td>
<td>833. 8.</td>
</tr>
<tr>
<td>34</td>
<td>64850.</td>
</tr>
<tr>
<td>35</td>
<td>10. 84.</td>
</tr>
<tr>
<td>36</td>
<td>2. 746, 000.</td>
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<tr>
<td>37</td>
<td>1. 604.</td>
</tr>
<tr>
<td>38</td>
<td>0. 1581.</td>
</tr>
<tr>
<td>39</td>
<td>0. 8076.</td>
</tr>
<tr>
<td>40</td>
<td>2. 802.</td>
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</table>

**Page 558**

<table>
<thead>
<tr>
<th>D 24</th>
<th></th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>27. 0 in.; 45. 2 cm.; 628 yd.</td>
</tr>
<tr>
<td>2</td>
<td>3. 82 in.; 1. 11 m.; 0. 318 ml.</td>
</tr>
<tr>
<td>3</td>
<td>208 sq. in.; 31. 2 sq. cm.; 15400 sq. yd.</td>
</tr>
<tr>
<td>4</td>
<td>50. 9 sq. in.</td>
</tr>
<tr>
<td>5</td>
<td>136 cu. in.; 95. 4 cu. cm.; 38. 2 cu. in.; 19. 7 cu. cm.</td>
</tr>
<tr>
<td>6</td>
<td>113 sq. in.; 42. 4 sq. cm.; 48 sq. in.; 39. 5 sq. cm.</td>
</tr>
<tr>
<td>7</td>
<td>283 sq. cm.</td>
</tr>
<tr>
<td>8</td>
<td>151 oz.</td>
</tr>
<tr>
<td>9</td>
<td>361 sq. ft.; 19. 7 tons.</td>
</tr>
<tr>
<td>10</td>
<td>136 cu. in.</td>
</tr>
<tr>
<td>11</td>
<td>1360 sq. in.</td>
</tr>
<tr>
<td>12</td>
<td>28. 2 ft.</td>
</tr>
<tr>
<td>13</td>
<td>2640 sq. ft.</td>
</tr>
<tr>
<td>14</td>
<td>4. 25 cu. ft.</td>
</tr>
<tr>
<td>15</td>
<td>16. 79. 5 in.</td>
</tr>
<tr>
<td>16</td>
<td>0. 359 in.</td>
</tr>
<tr>
<td>17</td>
<td>2. 21 ft.</td>
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<tr>
<td>18</td>
<td>30. 3 cu. in.; 28. 7 cu. in.; 314 cu. in.</td>
</tr>
<tr>
<td>19</td>
<td>251 sq. in.; 62. 8 sq. in.; 27. 1 sq. cm.</td>
</tr>
<tr>
<td>20</td>
<td>20. 9 sq. in.; 1. 67 in.; 3. 64 in.</td>
</tr>
<tr>
<td>21</td>
<td>638 cu. in.; 358 sq. in.</td>
</tr>
<tr>
<td>22</td>
<td>7-44 in.</td>
</tr>
<tr>
<td>23</td>
<td>25. 5 cu. cm.</td>
</tr>
<tr>
<td>24</td>
<td>37500.</td>
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<tr>
<td>25</td>
<td>64. 6 lb.</td>
</tr>
<tr>
<td>26</td>
<td>648 gm.</td>
</tr>
<tr>
<td>27</td>
<td>38. 5 cm.</td>
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**Page 560**

<table>
<thead>
<tr>
<th>D 25</th>
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<tbody>
<tr>
<td>1</td>
<td>£324 9s. 7d.</td>
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<tr>
<td>2</td>
<td>£275 12s. 6d.</td>
</tr>
<tr>
<td>3</td>
<td>£92 12s. 2d.</td>
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<tr>
<td>4</td>
<td>£843 13s.</td>
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<td>5</td>
<td>£484 13s. 10d.</td>
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<td>£1020 12s. 2d.</td>
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<td>8</td>
<td>£82 14s. 11d.</td>
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<td>£266 1s. 10d.</td>
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<td>£678 18s. 3d.</td>
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<td>12</td>
<td>£11, 165 19s. 1d.</td>
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<td>13</td>
<td>£36 14s. 6d.</td>
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<td>14</td>
<td>£14 19s.</td>
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<td>£4089 7s. 4d.</td>
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<td>£926 11s. 1d.</td>
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<td>£50 12s. 4d.</td>
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<td>£13 15s. 1d.</td>
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<td>27</td>
<td>£81 12s. 11d.</td>
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### Revision Papers A 49–56

**Page 562**

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<tbody>
<tr>
<td>1</td>
<td>74; 1. 77.</td>
</tr>
<tr>
<td>2</td>
<td>0. 249; 0. 534(5).</td>
</tr>
<tr>
<td>3</td>
<td>14. 4 m.p.h.</td>
</tr>
<tr>
<td>4</td>
<td>£175. 413s. 4d.</td>
</tr>
<tr>
<td>5</td>
<td>31. 3 cu. in.</td>
</tr>
<tr>
<td>6</td>
<td>£61 18s.</td>
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</table>

**Page 563**

<table>
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<th>Paper A 50</th>
<th></th>
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<tbody>
<tr>
<td>1</td>
<td>25 payments, 12s. 6d.; 0. 187.</td>
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<tr>
<td>2</td>
<td>4. 17; 0. 884.</td>
</tr>
<tr>
<td>3</td>
<td>3. 141 in.</td>
</tr>
<tr>
<td>4</td>
<td>£5 16s. 7d.</td>
</tr>
<tr>
<td>5</td>
<td>96. 3 ac.</td>
</tr>
<tr>
<td>6</td>
<td>7. 19, 6. 95 in.</td>
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</tbody>
</table>

**Page 563**

<table>
<thead>
<tr>
<th>Paper A 51</th>
<th></th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>1 ton 10 cwt. 98 lb.; 308. 8.</td>
</tr>
<tr>
<td>2</td>
<td>0. 00103; 25. 1</td>
</tr>
<tr>
<td>3</td>
<td>8. 41%; 13. 6%.</td>
</tr>
<tr>
<td>4</td>
<td>22. 2 hr.</td>
</tr>
<tr>
<td>5</td>
<td>80 cm.; 75. 4 cm.</td>
</tr>
<tr>
<td>6</td>
<td>217 sq. in.; 68. 7 lb.</td>
</tr>
</tbody>
</table>
Page 564  Paper A 52
1. £779 3s. 4d.; 1-77.
2. 213; 0-991.
3. 160,000,000 bush.
4. 3\(^\frac{1}{2}\)%.
5. 0-95 mm.
6. 40-8 oz.

Page 564  Paper A 53
1. 0-730 m.; £1050.
2. 0-396; 57-7.
3. 8-84 fr.
4. 5335.
5. 379 lb.
6. 47-6%.

Page 565  Paper A 54
1. 0-3097; 3 cwt. 1 qr.
2. 0-000,179; 2-59.
3. 35 m.p.h.
4. £115; £161.
5. 285.
6. 47-1 sq. in.; 4 in.

Page 566  Paper A 55
1. 2s. 4d.; 49.
2. 1-23; 3-69.
3. 33-6%.
4. 54,000 gall.; 6750 gall.
5. 8%.
6. 2-45 in.

Page 566  Paper A 56
1. £7 0s. 3d.; 59.
2. 6-60; 36.
3. 78-1%; 3-65: 1.
4. £4 17s. 6d.
5. 4 ft. 5-5 in.
6. 254 sq. in.; 251 cu. in.

Page 567  Paper A 57
1. 87-74 fr.; 4\(^{1/2}\)%.
2. 0-0445; 132.
3. 15%.
4. 1-31 kg.
5. £20 12s. 2d.
6. £187 10s.; £125.

Page 568  Paper A 58
1. 40\(^\frac{1}{2}\)%; 11\(^{1/2}\)%; 0-84.
2. 4-63; 3-05.
3. 0-88.
4. 6-90(5) cm.
5. 29s. (28-13).
6. £3 18s. 9d.

Page 568  Paper A 59
1. £875 8s. 4d.; 87\(^{1/2}\) sec.
2. 12-5; 3-15 ac.
3. 11,100 lb.
4. £39 0s. 1d.
5. 10 kg.
6. £27 2s. 6d.; 4-5%.

Page 569  Paper A 60
1. 1-1430 km.; 31\(^{1/2}\)%.
2. 0-014,1(5); 8-79.
3. 70 gall.; 20-7 cm.
4. 92\(^{1/2}\)%.
5. 4-1.
6. £3 2s. 6d. less.