REVISION COURSE IN GENERAL MATHEMATICS

BY

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PREFACE

Most pupils between the ages 11+ and 15+ work through a course of what is now called "mathematics at ordinary level" by English examining bodies, and the two terms preceding the examination for the General Certificate of Education are normally devoted by such pupils to reviewing and consolidating the ground covered in the previous four years. The purpose of this book is to provide a suitable two-term revision course for those who take this examination at "the ordinary level", but it is believed that it will also prove of value in preparation for any examination of similar standard.

The subject-matter is arranged in topics, with illustrative examples, statements of facts and formulæ, and exercises which include all main types of questions associated with the topic. Since the purpose of an exercise is to remind the pupil of ideas and methods previously encountered, the exercises are usually short, particularly if most of the "bracketed numbers" are found not to be needed. In this way, it is hoped that a comprehensive revision of the work of the preceding four years can be achieved in two terms.

There are two sets of revision papers: the questions in the A papers are of a routine type, those in the B papers are of greater length and difficulty. Since many teachers will wish to test their pupils by use of actual papers set in previous examinations, the major portion of the book is devoted to topics.

C. V. D.

February, 1952
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Logarithms: anti-logarithms; logarithmic sines, cosines, tangents; natural sines, cosines, tangents; cotangents, cosecants, secants, squares, square roots.

Arithmetic

Easy Miscellaneous Examples

Example. How many marking tapes, each 1½ in. long, can be cut from a strip 5 feet long and how much remains?

5 feet = 60 in.
60 in. - 1½ in. - 60 = 7 - 60 x 4 = 240
7 7 7
7 - 34½ in.

∴ 34 tapes can be cut from the strip, and the remainder is 5 7 in.

∴ remainder - 5 7 of 1½ in. = 5 7 - 1 2 in.

∴ 34 tapes can be obtained and 5 7 in. remains over.

Exercise 1

1. A man is cycling at 15 m.p.h.; how many yards does he travel in 3 seconds?

2. 2 dozen eggs cost 9 shillings; how many can I buy for 7s. 6d.?

3. How many escudos do I get for 13 shillings, if the rate of exchange is 80 escudos to the £?

4. Express 10,000 lb. in tons, cwt., lb.

5. Find the least number which must be added to 1250 to make the sum exactly divisible by 79.

6. 754 tickets at 9d. each are sold for a raffle. The prize is 15 guineas. What profit is made?

7. Find the cost of 13 yd. 2 ft. of material at 10½d. per yard.

8. Find in tons to the nearest ton how much extra meat is needed each week to increase the meat-ration of 52 million people by ¼ ounce per week.

9. Find the average speed in miles per hour, to nearest tenth of a mile, of an athlete who runs 100 yards in 10 seconds.

10. Express 0·7436 yards in ft., in., to nearest ¾ inch.

11. A strip of cloth is 0·6 metres long. Express the length in feet, inches to nearest inch. [1 metre = 39·37 in.]
ARITHMETIC

12. Express 4$\frac{1}{2}$ inches in cm., mm., to nearest mm.
1 inch = 2.54 cm.

13. The daily cost of school dinners for 1750 children is £115. Find the cost of a child's dinner to nearest farthing.

14. A visitor's hotel bill in Madeira is 100 escudos per day and other expenses average 8 escudos a day. Find in £ s. the cost of a stay of 6 weeks. (£1 = 80 escudos.)

15. Find the rates payable on a house of rateable value £64, if the rate is 1s. 6d. in the £.

16. Express 78 yards 8 inches as a fraction of a mile.

17. A boy is paid 1s. 10$\frac{1}{2}$d. per hour. How many hours does he work if he earns £4 10s.?

18. A car travels 21.5 km. in 22$\frac{1}{2}$ minutes. Find the average speed in metres per second to 1 place of decimals.

19. Find the total cost of the following: 8 eggs at 3s. per dozen, 6 quarts of milk at 3$\frac{1}{2}$d. per pint, 12 oz. of butter at 2s. 4d. per lb., 1 lb. 4 oz. of margarine at 10d. per lb.

20. Find the total cost of insurance stamps as follows: 14 at 4s. 2d. each; 15 at 3s. 3d. each; 24 at 1s. 11d. each.

21. Find the value of $\frac{3}{4}$ of £1 17s. 1$\frac{1}{2}$d.

22. A liner travels 2938 miles from America to Britain in 3 days 20 hours 42 minutes. Find the average speed in miles per hour to one place of decimals.

23. A bookshelf 4 ft. 6 in. long, contains 11 books each 2$\frac{1}{4}$ in. thick and 9 books each 1$\frac{1}{2}$ in. thick. How many more books each 1$\frac{1}{4}$ in. thick can be fitted in?

24. If the price of rubber rises by £21 10s. per ton, find the least rise in pence, farthings per lb. to cover this increase.

25. How many 5-lb. bags of sugar can be filled from a sack containing 1 cwt. 2 qr. 10 lb. of sugar, if half an ounce is wasted in filling each bag? How much is left over?

26. How many articles at 5$\frac{1}{4}$d. each can be bought for £1 10s. and how much money is over?

27. How many crates each weighing 7 cwt. 2 qr. can be carried by a truck if the total load must not exceed 10 tons?

28. The rateable value of a town is £135,000. How much is produced by a rate of 1d. in the £? What rate in pence to the £ is required to cover the expenditure of £1800 on traffic control?

29. I take 24 minutes to walk from my house to the station if I walk at 3 miles an hour. How long shall I take if I walk at 4 miles an hour?

30. If 2.86 kronen per kg. is the same price as 1s. 4d. per lb., find the rate of exchange in kronen to the £, taking 1 kg. = 2.2 lb.

PROPORTION, AVERAGES AND PRACTICE METHODS

Example. A man whose liabilities are £2833 goes bankrupt. His assets are £874. At what rate in s. d. to the £, to the nearest penny, are his creditors paid off?

For debts £2833, he can only pay £874.

$\therefore$ for a debt of £1, he can only pay £874

$\therefore$ for a debt of £2833, he can only pay £874

that is, 874

$\therefore$ for a debt of £2833, he can only pay 874

6s. 2d.

2833|17480s.
16998
4828.
5784d.
5666
118

$\therefore$ the creditors are paid at the rate of 6s. 2d. in the £, to the nearest penny.

EXERCISE 2

1. A car travels 48 miles in 1$\frac{1}{2}$ hours; how far will it travel in 2$\frac{1}{2}$ hours at the same rate?

2. 18 men can repair a road in 14 days; how long will 21 men take to do so if all work at the same rate?

3. 44 feet per second is the same speed as 30 m.p.h.; express a speed of 22$\frac{1}{2}$ m.p.h. in feet per second.

4. A comet aircraft is timed to take 3 hours 15 minutes for a journey of 1495 miles; find the average speed in m.p.h.
5. A car runs 24 miles on a gallon of petrol. Find the cost of a journey of 280 miles if petrol costs 3s. 6d. per gallon.

6. A swimming bath holds 37,500 gallons; find the cost of filling it at 1s. 4d. per 1000 gallons.

7. A boy earns 5 shillings for 3 hours' work; how long will he take to earn 8s. 4d.?

8. A strip of matting $\frac{3}{5}$ yard long costs 8s. 9d.; find the cost of a strip of the same width 5 yd. 2 ft. long.

9. I pay £1 11s. 6d. for 8 cwt. of coal; how much shall I obtain for £3 18s. 9d.?

10. Telegraph poles are situated at intervals of 66 yards along a railway. Find in m.p.h. the speed of a train which covers 16 of these intervals per minute.

11. From a reel of wire a piece 34-2 cm. is cut off and found to weigh 5.7 grams. Find in metres the length of wire which weighs 0.75 kg.

12. Linoleum for a passage 8 yards long, 5 feet wide, costs £3 8s. Find the cost for a passage 10 yards long, 6 feet wide.

13. A bicycle has a back wheel of diameter 28 inches and a front wheel of diameter 26 inches. How many revolutions are made by the front wheel for a journey in which the back wheel makes 728 revolutions?

14. How many gallons of water are required for 22 men for 12 days on a desert journey if each man needs 18 oz. of water a day, given that 1 gallon of water weighs 10 lb.?

15. Find the rate demanded from a man whose house is assessed at £34, if the rate is 12s. 6d. in the £.

16. A bankrupt owes £2750, but is only able to pay at the rate of 8s. 4d. in the £. Find his assets.

17. A local rate of 1d. in the £ yields £4580. Health services require a rate of 1s. 9d. in the £; find their cost. A householder is rated at £36; how much does he pay towards the health services? Salvage collection costs £29,790; what rate in pence per £ to the nearest farthing is required to meet it?

18. The rateable value of a town is £624,000 and the cost of the public library service is £3600. Find the rate in pence per £, correct to one place of decimals, required for the library service. How much to the nearest penny does a man who lives in a house rated at £40 pay towards the service?

19. The average of seven numbers is 39, and the average of three of them is 27, find the average of the other four.

20. The monthly outputs of coal from a mine for the first five months of a year were 4219, 4623, 4905, 5116, 5228, tons. What must be the output in the sixth month to make the monthly average 5000 tons?

21. A motorist averages 45 m.p.h. for the first 2 hours and 35 m.p.h. for the next 3 hours. Find the average speed for the whole journey.

22. A motorist averages 40 m.p.h. for the first 70 miles and 32 m.p.h. for the remaining 40 miles. Find the average speed for the whole journey.

23. A man's weekly wages are £6 for the first 20 weeks and £6 10s. for the next 24 weeks of a year. Find the weekly average for the other weeks of the year, if the average for the whole year, 52 weeks, is £6 5s.

24. 24 sacks of coal weigh 1 ton 5 cwt. 2 qr. Find in cwt., lb. the average weight of a sack of coal.

25. 550 yards of an embankment is constructed in 10 days by 250 men. The total length to be constructed is half a mile and the total time available is 14 days. How many extra men must be engaged to complete the work in the time?

26. A cyclist starts from A at 1 p.m. and rides towards B, 54 miles away, at 9 m.p.h.; a motorist leaves B at 3 p.m. and drives to A at 36 m.p.h. How far from A do they meet and at what time?

27. Taking 1 lb. = 0.4536 kg., express in grams to the nearest gram (i) 4 ounces, (ii) 6 ounces, (iii) 7 ounces.

28. Taking 1 yd. = 0.9144 metres, express in cm., mm., correct to nearest mm., (i) 1 ft., (ii) 2 ft. 4 in.

29. Taking 1 gallon = 277-3 cu. in., express 2½ pints in cu. in. to one place of decimals.

30. Taking 1 mile = 1.6093 km., express 1100 yards in metres to nearest metre.
31. Find the cost of 150 knives at 2s. 4½d. each.

32. Find the amount of 365 days' pay at 16s. 4d. a day.

33. Find by a subtraction method the cost of 92 yards at 11½d. a yard.

34. Multiply 2 yd. 1 ft. 6 in. by 2½, using a practice method.

35. Find the cost of 4 cwt. 3 qr. at £1 6s. per cwt.

36. Find the cost of 12 yd. 2 ft. 6 in. at £1 14s. 9d. per yard.

RATIO, SCALES AND PROPORTIONAL PARTS

Example 1. An insurance company deducts tax at 9s. in the £ from an annuity. Find the gross value of the annuity if the net annuity is £286.

After deducting 9s. from 20s., 11s. remains; and so

\[ \text{gross annuity} = \frac{11}{9} \times £286 = £20 \times 26 \]

= £520.

Example 2. The scale of a map is 2 inches to the mile. Find in acres the area of an estate represented by an area of 2¼ sq. in. on the map.

A square, side 2 in., represents 1 sq. mile,
that is,
\[ 4 \text{ sq. in. represent } 640 \text{ ac.} \]
\[ \frac{2}{4} \text{ sq. in. represent } \frac{640 \times 2}{4} = 320 \text{ ac.} \]

that is,
\[ (64 \times 6) \text{ ac. or } 384 \text{ ac.} \]

Example 3. A prize of 30s. is shared between A, B, C in the ratios 4 : 3 : 2 respectively. How much does A receive?

If A receives 4s., B receives 3s. and C receives 2s., and then \(4 + 3 + 2\) has been distributed.

\[ A's \ share = \frac{4}{4+3+2} \ of \ 30s. = \frac{4 \times 30}{9}s. = 3 \frac{2}{3} = 13s. 4d. \]

Exercise 3

1. Find the ratio of a speed of 400 yards a minute to a speed of 30 feet per second.

2. If \(\frac{3}{4}\) of a number is 66, find the number.

3. After 3d. in the shilling has been deducted from a bill, there remains 15s. 6d. How much was the bill?

4. After 3d. in the shilling has been added to the price of a seat at a cinema for entertainment tax, the cost is 7s. 6d. How much has been added for tax?

5. After driving \(\frac{1}{3}\) of the distance from A to B, a man has still 99 miles to go. How far is B from A?

6. After tax at the rate of 9s. in the £ has been deducted from a dividend, there remains £13 15s. How much was the dividend?

7. The edges of two cubes are 4½ inches, 6 inches. Find the ratio of (i) the areas of their surfaces, (ii) their volumes.

8. A solid metal cube, edge 1 inch, weighs 6½ oz. Find in lb. the weight of a solid cube of the same metal, edge 2 inches.

9. The scale of a map is 4 in. to the mile. Find in sq. miles the area of an estate represented by an area 32 sq. in. on the map.

10. The scale of a map is 2½ in. to the mile. Find in sq. in. on the map which represents a farm of 128 acres. \[640 \text{ ac.} = 1 \text{ sq. mile.}\]

11. The scale of a map is 5 miles to the inch. What area in sq. miles is represented by a square, side 3 in., on the map? What is the length of the side of a square on the map which represents an area of 506½ sq. miles?

12. The frontage, 12 yards, of a house is represented on a plan by a line 2 ft. 3 in. long. On this plan, a room measures 9 in. by 6 in.; find the area of the floor of the room in sq. ft.

13. The length of a scale model of a ship is 4 ft. 4 in. and there is a mast on the model 4½ in. high. If the actual height of the mast is 60 feet, find the length of the ship.

14. An estate represented by a rectangle 3½ in. by 5½ in. on a map, scale 6 in. to the mile, is sold for £9350. Find the price per acre.
15. A solid metal cylinder, radius 2 cm., height 6 cm., weighs 600 gm. Find in kg. the weight of a solid cylinder of the same metal, radius 3 cm., height 9 cm.

16. Two tins of paint at 3s. a tin are needed to paint the surface of a model of a machine on a scale of 1 in. to the foot. Find the cost of painting a model of the same machine on a scale of 1 1/2 in. to the foot.

[17] Divide a field of area 2700 sq. yd. into two allotments in the ratio 4 : 5.


[19] The lengths of the sides of a triangle are in the ratios 4 : 7 : 10 and its perimeter is 31.5 cm. Find the length of each side.

20. Divide 8s. 2d. into two shares so that one share is 2 1/2 times the other.

21. Coal at £4 10s. a ton is mixed with coke at £4 a ton in the ratio 7 : 3. How much money is saved by using 5 tons of the mixture in place of 6 tons of coal?

[22] Three ingredients costing 4s., 5s., 1s. 6d. per lb. are mixed so that their weights are proportional to 6, 5, 9. Find the cost of 1 lb. of the mixture.

[23] A, B, C provide £400, £350, £250 respectively for a business and share the profits £120 in the ratios of the capital each provides. What does each receive?

24. A puts £800 for 9 months and B puts £500 for 8 months into a business. The profits shared between A and B are £35; how much should each receive?

PERCENTAGE AND SIMPLE INTEREST

The percentage, R per cent., is the ratio R : 100 and is represented by the fraction $\frac{R}{100}$.

Example 1. Express £2 16s. as a percentage of £80.

\[ \frac{\text{£}2 \hspace{1em} 16s.}{\text{£}80} = \frac{38}{80} = \frac{19}{40} = \frac{1+\frac{1}{4}}{2} = 1+\frac{1}{4} = \frac{5}{4} \times 100\% = 125\% \]

Example 2. After the price of an article has been increased by 8%, the article costs 13s. 6d. Find the original price.

\[ \text{Old price} : \text{new price} = 100 : 108, \]
\[ \therefore \text{old price} = 13s. 6d. \times \frac{100}{108} = \frac{1627}{108} = \frac{20}{13} \times 100s. \]
\[ = \frac{50}{13} s. - 12s. 6d. \]

The phrase, gain or loss per cent., is taken to mean the percentage that the gain or loss is of the cost price.

Example 3. An article costing 7s. is sold for 10s. Find the gain per cent.

\[ \text{Cost price} = 7s., \text{gain} = 3s., \]
\[ \therefore \text{the gain is } \frac{3}{7} \text{ of the cost price,} \]
\[ \therefore \text{the gain per cent.} = \left( \frac{3}{7} \times 100 \right) \text{ per cent.} \]
\[ = 42\% \]

Simple Interest

If $P$ is the simple interest on $P$ for $T$ years at $R\%$ per annum

\[ i = \frac{P \times R \times T}{100} \]

Hence also, since $P \times R \times T = 100 \times 1$,

\[ R = \frac{100 \times 1}{P \times T}; \quad T = \frac{100 \times 1}{P \times R}; \quad P = \frac{100 \times 1}{R \times T} \]

Example 4. A man borrows £40 on the condition that he pays back £44 at the end of 8 months. At what rate % p.a. is interest charged?

\[ P = 40, 44 - 40 = 4, T = \frac{8}{12} = \frac{2}{3}, \]
\[ R = \frac{100 \times 1}{P \times T} = \frac{100 \times 4}{40 \times \frac{2}{3}} = 10 \times \frac{2}{1} = 15; \]
\[ \therefore \text{interest is charged at } 15\% \text{ p.a.} \]

EXERCISE 4

1. It costs £20 to make a book-case; 62 1/2% of the cost is for labour; how much is this?

[2] A man receives £840 a year and spends £780 a year. What percentage of his income does he save?
ARITHMETIC

8. Housekeeping bills which were formerly £6 5s. a week have now increased by 12%. How much are they now?

4. The population of a village increased by 8% between 1941 and 1951. It was 810 in 1951; what was it in 1941?

5. I bought an overcoat for ten guineas and sold it to a friend for £8 15s.; find my loss per cent.

6. By selling a radio set for £35, a dealer made a profit of 40%; what did he pay for it?

7. 15% of a farm is under grass and 78% sown for crops. What percentage of the farm remains?

8. A carpet 15 ft. 9 in. by 12 ft. is placed in a room 18 ft. by 15 ft.; what percentage of the floor is uncovered?

9. Express (i) 3s. 4½d. as a percentage of 5s.; (ii) 16.5 grams as a percentage of 1½ kg.

10. Coal output fell from 231,000,000 tons in 1939 to 184,000,000 tons in 1944. Find the decrease per cent. to 2 figures.

11. A prefabricated house, estimated to cost £954, is found to cost £1116. Find the percentage increase to nearest unit.

12. A house-agent charges as commission on the sale of a house, 4% on the first £500 of the price, 2½% on the next £2000, and 1% on the remainder. Find his commission on the sale of a house for £4000.

13. A salesman charges £840 for a car together with £580 for purchase tax. Express the tax as a percentage of the total amount paid, correct to 3 figures.

14. Goods from a bankrupt stock are sold for £68 at a loss of 15%. What did they cost?

15. A retailer buys goods at £9 6s. 8d. per cwt. and sells at £s. 11½d. per lb. Find his gain per cent.

16. A ring is marked at a price 67½% above cost. If 20% discount is allowed on the marked price, find the net percentage profit.

17. The receipts from income tax increased during the years 1938 to 1944 by 290%. In 1938 the receipts were 336 million pounds; find to 3 figures the receipts in 1944.

18. 66⅔% of the price of a car is added for purchase tax. Find the original price if the total amount paid is £883 6s. 8d.

19. A commercial traveller is paid a commission of 7½% of the value of the goods he sells. Find the value of the goods on which the commission is £14 5s.

20. Goods are sold for £33 12s. at a profit of 26% of the cost price. Find the profit.

21. Find the simple interest on £260 for 4 years at 3½% per annum.

22. Find the simple interest on £256 for 15 months at 2½% per annum.

23. Find to the nearest penny the simple interest on £182 for 3 years at 4½% per annum.

24. Find to the nearest penny the simple interest on £140 for 20 days at 4⅜% per annum, taking 1 year = 365 days.

25. The interest on a loan at 4½% per annum for 2½ years is £108. Find the amount of the loan.

26. If the interest on £65 for 1½ years is £4 11s., find the rate per cent per annum.

27. A moneylender charges 8d. interest on a loan of £1 for 1 month. To what rate per cent per annum simple interest is this equivalent?

28. The interest on a loan of £375 at 5½% per annum is £36. For how long is the money lent?

PERCENTAGE PROBLEMS

Example 1. If the price of petrol rises by 20%, find the percentage reduction a man must make in his consumption if his expenditure rises only by 8%.

The first cost of x gall. at y pence per gall. = xy pence.

If new cost of 1 gall. is 120 100 pence and if new expenditure is 108 100 pence, new consumption = 108×x 120 ×x = 9x 10 gallon.

.: reduction in consumption = (x - 9x) = x 10 gallon.

.: the percentage reduction is 10%.
Example 2. A retailer makes a profit of 33\(\frac{1}{3}\)% by selling refrigerators at £96 each. When the wholesale price rises by £18, he sells at a price which gives himself only a profit of 30%, but the number he sells per year fall by 15%. Find the percentage decrease in his annual profit on refrigerator-sales.

The retailer's first cost price = \(\frac{100}{133\frac{1}{3}}\) of £96 = \(\frac{3}{4}\) of £96 = £72.

His first profit on each = £96 - £72 = £24.

His second cost price = £72 + £18 = £90.

His second profit on each = \(\frac{10}{9}\) of £90 = £10.

The number sold annually is reduced in the ratio 100 : 85 or 20 : 17; his former profit on sale of 20 refrigerators = £24 \times 20 = £480,

his new profit on sale of 17 refrigerators = £10 \times 17 = £170.

\[\text{the percentage decrease in annual profit} = \frac{480 - 170}{480} \times 100\% = 64\%\]

EXERCISE 5

1. In an examination, a boy obtains 64 out of 80 for arithmetic, 77 out of 110 for algebra, 44 out of 110 for geometry. What percentage of the total does he obtain? If each paper had been marked out of 100, find how many marks he would have obtained on each paper and his percentage of the total.

2. The cost of manufacture of a radio set is £45 and is sold at a profit of 65%; 25% of this profit is put aside for overheads. Find the nett profit and express it as a percentage of the cost price.

3. A man's income is £750 a year. The first £110 is free of tax; the next £50 is taxed at 3s. in the £, the next £75 at 6s. in the £, and the rest at 9s. in the £. What percentage of his income is taken for tax?

4. Purchase tax on an article is raised from 66\(\frac{2}{3}\)% to 75% of its cost price. If the price with tax was originally 16s. 8d., find the new price with tax.

5. Full marks for two examination papers are in the ratio 3 : 4; a candidate obtains 70% on the first paper and 55% on the second paper. Find the percentage of the total obtained.

6. A market gardener sells to a retailer 50 dozen plants at 37s. 6d. per dozen less 20% discount. The retailer finds that 3 dozen are unsaleable; find the price per dozen at which he must sell the others to make a profit of 45%. [Answer to nearest penny.]

7. A greengrocer sells strawberries at 3s. 4d. per lb. at a profit of 60% on the cost price. After selling at 5s. 4d. per lb. three-quarters of what he has bought, he reduces the price of the remainder to 1s. 10d. per lb. Find his profit per cent. on the whole transaction.

8. The shop price of an article is made up as follows: materials 11s. 3d., labour 22s. 6d., transport 3s. 9d., profit 7s. 6d. If cost of material rises by 20%, labour by 30%, transport by 40%, profit by 20%, find the percentage increase in the shop price.

9. A dealer buys articles at invoiced price of £2 10s. a dozen plus purchase tax at 66\(\frac{2}{3}\)% of invoiced price. He is given a discount of 1s. 8d. in the £ on the invoiced price. If he sells the articles at 7s. 11d. each, find the percentage profit on his outlay.

10. 3272 copies of a book are sold in the home market at 8s. 6d. each. The author receives 10% of the price on the first 2000 copies and 12\(\frac{1}{2}\)% on the others. There is also an export sale of 500 copies at 6s. each on which the author receives 7\(\frac{1}{2}\)% of the export price. How much did the author receive altogether?

11. A garage sold an average of 435 gallons of petrol a day when the price was 2s. 10d. per gallon. When the price was raised by 4d. per gallon, the average amount of petrol sold daily fell by 20%; but after 20 days it returned to normal. Find the change between the proceeds of sales for the 30 days before the rise in price and the proceeds for the 30 days after it.

12. A manufacturer sells an article to a wholesaler at a profit of 20%; the wholesaler sells it to the retailer at a profit of 33\(\frac{1}{3}\)%; the retailer sells it to a customer at a profit of 40%. In each transaction, profit is calculated as a percentage of the seller’s cost price. If the customer pays £5 12s., what was the cost to the manufacturer?

13. A man buys a house for £900 and spends £150 on repairs. The annual charges are ground rent £12, upkeep £25, and rates at 13s. 8d. in the £ on a rateable value of £45. At what weekly rent must he let the house so that he receives for 52 weeks enough to cover the annual charges and give him 5% interest on the money invested?
14. A dealer sells radio sets at £35 at a profit of 40%. When he has to pay £5 more for each set, he raises the selling price so as to make a profit of only 35% on the new cost price, but he finds that the number of sets he sells falls by 20%. Express his new total profit as a percentage of his former total profit.

15. In 1939, a rate of 12s. 6d. in the £ was required to meet annual expenses. In 1949, the rateable value was 10% lower than in 1939 and annual expenses were 26% higher. What rate in s. d. to the £ was needed in 1949 to meet expenses?

16. In an examination, every candidate must take either French or German and may take both. If 83.6% of the candidates took French and 31.2% of the candidates took German, what percentage took both? If 370 candidates took both, how many candidates were there altogether?

LENGTH AND AREA

Rectangle. If the area of a rectangle, \( l \) in. long, \( b \) in. broad, is \( A \) sq. in.,

\[ A = l \times b; \quad l = \frac{A}{b}; \quad b = \frac{A}{l}\]

Circle. If the length of the circumference of a circle, radius \( r \) in diameter \( d \) in., is \( C \) in., and if the area of the circle is \( A \) sq. in.,

\[ d = 2r; \quad r = \frac{d}{2}\]

\[ C = 2\pi r = \pi d; \quad r = \frac{C}{2\pi}; \quad d = \frac{C}{\pi}\]

\[ A = \pi r^2 = \frac{1}{4}\pi d^2; \quad r = \sqrt{\frac{A}{\pi}}; \quad d = \sqrt{\frac{4A}{\pi}}\]

\[ \pi \approx 3.1416\text{ or } \pi = 3.1416\]

Annulus. If a figure is bounded by two concentric circles, radii \( R \) in., \( r \) in., (\( R > r \)),

\[ \text{area} = \pi (R^2 - r^2) \text{ sq. in.} \]

\[ = \pi (R + r)(R - r) \text{ sq. in.} \]

Circular Cylinder. If the radius of a cylinder is \( r \) in. and the height of the cylinder is \( h \) in.,

\[ \text{area of curved surface of cylinder} = 2\pi rh \text{ sq. in.} \]

Measures. 1 mile = 1760 yards; 1 acre = 4840 sq. yards; 1 sq. mile = 640 acres.

EXERCISE 6

[Take \( \pi = \frac{22}{7} \) unless otherwise stated.]

1. A rectangular plot of land, 270 ft. by 40 ft. is sold for £75. Find the price per acre.

2. A rectangular field of area \( \frac{7}{4} \) acres is 165 yards wide. Find the length of the fence which encloses it.

3. Find the number of tiles, each 1 ft. 3 in. by 9 in., required for the floor of a hall, \( 15\frac{3}{4} \) yards long, 22\( \frac{1}{4} \) feet wide.

4. A kitchen is 15 ft. long, 11 ft. wide, 7 ft. 6 in. high. The walls and ceiling, excluding 60 sq. ft. of window-space, are painted at a cost of 2s. 3d. per sq. yd. Find the cost.

5. Find the cost of the material for a carpet, 13 ft. 6 in. by 11 ft. 8 in., if the carpet is made up from a roll 28 in. wide at 15s. per yard length.

6. Find the area of a path running all round a lawn, 45 ft. long, 30 ft. wide, if the path is 5 ft. wide along the longer sides and 3 ft. wide along the shorter sides.

7. A closed rectangular tin has a square base and is 18 in. high. A rectangular sheet of paper, 3 ft. long, just covers the sides of the tin as a label. Find in sq. ft. the area of sheet metal used to make the tin.

8. A courtyard, 65 ft. long, 20 ft. wide, is paved with black and white tiles each 5 in. square. There is a band \( 2\frac{1}{2} \) ft. wide of black tiles all round the edge and the rest are white. How many tiles of each colour are used?

9. The length of the diagonal of a square field is a quarter of a mile. Find the area of the field in acres.

10. A metal sheet is a quadrant of a circle of radius 10\( \frac{1}{2} \) in. Find (i) its perimeter, (ii) its area.

11. A semi-circular table-cover is 5 ft. 3 in. in diameter. Find the cost of binding the edge of the cover at 1s. 4d. per yard.

12. A bicycle wheel, diameter 28 in., is making 2 revolutions per second. Find the speed of the bicycle in m.p.h.

13. The driving-wheel of an engine is 5 ft. 10 in. in diameter. How many revolutions does it make per mile?
14. A circular disc, radius 4 in., is cut out of a metal sheet which weighs \( \frac{2}{3} \) oz. per sq. in. of surface area. Find the weight of the disc in lb., correct to \( \frac{1}{10} \) lb.

15. Find in acres the area enclosed by a circular running track whose length is one-quarter of a mile.

16. A gas holder is a vertical circular cylinder with a flat top. It is 18 ft. high and its circumference is 66 ft. Find in sq. yd. the total area of the curved surface and top.

17. A closed cylindrical tin is made of thin sheet metal and the lid overlaps the curved surface to a depth of half an inch. The tin is 6 inches in diameter and 7\( \frac{1}{2} \) inches high. Find to 3 figures the area of sheet metal used to make the tin. [Take \( \pi = 3.142 \).]

18. From a piece of material 5 ft. wide, 10 ft. long, is cut out a circle of diameter 5 ft. and four circles each of diameter 2 ft. 6 in. What area of material in sq. ft., to 3 figures, remains? [Take \( \pi = 3.142 \).]

19. A rectangular lawn is 87\( \frac{1}{2} \) ft. long, 77 ft. wide. Quadrants of radius 10\( \frac{1}{2} \) ft. are cut away at each corner and the remainder is treated with lawn mixture at 2 oz. per sq. yd. costing 2s. 6d. per lb. Find the cost to the nearest shilling.

20. Find the cost of lining the whole of the inside of a closed cylinder, internal diameter 21 ft., internal length 12 ft. 6 in., at 6d. per sq. ft., to the nearest £.

**VOLUME**

Cuboid. If the volume of a rectangular block, \( l \) in. long, \( b \) in. broad, \( h \) in. high, is \( V \) cu. in.,

\[
V = l \times b \times h; \quad h = \frac{V}{l \times b}; \quad l = \frac{V}{h \times b}.
\]

Solid of Uniform Cross-section. If the volume of a right prism, whose cross-sectional area is \( A \) sq. in. and whose height is \( h \) in., is \( V \) cu. in.,

\[
V = A \times h; \quad h = \frac{V}{A}; \quad A = \frac{V}{h}.
\]

Circular Cylinder. If the volume of a circular cylinder, base-radius \( r \) in., height \( h \) in., is \( V \) cu. in.,

\[
V = \pi r^2 h; \quad h = \frac{V}{\pi r^2}; \quad r = \sqrt[3]{\frac{V}{\pi h}}.
\]

**EXERCISE 7**

[Take \( \pi = \frac{22}{7} \) unless otherwise stated; assume 1 cu. ft. = 7\( \frac{1}{10} \) gal.]

1. A rectangular tank is 11 ft. 3 in. long, 4 ft. 8 in. wide, 4 ft. 6 in. deep. Find its capacity in cu. yd.

2. Find the capacity in gallons of a tank 5 ft. 4 in. long, 4 ft. 6 in. wide, 3 ft. 4 in. deep.

3. Find the weight of a beam 18 ft. 8 in. long, 9 in. wide, 1\( \frac{1}{2} \) in. deep, if 1 cu. ft. of the timber weighs 56 lb.

4. A sheet of metal is 9 in. square and weighs 3 lb. Find its average thickness, correct to \( \frac{1}{100} \) in., given that 1 cu. ft. of the metal weighs 488 lb.

5. A solid circular cylinder is 3 ft. long and of radius 1 ft. 4 in. Calculate, to 3 figures, (i) the total surface area in sq. ft., (ii) the volume in cu. ft. [\( \pi = 3.142 \).]

6. A cylindrical tank is 3 ft. 6 in. deep and its base-radius is 1 ft. 8 in., internal measurements. How many gallons, to the nearest gallon, will it hold?

7. The volume of a circular cylinder, radius 3 cm., is 198 c.c.; find its height.

8. The volume of a circular cylinder, height 5-6 in., is 39-6 cu. in.; find its radius.

9. A petrol storage tank is a vertical cylinder of diameter 21 ft., height 12 ft. Find in tons, to 2 figures, the weight of petrol it will hold, assuming that petrol is 0.9 times as heavy as water and that 1 cu. ft. of water weighs 1000 oz.

10. A rectangular swimming-bath, 60 yd. long, 40 ft. wide, is being filled by 4 pipes which together deliver 450 gallons per minute. Find the time taken for the water-level to rise 4 inches.

11. A thin rectangular aluminium sheet is 25 cm. by 15 cm. From each corner a square of side 2-5 cm. is cut away and the remainder is then bent to form an open rectangular pan. Express in litres the volume of milk the pan will hold.

12. A closed box is made of wood \( \frac{1}{2} \) in. thick; the outside measurements are 11 in. by 10 in. by 7 in. Find the volume of wood used for the box.
ARITHMETIC

13. Find, to the nearest \( \frac{1}{10} \) cu. in., the volume of metal in a tube 1 ft. long, open at both ends, whose bore (i.e. internal diameter) is 3·2 in., the thickness of the metal being 0·3 in. [Take \( \pi = 3·142 \).]

[14] Rainwater from a flat roof 16 ft. long, 15 ft. wide, is collected in a cylindrical tank, axis vertical, internal diameter 5 ft. Find to \( \frac{1}{10} \) in. the rise in level of the water in the tank due to a rainfall of \( \frac{1}{2} \) in.

15. A firm used to deliver coffee in closed rectangular tin containers 10 in. high, with base 7 in. by 5\( \frac{1}{2} \) in. It is now packed in closed cylindrical tin containers 10 in. high, containing the same amount as before. Find the diameter of the new container and the percentage saving of tin-sheeting to 2 figures.

16. A shallow circular metal pan has internal diameter 18 cm. and external diameter 20 cm.; the internal depth is 1·5 cm. and the base is 0·5 cm. thick. The metal weighs 8 gm. per c.c.; find the weight of the pan in kg. to 3 figures.

17. The internal and external diameters of a cylindrical iron pipe 12 ft. long are 1 ft. and 1 ft. 2 in. and 1 cu. ft. of iron weighs 480 lb. Find the weight of the pipe in lb. to 3 figures. [Take \( \pi = 3·142 \).] If the internal diameter is increased by half an inch and the external diameter is unchanged, find the ratio of the new weight to the former weight.

18. The vertical cross-section of a feeding trough, closed at each end, is an isosceles triangle, base 2 ft., height 1 ft. 4 in., and the trough is 8 ft. long, internal measurements. Find (i) the cubic content of the trough, (ii) the area of its internal surface, including the triangular ends.

19. The base of a lean-to shed is a rectangle 15 ft. by 12 ft. 6 in.; the shed is 7 ft. 3 in. high in front and 8 ft. 9 in. high at the back; all measurements are internal. Find the capacity of the shed in cu. ft.

20. A swimming bath, 80 ft. long, 35 ft. wide, is 3 ft. deep at one end and 8 ft. deep at the other and the floor slopes uniformly. The bath is filled at the rate of 7500 gallons per hour by means of a pipe of radius 2·1 inches. Find (i) the number of gallons the bath holds, (ii) the time taken to fill the bath, (iii) the speed in ft. per sec., to \( \frac{1}{10} \) ft. per sec., of the water in the pipe, assuming the pipe remains full.

FRACTIONS AND DECIMALS

[21] A hay-rick is in the form of a prism whose long edges are horizontal and whose section perpendicular to these edges is a rectangle surmounted by a triangle. The rick is 28 ft. long, 16 ft. wide; the height to the eaves is 12 ft. and to the top is 18 ft. Given that 1 cu. ft. of hay weighs \( \frac{7}{2} \) lb., and that the hay is sold at \( £15 \) per ton, find the value of the rick.

22. A container is formed by a circular cylinder closed at its base surmounted by a circular cone. The diameter of the base is 8 ft. and the heights of the cylinder and cone are each 3 ft. Calculate to 3 figures (i) the volume, (ii) the total surface area of the container. [If the base-radius of a cone is \( r \), and height is \( h \) and slant height is \( l \), the volume is \( \frac{1}{3} \pi r^2 h \) and the area of the curved surface is \( \pi rl \).]

23. A hollow right circular cone, fitted with a base, is 1 ft. high and the base-radius is 1 ft. 4 in., internal measurements. It stands with its base on a horizontal table and contains water to a depth of 9 inches. Calculate (i) the radius of the water-surface, (ii) the volume in cu. in. of water to 3 figures, (iii) the area of the total internal surface of the cone to 3 figures. [Use the formulae in No. 22 and take \( \pi = 3·142 \).]

24. A cistern is 16 in. long, 7\( \frac{1}{2} \) in. wide. A ball-cock is a sphere of diameter 6 in. and is half under water. By how much does the water-level fall when the ball is lifted out of the water? If the depth of water in the cistern is 9·3 in. when the ball is out of the water, find the depth when the ball is pushed completely under the water. Give answers correct to \( \frac{1}{10} \) inch. [Volume of sphere, radius \( r \), is \( \frac{4}{3} \pi r^3 \).]

FRACTIONS AND DECIMALS

The following exercise is included in case the previous work has shown that some additional practice in working with fractions and decimals is required.

EXERCISE 8

Simplify the following:

1. \( 4\frac{1}{2} - 2\frac{1}{4} - \frac{5}{8} \)
2. \( 1\frac{7}{12} + 2\frac{1}{3} - 4\frac{3}{10} \)
3. \( 2\frac{1}{4} - 1\frac{3}{4} + 1\frac{1}{2} \)
4. \( \frac{3}{4} \times \frac{2}{5} \times (1\frac{3}{4})^3 \)
5. \( (1\frac{1}{6} + 1\frac{1}{4}) \times 1\frac{1}{4} \)
6. \( (3\frac{5}{8})^2 - (2\frac{3}{8})^2 \)
7. \( 3\frac{3}{4} \times 2\frac{1}{4} - 1\frac{1}{4} + \frac{1}{4} \)
8. \( \frac{3}{8} \) of \( (2\frac{1}{2} + 2\frac{1}{4}) + \frac{5}{8} \)
ARITHMETIC

9. \[ \frac{2}{3} + \frac{2}{3} = \frac{4}{3} \]
10. \[ 1 + \frac{1}{3} = \frac{4}{3} \]
11. \[ \frac{3\frac{1}{2}}{3\frac{1}{2}} + \frac{1}{3} \]
12. \[ 10.24 \times 0.0625 = 0.625 \]
13. \[ 0.07344 \times 0.34 = 0.025 \]
14. \[ 3.142(2.7^2 - 2.3^2) \]
15. \[ 7.005 \times 0.408 = 2.874 \]
16. \[ 1.725 \times 0.046 = 0.079 \]
17. \[ 10.25 \times 1.7 = 17.425 \]
18. \[ 7.5 \times 17.5 = 131.25 \]
19. \[ 4.5^2 \times 2^4 = 48 \]
20. \[ 20 \times (0.02)^2 \]

21. Express 4·0051 correct to (i) 2 places of decimals, (ii) 2 significant figures.

22. Express \[ \frac{1}{10} \] as a decimal correct to (i) 3 places of decimals, (ii) 3 significant figures.

23. Express £2·6094 in £ s. d. to the nearest farthing.

24. Express 8s. 5d. as a decimal of £1 correct to 4 figures.

25. A boy weighs 58·32 kg. Find his weight in stones, lbs. correct to \( \frac{1}{2} \) lb., taking 1 kg. = 2·205 lb.

26. Find to the nearest penny the value of \( 0·8072 \times 190 \).

LOGARITHMS

Example 1. Evaluate, to 3 figures, \( \sqrt[3]{0.782^2} \).

<table>
<thead>
<tr>
<th>Number</th>
<th>Logarithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.782</td>
<td>0.8932</td>
</tr>
<tr>
<td>0.782</td>
<td>1.7864</td>
</tr>
</tbody>
</table>

\[ \therefore \text{expression} = 0.8488 = 0.849 \text{ to 3 figures.} \]

Example 2. Evaluate, to 3 figures, \( \sqrt[3]{0.782^2} \).

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<td>1.7864</td>
</tr>
</tbody>
</table>

\[ \therefore \text{expression} = 0.8488 = 0.849 \text{ to 3 figures.} \]

EXERCISE 9

Find by logarithms the values to 3 figures of the following:

1. \[ 8·042 \times 39·07 \]  2. \[ 16·41 \times 0·7682 \]  3. \[ 0·0637 \times 0·1045 \]

4. \[ 5126 \div 76·48 \]  5. \[ 0·01 \div 87·3 \]  6. \[ 0·5629 \div 0·0074 \]

7. \[ 3784^2 \]  8. \[ 0·0008^2 \]  9. \[ 0·7251^3 \]  10. \[ 0·586^4 \]

11. \[ \sqrt[3]{281·7} \]  12. \[ \sqrt[3]{17706} \]  13. \[ \sqrt[3]{816·2} \]  14. \[ \sqrt[3]{0·6092} \]

15. \[ 27·5 \times 0·316 \]  16. \[ 0·00692 \]  17. \[ 18·53 \times 0·384 \]

18. \[ 87·16^2 \times 0·0493 \]  19. \[ 0·637^4 + \frac{4}{3} \]  20. \[ 10^3 \cdot 7856 \]

21. \[ 3^4 \]  22. \[ (0·4)^3 \]  23. \[ (0·28)^3 \]

24. \[ \sqrt[3]{16·3 \times 0·0719} \]  25. \[ \sqrt[3]{3·142} \]

26. \[ \frac{\sqrt{28·65^2 - 21·45^2}}{\sqrt{47·1 \times 0·828}} \]  27. \[ \sqrt[3]{0·3 \times 1·142(0·767^2 - 0·481^2)} \]

28. Find the value of \( xy^2 \) if \( x = 1·65 \times 10^4 \), \( y = 3·8 \times 10^{-3} \)

29. Find the value of \( \sqrt[3]{\frac{1}{c}} \) if \( a = 71·8 \), \( b = 0·723 \), \( c = 18·57 \)

30. Find the value of \( \sqrt[3]{\sqrt{\frac{3V}{4\pi}}} \) if \( V = 57·5 \), \( \pi = 3·142 \)

31. If \( V = \frac{1}{4}\pi a^2 \), find the value of \( d \) when \( V = 3500 \), \( \pi = 3·142 \)

32. Express \( \frac{650d^2}{m} \) in the form \( a \times 10^b \) where \( a \) is between 1 and 10 and \( b \) is an integer, when \( d = 8\times14 \times 10^{-2} \), \( m = 7·64 \times 10^4 \).
USE OF LOGARITHMS

Compound Interest Formula

The interest on £P for 1 year at r\% p.a. is £ \frac{Pr}{100}.

\[ \therefore \text{£P amounts to} \left( \text{£P} + \frac{Pr}{100} \right), \text{that is,} \left( \frac{1 + \frac{r}{100}}{1} \right)^1 \text{after 1 year;} \]

\[ \therefore \text{at compound interest} r\% \text{p.a., payable yearly,} \text{£P amounts to} \left( \frac{1 + \frac{r}{100}}{1} \right)^2 \text{after 2 years, and to} \left( \frac{1 + \frac{r}{100}}{1} \right)^3 \text{after 3 years, and so on.} \]

\[ \therefore \text{the amount of £P for} \ n \ \text{years at} \ r\% \ \text{p.a. compound interest,} \text{payable yearly, is} \text{£P} \left( 1 + \frac{r}{100} \right)^n. \]

**Example 1.** Find by logarithms the approximate amount at compound interest of £316 for 12 years at 3\%{\text{\%}} p.a., payable yearly.

\[
\text{Amount} = £316 \left( 1 + \frac{3{\text{\%}}}{100} \right)^{12} = £316 \left( 1.03 \right)^{12}
\]

\[
\begin{array}{c|c|c}
\text{Amount} & 1.035 & 0.0149 \\
\hline
1.035^{12} & 1.0788 & \\
316 & 2.6785 & \\
\hline
\end{array}
\]

Since the logarithm of 1.035 has been multiplied by 12, the logarithm of the result is not reliable to 3 figures.

From 5-figure tables, log 1.035 = 0.01494,

\[ \therefore \text{log} \ 1.035^{12} = 0.1793, \text{to 4 figures,} \]

\[ \text{hence amount} = £477.0. \]

For a higher degree of accuracy, either 7-figure tables must be used or the amounts at end of each successive year must be calculated, as in the following example.

**Example 2.** Find to the nearest penny the amount at compound interest of £316 for 3 years at 3\%{\text{\%}} p.a., payable yearly, and find the compound interest for this period.

<table>
<thead>
<tr>
<th>Year</th>
<th>Amount</th>
<th>Int.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>316.00</td>
<td>9.48</td>
</tr>
<tr>
<td>2nd</td>
<td>327.06</td>
<td>1.58</td>
</tr>
<tr>
<td>3rd</td>
<td>338.57</td>
<td>1.63</td>
</tr>
</tbody>
</table>

\[ \text{Amount after 3 years} = £350.355 \text{to 3 figures} \]

\[ = £350 \ 7s. \ 1d. \text{to nearest penny.} \]

Hence also, compound interest = £350 7s. 1d. - £316 = £34 7s. 1d. to nearest penny.

**EXERCISE 10**

[Give answers to 3 figures. \( \pi = 3.142 \)]

1. A French car runs 100 km. on 13.7 litres of petrol. Find how many miles it runs on a gallon of petrol.

[Take 1 mile = 1.609 km., 1 gallon = 4.546 litres.]

2. A cylindrical vessel is 19.5 cm. in diameter and holds 4.5 litres. Find its depth in cm. [1 litre = 1000 c.c.]

3. The population of Switzerland is 4,250,000 and the area is 15,950 sq. km. Find the average population per sq. km. [Take 1 mile = 1.609 km.]

4. 1 c.c. of copper weighs 8.85 gm. Find the weight in lb. of 1 cu. ft. of copper. [Take 1 in. = 2.540 cm., 1 lb. = 453.6 gm.]
6. A square lead sheet, 4.5 mm. thick, weighs 1.565 kg. Find the length of the sheet given that 1 c.c. of lead weighs 11.3 gm.

7. A swimming bath 84 ft. long, 56 ft. wide, is 3 ft. deep at one end, 9 ft. deep at the other, and the floor slopes uniformly. The bath is filled by water flowing through a pipe of diameter 3.5 in. at a speed of $2\frac{3}{8}$ m.p.h. If at the start the bath is empty, find in hours the water taken (i) for the floor to be submerged, (ii) for the bath to be filled.

8. A solid metal sphere weighs 20 lb.; the metal weighs 550 lb. per cu. ft. Find the diameter of the sphere in inches. [Volume of sphere, radius $r$, is $\frac{4}{3}\pi r^3$.]

9. The external diameter of a spherical copper shell is 1 ft., and the thickness of the shell is $\frac{1}{2}$ inch. Find the weight in lb. of the shell, given that 1 cu. ft. of copper weighs $(8.85 \times 62.5)$ lb. [Volume of sphere, radius $r$, is $\frac{4}{3}\pi r^3$.]

10. Sugar is imported in cylindrical drums, base-diameter 1.55 metres, height 1.85 metres, and is repacked in rectangular cartons measuring $2\frac{1}{2}$ in. by $3\frac{1}{4}$ in. by 7.5 in. How many cartons can be completely filled from one drum? [1 in. $= 2.54$ cm.]

11. The bowl of a wine-glass is an inverted cone, height 2.18 in., base-diameter 1.95 in. How many full glasses can be obtained from 3 dozen bottles each containing $1\frac{1}{2}$ pints? [1 gallon $= 0.1605$ cu. ft.; volume of cone, height $h$, base-radius $r$, is $\frac{1}{3}\pi r^2 h$.]

12. A thread of mercury 16.8 cm. long is run out of a cylindrical tube and found to weigh 0.116 gm. Find the diameter of the tube, given that 1 c.c. of mercury weighs 13.6 gm.

13. Find to the nearest £ the amount of £268 for 10 years at 4% p.a. compound interest, payable yearly. [Take log 1.04 = 0.017035.]

14. Find to the nearest £ the amount of £426 for 15 years at 2% p.a. compound interest, payable yearly. [Take log 1.025 = 0.010724.]

15. Find to the nearest penny the amount of £580 for 3 years at 3% p.a. compound interest, payable yearly.

16. Find to the nearest penny the amount of £620 for 3 years at 4% p.a. compound interest, payable yearly and the compound interest.

SHARES AND STOCKS

Shares. The statement that a dividend of 8% is paid on the (2s.) shares of a company, standing at 3s., means:

The dividend on one (2s.) share is $\frac{8}{100}$ of 2s., and the cost of one (2s.) share is 3s.

Stocks. The statement that a 4% stock stands at 92\frac{1}{2} means:

The dividend on £100 stock is £4, and the cost of £100 stock is £92\frac{1}{2} cash.

Example 1. By investing £150 in (10s.) shares paying 8%, a man obtained a dividend of £10. At what price did the shares stand?

The dividend from 1 (10s.) share is $\frac{8}{100}$ of 10s., $= \frac{4}{5}$ of 10s.,

1. a dividend of $(10 \times 20)$ is obtained from $(10 \times 20 \div \frac{4}{5})$ shares,

2. 1 share costs $\frac{150 \div 4}{150 \times 4} = \frac{10 \times 20 \times 5}{4}$

3. the shares stand at 12s.

Example 2. Find the yield derived from investment in (2s.) shares at 3s. 6d., paying 10\frac{1}{2}%.

1 (2s.) share costs $3\frac{3}{4}$ s. and the dividend on 1 share is $\frac{10\frac{1}{2}}{100}$ of 2s.

1. an investment of $\frac{7}{8}$ s. cash yields $\frac{2}{100}$ s. interest.

2. an investment of 100s. cash yields $\frac{2}{100} \times \frac{7}{8} \times \frac{1000}{12}$. interest, that is, 6s. interest.

3. the yield from the investment is 6%.
Example 3. A man sold £800 of 24\% stock at 65 and invested the proceeds in 7\% stock at 120. Find the net change of income, allowing for deduction of tax at 9s. 6d. in the £.

First income. £100 stock yields £2 1\frac{1}{2} gross income.
\[ \therefore £800 \text{ stock yields } £2 \frac{1}{2} \times 8, = £20, \text{ gross income.} \]

Selling out. He sells £100 stock for £65 cash,
\[ \therefore \text{he sells } £800 \text{ stock for } (£65 \times 8) \text{ cash,} \]
\[ \therefore \text{he invests } (£65 \times 8) \text{ cash in } 7\% \text{ stock at } 120. \]

Second income. £120 cash invested yields £7 gross income,
\[ \therefore (£65 \times 8) \text{ cash invested yields } (£7 \times 65 \times 8 \times \frac{1}{2}e), \text{ gross income.} \]
That is, £3 \frac{1}{3}, = £30 \frac{1}{3}, \text{ gross income.}

Change of income. The increase of gross income = £30 \frac{1}{3} - £20, = £10 \frac{1}{3}.
From a gross income of 20s., 9\frac{1}{2}s. tax is deducted
and so the net income is (20 - 9\frac{1}{2})8s., = 10\frac{1}{2}s.
\[ \therefore \text{net income} = \frac{10\frac{1}{2}}{20} \text{ of gross income.} \]
\[ \therefore \text{increase of net income} = \frac{10\frac{1}{2}}{20} \text{ of } £10 \frac{1}{3} \]
\[ = (£10 \frac{1}{3} \times \frac{3}{4}), = £2 \frac{1}{4}. \]
\[ = £2 \frac{1}{4} = £5 \text{ 8s. 6d.} \]

EXERCISE 11

1. Find the cost of, and the income derived from, £750 of 5\% stock, standing at 110.

2. Find the cost of, and the income derived from, 240 (10\%) shares paying 7\%, standing at 12s. 4d.

3. How much stock can be bought by investing £350 in a 5\% stock at 112\%? Find the net income derived from the investment if tax is deducted at 9s. in the £.

4. Find the number of shares obtained by investing £1650 in £1 shares at 2\frac{1}{2}, paying 10\%. Find the net income derived from the investment if tax is deducted at 9s. 6d. in the £.

5. A man sold 1760 (5\%) shares for £748. At what price did the shares stand?

6. Find the yield obtained by investing in £1 shares, standing at 3\frac{1}{2} and paying a dividend of 20\%.

SHARES AND STOCKS

7. Find the yield obtained by buying 10\% shares at 32s. 6d., paying a dividend of 15\%.

8. Find in the form £ s. d. per cent., to the nearest penny, the yield from 2\frac{3}{4} stock at 82\frac{3}{4}.

9. Find the price of a 3\frac{1}{4}\% stock if it yields 4\frac{1}{2}\% on an investment.

10. A man invested £1200 in 3\frac{3}{4}\% stock at 96 and sold out at 102. Find his profit.

11. A man invests £680. Find the difference of income according as he invests in 5\% shares at 21s. 3d. paying a dividend of 7\frac{1}{4}d. a share or in a 3\frac{3}{4}\% stock at 102. Find the price of a 4\% stock which gives him the same income as the 5\% shares.

12. What is the price of a 4\frac{1}{4}\% stock if it gives the same yield as a 5\frac{1}{4}\% stock at 112?

13. A man invests £462 in 5\% shares at 16s. 6d. After receiving a dividend of 10d. a share he sells out at 17s. 2d. Find his total profit including the dividend.

14. Which investment gives the higher yield, a 3\frac{3}{4}\% stock at 85 or a 5\% stock at 135?

15. A man invests £850 10\%, in a 5\% stock at 126. After receiving a year’s dividend less tax at 8s. in the £, he sells out at 122. Find his loss on the whole transaction.

16. A man’s holding of a 5\frac{1}{2}\% stock gives him an income of £315. How much stock does he hold? He sells his holding at 112 and invests the proceeds in a 4\frac{1}{2}\% stock at 96. Find the change in income.

17. A man sells £650 of a 4\% stock at 96 and invests the proceeds in a 5\frac{1}{2}\% stock at 104. Find the net change in income after paying tax at 9s. in the £ in each case.

18. A man bought 300 (5\%) shares at 7s. 4d. He sold 100 of the shares when they had risen to 9s. At what price does he sell the rest of the shares if he neither gains nor loses on the whole transaction?

19. A man sells out enough of his holding of a 4\% stock at 97\frac{1}{4} in order to obtain £3900 to pay for a house. How much stock does he sell and what is the net loss of income if tax is deducted at the rate of 9s. in the £?
20. By selling some 24% stock at 77 and investing the proceeds in 6% stock at 110, a man increases his income by £78 a year. How much stock did he sell?

21. A man receives an income of £132 from his holding of a 24% stock. He sells out at 85 and invests the proceeds in a 34% stock, thereby increasing his income by £8. How much does he pay for each £100 of the 34% stock?

MISCELLANEOUS ARITHMETIC

EXERCISE 12

1. An alloy is formed of copper and zinc in the ratio 13 : 3 by weight. Find (i) the weight of zinc in 1 cwt. 3 qr. of the alloy; (ii) the weight of a lump of the alloy which contains 35 lb. more copper than zinc.

2. A man proposed to stay 40 days in Portugal on £40. On arrival, he changed £25 into escudos at 100 to the £ and spent 120 escudos a day for the first 18 days. When he changed the rest of his money, the rate of exchange had fallen to 80 escudos to the £. How much a day could he spend for the remainder of his visit?

3. The population of a town is expected to increase in each 10-yearly period by 12% of its population at the beginning of that period. If the population is 165,000 in 1951, find to the nearest hundred the expected population in 1981.

4. English manufactured goods are delivered in America at £12 15s. per ton together with an import duty of 16·8 dollars per ton. If manufactured in America, the cost is 52·5 dollars per short ton of 2000 lb. Taking £1 = 2·80 dollars, find in dollars the difference between the cost of 1 ton of goods when imported from England and their cost when manufactured in America. Also express this difference in £ s. d.

5. A building is shared by two typewriting agencies A, B; A occupies 14 rooms, B occupies 6 rooms, all alike. The fuel required for heating the rooms for 20 weeks costs £150. After 12 weeks of this period, B gives up 2 rooms to A. How much of the fuel bill should A pay?

6. A man borrows £1000 at 4% per annum compound interest. He repays £300 at the end of each year. How much to the nearest penny does he owe at the beginning of the fourth year?

7. The value of a car depreciates each year by 20% of its value at the beginning of the year. By what percentage has its value decreased at the end of 4 years?

8. A man is cycling at 9 m.p.h. along a straight road on a misty day when visibility is down to 110 yards. A car passes him and disappears in the mist 15 seconds later. At what rate is the car travelling?

9. Gas costs 1s. 4d. per therm, 1 therm being equivalent to 200 cu. ft. In one quarter, I use 8400 cu. ft. of gas for cooking, 160 units of electricity for light at 7½d. per unit and 48 units of electricity for power at 2½d. per unit. In the next I use electricity for all purposes, 560 units for cooking, 184 units for light, 136 units for power and pay 1½d. per unit over all, together with a fixed charge of £2 15s. Find the difference between the quarterly bills.

10. The gas meter readings in hundreds of cu. ft. at Christmas and Lady Day were 4758 and 4843, and the electricity meter readings in units at the same times were as follows: lighting, 36486 and 36612; power, 07813 and 07967. The gas costs 1s. 8d. per therm and 1000 cu. ft. of gas = 4 therms; the electricity costs 8d. per unit for light and 1½d. per unit for power. Find the total charges for gas and electricity for this quarter.

11. The pre-war cost of manufacture of an article was £2 12s. 6d., made up of labour, materials, overheads in the ratios 8 : 10 : 3 respectively. These costs have now increased by 75%, 80%, 50% respectively. What is the present cost of manufacture? Find to 3 figures the percentage increase in cost.

12. An alloy is formed of tin and copper in the ratio 7 : 5 by weight, and a second alloy of tin and copper in the ratio 3 : 1 by weight. Find the ratio of tin and copper in a third alloy formed from equal weights of the two first alloys.

13. A tramp-steamer was chartered by three firms A, B, C to carry wheat. 4 of the cargo belonged to A, 3 of it to B and the rest to C. The boat was driven ashore and 3 of A's wheat, 2 of B's wheat and all of C's wheat was lost. The underwriters paid £24,000 for the damage to the cargo. How much does each firm receive?

14. A and B embroider handkerchiefs; A does 22 in the time B takes to do 15. A works 8 hours a day for 5 days a week and B works 6 hours a day for 5½ days a week; and 4s. is deducted weekly for insurance. Working on piece rates, A receives £4 12s. for a week's work: how much does B receive?
15. An alloy is composed of metals A, B, C in the ratios 2 : 5 : 9 by weight, respectively. The weights of 1 cu. ft. of A, B, C are 220 lb., 480 lb., 560 lb. respectively. Find (i) the volume of 160 lb. weight of the alloy to 3 figures, (ii) the weight of 1 cu. ft. of the alloy to 2 figures.

16. I invested £300 in Savings Certificates at 15s. each ten years ago; they are now worth 20s. 6d. each and in 1 year’s time will be worth 21s. 13d. each and are free of tax. If I cash the certificates now and lend the money on mortgage at 5 per cent. per annum interest, 9s. in the £ is deducted for tax from the interest. Find the net total value in 1 year’s time. Find also the total value of the certificates in 1 year’s time if I do not cash them now.

[17] A and B buy a mushroom farm; A invests £7500 and B invests £10,000. After 6 months C joins the partnership and invests £2500. The profits for the first year are £2800. From this, A and B draw salaries of £250, £350 respectively and 20% of the remainder is put to reserve. The balance is shared between A, B, C in proportion to the capital invested and the time it was in use. Find the total amounts received by A, B, C.

18. The horizontal base of a circular cylinder is 6 inches in diameter and contains water to a depth of 6 inches. When a metal sphere is totally submerged in it, the water-level rises 2 inches. Find to 3 figures the radius of the sphere. [Volume of sphere, radius $r$, is $\frac{4}{3}\pi r^3$.] What depth of water would there have been originally in the cylinder if this sphere is just covered by the water when placed in the cylinder?

19. The rateable value of a borough is £405,000 and the contribution of the rates towards the primary education of 4100 children in the borough is £29,700. Find (i) the necessary education rate in pence to the £, (ii) the contribution paid by a man who lives in a house rated at £45.

If the man has three children at school, what percentage of the cost to the borough of their education is met by the education rate he pays?

20. A rectangular tank 12 feet deep can be filled by water from a tap in $2\frac{1}{2}$ hours, and water is drawn off through another tap at the rate of 8 gallons per minute. If both taps are open, the depth of water rises from 5 ft. to 8 ft. in 1 hour 40 minutes. Find how many gallons per minute are supplied by the first tap.
21. Multiply \( a^3 + a - \frac{1}{a} \) by \( a - \frac{1}{a} \).

22. (i) Find the square of \( x + \frac{1}{x} \).

(ii) If \( y + \frac{1}{y} = 3 \), find the value of \( y^2 + \frac{1}{y^2} \).

23. If a man's height is \( x \) inches, his weight should be \( 5\frac{1}{4}(x - 60) + 110 \) pounds. How much ought a man, 5 ft. 10 in. tall, to weigh?

24. Simplify \((x - 1)(x + 2)(x - 3) - (x + 1)(x - 2)(x + 3)\).

**SIMPLE EQUATIONS AND PROBLEMS**

Example. Solve \( 2 - \frac{2x - 1}{5} = x - \frac{1}{2}(1 + x) \).

Multiply each side by 10,

\[
20 - \frac{10(2x - 1)}{5} = 10x - \frac{10}{2}(1 + x),
\]

\[
\therefore\ 20 - 2(2x - 1) = 10x - 5(1 + x),
\]

\[
\therefore\ 20 - (4x - 2) = 10x - 5 + 5x,
\]

\[
\therefore\ 20 - 4x + 2 = 10x - 5 + 5x,
\]

\[
\therefore\ 22 - 4x = 5x - 5
\]

\[
\therefore\ 5x - 5 = 22 - 4x
\]

\[
\therefore\ 5x + 4x = 22 + 5
\]

\[
\therefore\ 9x = 27
\]

\[
\therefore\ x = 3.
\]

**Check:**

When \( x = 3 \),

left side \( = 2 - \frac{6 - 1}{5} = 2 - \frac{5}{5} = 2 - 1 = 1 \),

right side \( = 3 - \frac{1}{2}(1 + 3) = 3 - \frac{1}{2}(4) = 3 - 2 = 1 \),

\( \therefore \) when \( x = 3 \), left side \( = \) right side.

**EXERCISE 14**

Solve the following equations:

1. \( 2(x - 3) - 4(2x + 1) = 3x \).

2. \( y - 5(100 - \frac{1}{2}y) = 130 \).

3. \( 2 \cdot 6(l - 2) = 3(3 \cdot 4 - l) \).

4. \( (h + 3)^2 + (h - 2)^2 = 2h^2 \).
24. If I walk from my house to the post-office at 3\(\frac{1}{2}\) miles an hour, I take 32 minutes longer than if I bicycle at 10\(\frac{1}{2}\) miles an hour. How far is it to the post office?

[25] Two numbers are in the ratio 3 : 5. When 49 is added to each number, the ratio is changed to 5 : 6. Find the numbers.

26. A man starts at 2.20 p.m. and walks at uniform speed along a road until he is picked up by a lorry travelling at 12 miles an hour. He notices that he has gone 2\(\frac{1}{2}\) miles at 3 p.m. and 9 miles at 3:49 p.m. At what time was he picked up by the lorry?

27. There is a misprint in the coefficient of \(x\) in one of the equations, \(x + 3y + 2z = 5\), \(3x - y - 4z = 5\), \(5x + 4y - z = 2\). The correct answer is \(x = 4, y = -1, z = 2\). What is the misprint?

SIMULTANEOUS EQUATIONS AND PROBLEMS

Example. The resistance \(R\) lb. to a train of weight 100 tons running at \(v\) miles an hour is given by the formula, \(R = a + bv^2\), where \(a\) and \(b\) are independent of the speed. At 20 m.p.h., the resistance is 960 lb.; at 50 m.p.h., it is 2850 lb. Find the values of \(a\), \(b\), and the resistance at 30 m.p.h.

\[R = a + bv^2.\]

When \(v = 20, R = 960,\)
\[960 = a + 20^2 = a + 400b\]  
\[\therefore 960 = a + 400b \quad \text{(i)}\]

When \(v = 50, R = 2850,\)
\[2850 = a + 50^2 = a + 2500b\]  
\[\therefore 2850 = a + 2500b \quad \text{(ii)}\]

From (ii) and (i), by subtraction,
\[2500b = 2850 - 960\]
\[\therefore 2500b = 1890\]
\[\therefore b = \frac{1890}{2500} = \frac{189}{250}\]

Substituting in (i) for \(b, a + 400 \times \frac{189}{250} = 960,\)
\[\therefore a = 960 - 360 = 600.\]

Hence \(R = 600 + 30^2 = 1410\) lb.

EXERCISE 15

Solve the following equations:

1. \[a + 3b = 5\]
   \[3a + b = -1\]

2. \[2x + y = 6\]
   \[4x - 3y = 17\]

3. \[p = 2g - 1\]
   \[q = 5 - 3p\]

4. \[\frac{y}{2} - 2 = \frac{z}{3}\]
   \[\frac{x}{3} - \frac{y}{12} = 1\]

5. \[\frac{u - 1}{4} = \frac{v + 3}{5}\]
   \[x + y = 42\]

6. \[\frac{r}{s - \frac{1}{2}} = 1\]
   \[5(s - 2) - 2(r - 1) = 0\]

7. \[0.3p - 0.4c = 0.45\]
   \[2c - m = 1\]

8. \[\frac{3p - 1}{2} = \frac{3p - 9}{2} = 2q - 7\]

9. \[\frac{3p - 1}{2} = 2p - 3q = 9 = 2q - 7\]

10. \[2x + y - z = 8, \ x - 2y + z = 5, \ x + 4y + 3z = 11\]

11. \[2a - b + 12 = 0, \ b + 3c - 20 = 0, \ a + 2b + 5c = 29\]

12. If \(x + 3y = 5a\) and \(2x + y = 0\), find \(x\) and \(y\) in terms of \(a\).

13. If \(px - qy = q^2\) and \(x + y = p\), find \(x\) and \(y\) in terms of \(p, q\).

14. If \(m_1g - T = m_1f\) and \(T - m_2g = m_2f\), find \(T\) and \(f\) in terms of \(m_1, m_2, g\).

15. If \(y = mx + c\) where \(m, c\) are constants, and if \(y = -10\) when \(x = 2\), and \(y = 15\) when \(x = -8\), find the values of \(m\) and \(c\). Also find, for what value of \(x\), the values of \(x\) and \(y\) are equal. For what value of \(x\) is \(y = 0\)?

16. If \(y = ax^2 + bx\) where \(a, b\) are constants, and if \(y = 8\) when \(x = 1\), and if \(y = 2\) when \(x = 2\), find the values of \(a\) and \(b\). For what values of \(x\) is \(y = 0\)?

17. The equation \(x^2 + y^2 + 2fx + 2gy = 12\), where \(f\) and \(g\) are constants, is satisfied by \(x = 6, y = 6\) and by \(x = -1, y = 7\). Find the values of \(f\) and \(g\).

Taking these values of \(f\) and \(g\), show that the equation can be expressed in the form \((x-h)^2 + (y-k)^2 = r^2\), and state the values of \(h, k, r\).

18. One year ago, a man was 4 times as old as his son then was, and in 3 years' time he will be 3 times as old as his son will then be. Find their present ages.
[19] A contractor starts by hiring 15 men and 20 boys at a cost of £5 10s. per hour; when he takes on another 18 men and discharges 8 of the boys, the cost rises by £3 an hour. How much does he pay to each man and to each boy per hour?

[20] The lengths of the sides of a triangle are \(x + 2y\), \(2(x - \frac{1}{2}y)\), \(7y - x + 1\), inches. If the triangle is equilateral, find the values of \(x, y\) and the perimeter of the triangle.

21. My quarterly bill for electricity is made up of a charge of \(p\) pence per unit and a fixed charge of \(r\) shillings. When I use 160 units, the bill is £2 15s., and when I use 400 units the bill is £5. Find the values of \(p\) and \(r\). How many units do I use if my bill works out at an average of 34d. per unit?

[22] The annual rates of subscription of a golf club have been raised from 4 guineas to £6 for men and from £3 to £4 10s. for women. The total receipts from subscriptions have risen from £870 to £1110 although now 10 fewer men and 20 fewer women belong to the club. How many men and how many women now belong?

23. The speedometer on my car is faulty. When the actual speed is \(v\) miles per hour, the speedometer reading is \((a + bv)\) miles per hour where \(a, b\) are constants. When the actual speeds are 20 m.p.h. and 50 m.p.h., the readings are 24 m.p.h. and 48 m.p.h. respectively. Find the values of \(a, b\). At what speed is the car travelling if the reading is correct?

24. If a train is running at \(v\) miles an hour, it can be stopped in a distance \(s\) yards, where \(s = av^2 + bv\) and \(a, b\) are constants. At speeds of 30 m.p.h. and 60 m.p.h. the distances are 105 yards and 390 yards respectively. Find the distance when the speed is 45 m.p.h.

[25] The stretched length of a spiral spring supporting a body weight of \(w\) grams is \((a + bw)\) cm, where \(a, b\) are constants. For loads of 100 grams and 150 grams, the stretched lengths are 38.9 cm. and 42.1 cm. respectively. Find the values of \(a, b\) and the stretched length for a load of 175 grams. What is the unstretched length of the spring?

[26] Find the numbers \(a, b\) if \(2(x - 1)^2 + a(x - 1) + b\) is equal to \(2x^2 - 7x\) for all values of \(x\).

[27] Find the values of \(a, b, c\) if \(a(x + 1)(x + 2) + b(x + 1) + c\) is equal to \(2x^2 - x - 4\) for all values of \(x\).

**FACTORS AND REMAINDER THEOREM**

**Factors.** First see if there is a common factor of each term. If so, write it down first and find the other factor by short division.

**Example 1.** Factorise \(3(2x - 1)^2 - 9(2x - 1)(x + 2)\).

\(3(2x - 1)\) is a common factor of each term. Divide each term by \(3(2x - 1)\),

\[
3(2x - 1)^2 - 9(2x - 1)(x + 2) = 3(2x - 1)[(2x - 1) - 3(x + 2)]
\]

\[
= 3(2x - 1)(2x - 1 - 3x - 6)
\]

\[
= 3(2x - 1)(-x - 7)
\]

\[
= -3(2x - 1)(x + 7).
\]

**Squares.**

\[
a^2 + 2ab + b^2 = (a + b)^2,
\]

\[
a^2 - 2ab + b^2 = (a - b)^2,
\]

\[
a^2 - b^2 = (a + b)(a - b).
\]

**Example 2.** What number must be added to \(x^2 - 5x\) to make the sum a perfect square?

\[
(x - \frac{5}{2})^2 = x^2 - 2(\frac{5}{2})x + \left(\frac{5}{2}\right)^2
\]

\[
- x^2 - 5x + \frac{25}{4};
\]

and so, if \(\frac{25}{4}\) is added to \(x^2 - 5x\), the sum is \((x - \frac{5}{2})^2\).

**Example 3.** Factorise \(9(2x - y)^2 - (x + y)^2\).

\[
9(2x - y)^2 - (x + y)^2 = [3(2x - y)]^2 - (x + y)^2
\]

\[
= [3(2x - y) - (x + y)](3(2x - y) + (x + y))
\]

\[
= (6x - 3y + x + y)(6x - 3y - x - y)
\]

\[
= (7x - 2y)(5x - 4y).
\]

**Example 4.** Factorise \(ad - ae + be - bd\).

\[
ad - ae + be - bd = (d - e)(a - d) + (d - c)(b - d),\] since \(c - d = -(d - c),
\]

\[
= (d - c)(a - b).
\]

Simple quadratic functions can often be factorised at sight. If this is done, the answer should be checked by multiplication.
Example 5. Factorise (i) \( x^2 - 7x - 30 \); (ii) \( 3 - 5y - 2y^2 \).

(i) 
\[ x^2 - 7x - 30 = (x - 10)(x + 3) \]
because \((x - 10)(x + 3) = x^2 + 3x - 10x - 30 = x^2 - 7x - 30\).

(ii) 
\[ 3 - 5y - 2y^2 = (3 + y)(1 - 2y) \]
because \((3 + y)(1 - 2y) = 3 - 6y + y - 2y^2 = 3 - 5y - 2y^2\).

If it is difficult to find the factors by inspection, the following method can be used.

Example 6. Factorise \( 15a^2 - 4ab - 4b^2 \).

Replace \(-4ab\) by two equivalent terms whose product is
\[ 15a^2 \times (-4b^2) = -60a^2b^2. \]

Since the product is negative, the two terms have opposite signs; since
\(-4\) is negative, the numerically larger coefficient is negative.
\[-60 = 2 \times (-30) = 3 \times (-20) = 4 \times (-15) = 5 \times (-12) = 6 \times (-10);\]
replace \(-4ab\) by \(6ab - 10ab\).

\[ 15a^2 - 4ab - 4b^2 = 15a^2 + 6ab - 10ab - 4b^2 \]
\[ = 3(a^2 + 2ab) - 2b(5a + 2b) \]
\[ = (a + 2b)(3a - 2b). \]

Remainder theorem. If \( ax^2 + bx + c \) is divided by \( x - k \), the value of the remainder is obtained by substituting \( x = k \) in
\[ ax^2 + bx + c \]
If the remainder is zero, \( x - k \) is a factor.

The value to be substituted for \( x \) is the value of \( x \) which makes the divisor zero.

Example 7. Factorise \( x^2 + 2x^2 - 5x - 6 \).

Try putting \( x = 1, -1, 2, -2, 3, -3, 6, -6 \) (these are the various factors of the constant term \( 6 \)) and see if the result is zero in any of these cases.

If \( x = 1, x^2 + 2x^2 - 5x - 6 = 1 + 2 - 5 - 6 = -8, \)
then \( x - 1 \) is not a factor.

If \( x = -1, x^2 + 2x^2 - 5x - 6 = -1 + 2 + 5 - 6 = 0, \)
then \( x + 1 \) is a factor.

Other factors can be found in the same way, but it is quicker to find the other factor by division,
\[ x^2 + 2x^2 - 5x - 6 = (x + 1)(x^2 + x - 6) \]
\[ = (x + 1)(x - 2)(x + 3). \]

EXERCISE 16

Find the factors of the following expressions:

1. \( a^2 - 7a + 6 \)
2. \( b^2 - 7b - 8 \)
3. \( 3a^2 + 7c - 6 \)
4. \( 4a^2 + 3d - 35 \)
5. \( 6e^2 - 5e - 6 \)
6. \( f^2 - 6f + 9 \)
7. \( 7m^2 - 7mn + 2n^2 \)
8. \( 5p^2 + pq - 6q^2 \)
9. \( 9r^2 - 6rs - 8s^2 \)
10. \( 1 + t - 6a^2 \)
11. \( 9 - 3x - 2x^2 \)
12. \( 10 + 33y - 7y^2 \)
13. \( 20a^2 - 5 \)
14. \( b^4 - b^2c^2 \)
15. \( c^2 - (c - 3d)^2 \)
16. \( ax^4 - bx^2 - c \)
17. \( (a + 2b)^2 - 8c \)
18. \( 4a^3 - 36b \)
19. \( p^3 - pr + 2pq - 2qr \)
20. \( st + 3s - 2t - 6 \)
21. \( t^2 - tx + t + x \)
22. \( y^3 + 2y^2z - yz^2 - 2z^2 \)
23. \( (2a - 3)^2 - (2a - 3)(a + 2) \)
24. \( 9b + 2c)^2 - (2b + c)^2 \)
25. \( x^3 - 2xy + y^2 - z^2 \)
26. \( r^2 - x^2 + 2r + x \)
27. \( a^2 - ab^2 \)
28. \( a^2 + 8b^2 \)
29. \( 1 - 2a \)
30. What numbers must be added to \( x = 12x, (ii) y^2 + y, (iii) 4x^2 - 24x \) to give perfect squares?
31. Factorise \( 2x^2 + 5x + 2 \); hence express \( 20502 \) in prime factors.
32. Simplify \( (a - 2)^3 - (a - 1)(a - 2)(a - 3) \)
33. If \( 2s = a + b + c \), express \( (s - a)^2 - (s - b)^2 \) as simply as possible in terms of \( a, b, c \).
34. Find the value of \( b \) if \( x - 5 \) is a factor of \( x^2 + bx - 15 \).
35. Find the value of \( c \) if \( x + 6 \) is a factor of \( 2x^2 + 9x + c \).
36. Find the remainder when \( x^3 - 4x^2 + 5x - 3 \) is divided by \( x - 2 \).
37. Prove that \( x - 1 \) is a factor of \( x^3 - 3x^2 - 13x + 15 \) and then factorise the expression completely.
38. Prove that \( x + 2 \) is a factor of \( 2x^3 - 3x^2 - 11x + 6 \) and then factorise the expression completely.

Solve the equation \( 2x^3 - 3x^2 - 11x + 6 = 0 \).
39. If \( x + 3 \) is a factor of \( x^3 + ax + 6 \), find the value of \( a \) and in this case factorise the expression completely.

40. If \( x - 3 \) and \( x + 1 \) are both factors of \( x^4 - x^3 - 3x^2 + bx + c \), find the values of \( b \) and \( c \) and in this case find the other factor.

41. Prove that \( a - b \) is a factor of \( a^3 + 3a^2b - 4b^3 \) and then factorise the expression completely.

42. If \( x + 1 \) is a factor of \( x^3 + px + q \) and if the remainder when \( x^3 + px + q \) is divided by \( x + 2 \) is \(-3\), find the values of \( p \) and \( q \).

**Fractions**

Example. Simplify \( \frac{a}{a-b} \frac{2ab}{a^2-b^2} \)

\[
\frac{a}{a-b} \cdot \frac{2ab}{a^2-b^2} = \frac{a(a+b)-2ab}{(a+b)(a-b)} = \frac{a^2+ab-2ab}{(a+b)(a-b)} = \frac{a^2-ab}{(a+b)(a-b)} = \frac{a}{a+b}
\]

**Exercise 17**

Copy and complete the following:

1. \( \frac{4a}{b} = \ldots \frac{3x}{y} = \ldots \frac{2a+2b}{a^2-b^2} = \ldots \frac{1}{(x-y)(x-z)} = \ldots \) (From \( \frac{3x}{y} = \ldots \frac{2a+2b}{a^2-b^2} = \ldots \frac{1}{(x-y)(x-z)} = \ldots \))

2. \( \frac{x-y}{x-z} = \ldots \frac{y-z}{y-x} = \ldots \frac{1}{x+y} = \ldots \frac{1}{x} = \ldots \) (From \( \frac{x-y}{x-z} = \ldots \frac{y-z}{y-x} = \ldots \frac{1}{x+y} = \ldots \frac{1}{x} = \ldots \))

5. Find the H.C.F. and L.C.M. of \( 6a^2b, 2bc^3, 12ab^2d \).

6. Find the L.C.M. of \( x^2 + x - 6 \) and \( x^2 + 4x + 3 \).

7. Find the L.C.M. of \( x^2 + 2xy + y^2, x^2 - y^2, x^2 - 2xy + y^2 \).

8. Express each of the following as a single fraction in its simplest form:

\[
\begin{align*}
9. \frac{a-1}{3} + \frac{1}{a+1} & = \frac{a+1}{6} \\
10. \frac{a^2 + 3a^2b - 4b^3}{6ab^2} & = \frac{a^2}{4a^2} \\
11. \frac{x^2 - xy}{xy} & = \frac{z}{z-y} \\
12. \frac{r^2 - s^2}{rs} & = \frac{x-y}{(y-x)^2} \\
13. \frac{a}{c} - \frac{b}{b-c} & = \frac{3}{4a} - \frac{3}{4} \\
14. \frac{a^2 + z}{x^2 + z} & = \frac{z}{z^2 - z} \\
15. \frac{1}{a + b} & = \frac{1}{a + b}
\end{align*}
\]

9. \( \frac{a - c}{b - c} \)

10. \( \frac{3}{a} - \frac{3}{a^2 - 3a} \)

11. \( \frac{3x + y}{4a - 3} \)

12. \( \frac{3}{a - 3} \)

13. \( \frac{1}{a - b} = \frac{1}{a - b} \)

14. \( \frac{1}{a + b} \)

15. \( \frac{1}{a + b} \)

16. \( \frac{1}{a + b} \)

17. \( \frac{1}{a + b} \)

18. \( \frac{1}{a + b} \)

19. \( \frac{1}{a + b} \)

20. \( \frac{1}{a + b} \)

21. \( \frac{1}{a + b} \)

22. \( \frac{1}{a + b} \)

23. \( \frac{1}{a + b} \)

24. \( \frac{1}{a + b} \)

25. \( \frac{1}{a + b} \)

26. If \( z = \frac{x^2 + 2}{2x + 3} \) and \( y = \frac{3x^2 + 2}{2x - 3} \), express \( z \) in terms of \( x \) in its simplest form.

27. If \( x = \frac{a + b}{a - b} \) and \( y = \frac{a - b}{a + b} \), express \( \frac{1}{1 + y} \) in terms of \( a \) and \( b \) as simply as possible.

**LITERAL RELATIONS**

Example. The length of a rectangular field is \( n \) times its breadth; the perimeter is \( p \) yards; find the breadth.

Let the breadth be \( x \) yards, then the length is \( nx \) yards;
\( \therefore \) the perimeter is \( 2nx + 2x \) yards; but this is \( p \) yards,
\( \therefore \) \( 2nx + 2x = p; \therefore 2x(n + 1) = p; \)
divide each side by \( 2(n + 1) \). \( \therefore \) \( x = \frac{p}{2(n + 1)}; \therefore \) the breadth is \( \frac{p}{2(n + 1)} \) yards.
EXERCISE 18

1. Find the difference in inches between \( \frac{3m}{4} \) yards and \( \frac{13n}{6} \) feet.

2. I buy 4 lb. of orange at 5d. per lb. and 6 lb. at 3d. per lb., find the average cost per lb.

3. Find the weight in lb. of a solid rectangular block, \( l \) yards long, \( b \) feet broad, \( t \) inches thick, made of material which weighs \( w \) lb. per cu. ft.

4. The length of each edge of a cubical block is \( 6t \) inches. Find in sq. ft. the total area of the surface.

5. An alloy is formed by mixing \( m \) cu. ft. of a substance whose density is \( x \) lb. per cu. ft. with \( n \) cu. ft. of a substance whose density is \( y \) lb. per cu. ft. Find the density of the alloy in lb. per cu. ft.

6. A man is \( 4 \frac{1}{2} \) times as old as his son. If the man is \( 18t \) years old, how old is his son?

7. Find in £ the total profit if \( w \) tons of coal is bought at £\( p \) per ton and is sold at \( r \) half-crowns per cwt.

8. An employer pays \( n \) workmen altogether \( p \) shillings for \( c \) hours' work. How many does he employ if he pays altogether \( q \) shillings for \( d \) hours' work, if each is paid the same amount per hour?

9. A rectangular box without a lid is \( l \) ft. long, \( b \) ft. broad, \( h \) ft. high, external measurements; find the total area of the external surface of the box.

10. The sum of 5 consecutive even numbers is 50n. Find in terms of \( n \) the greatest of these numbers.

11. Find in shillings the total value of 20 coins if \( 6t \) of them are half-crowns and the rest are sixpences.

12. \( w \) lb. of luggage is carried free and \( d \) pence per lb. is charged for every pound in excess of \( w \) lb. If I have to pay excess when my luggage weighs \( W \) cwt., express the charge in shillings.

13. A car runs \( s \) miles on a gallon of petrol costing \( p \) pence per gallon. If the average distance run is \( d \) miles each week-day, find the cost in shillings of the petrol used in 6 week-days.

14. A man will be \( 4t \) years old in \( n + 4 \) years' time; how old was he \( n - 4 \) years' ago?

15. Find the profit per cent. if a man buys \( nm \) plants at \( m \) for \( h \) shillings and sells them at \( n \) for \( c \) shillings.

16. A passage is \( l \) ft. long, \( h \) ft. high and the wall along one side of it is panelled to a height of 4 feet; the rest of this wall is painted at a cost of \( r \) shillings per sq. yard. Find in £ the cost of the painting.

17. A rectangular tank of height \( h \) ft. and base-area \( A \) sq. ft. holds \( n \) gallons. Assuming 1 cu. ft. = \( g \) gallons, find \( h \) in terms of \( A, g, n \).

18. A rectangular room is twice as long as it is wide; its perimeter is \( p \) feet. Find in terms of \( p \) the area of the floor.

19. A rectangular box without a lid is made of thin cardboard; the base of the box is a square of side \( x \) in. and the area of cardboard required is 120 sq. in.; prove that the volume of the box is \( 30x^2 - 4x^3 \) cu. in.

20. \( ABGD \) is a fenced rectangular field. It is divided in half by a fence parallel to \( AB \) and each portion is 100 sq. yd. in area. If \( AB = x \) yards, find the total length of fencing in terms of \( x \).

21. At the end of a term, the average age of a class of \( n \) boys is \( t \) years; \( c \) boys of average age \( (t + 1) \) years are then promoted and \( d \) boys of average age \( (t - 1) \) years join the class. If the new average age of the class is \( t + b \) years, find \( b \) in terms of \( n, c, d \).

22. State in algebraic form a general statement of which the following are special cases:

\[
8^2 - 2 \times 9^2 + 10^2 = 2, \quad 9^2 - 2 \times 10^2 + 11^2 = 2, \quad 10^2 - 2 \times 11^2 + 12^2 = 2.
\]

Prove that the general statement is true; hence write down the value of \( 999^2 + 1001^2 - 2 \).

23. State a problem about consecutive numbers which is solved by using the equation, \( n^2 + (n + 1)^2 = (n + 3)^2 + 2(n + 4) \).

24. An aircraft flies 150 miles in a straight line at \( x + 30 \) miles per hour and returns to its starting point along the same line at \( x - 30 \) miles per hour. Find a single expression for the total number of hours of flight.

25. The internal and external diameters of a hollow metal tube 1 foot long, with open ends, are \( d_1 \) inches, \( d_2 \) inches, and the metal weighs \( 48w \) lb. per cu. ft. Find (in factors) the weight of the tube in lb. [Area of circle, radius \( r \), is \( \pi r^2 \).]
### CHANGE OF SUBJECT OF A FORMULA

**Example 1.** If \( d = t \cdot \frac{n - 1}{n} \), find \( n \) in terms of \( d, t \).

\[
\frac{t(n-1)}{n} = d;
\]

Multiply each side by \( n \),

\[
(t(n-1)) = dn,
\]

\[\therefore \quad tn - t = dn, \quad \therefore \quad tn - dn = t,
\]

divide each side by \( t - d \),

\[\therefore \quad n = \frac{t}{t - d},
\]

\( n \) is called the **subject** of this formula.

**Example 2.** Make \( r \) the subject of the formula, \( V = \frac{4}{3} \pi r^3 \).

\[
\frac{4}{3} \pi r^3 = V, \quad \therefore \quad 4\pi r^3 = 3V,
\]

\[\therefore \quad r^3 = \frac{3V}{4\pi},
\]

take the cube root of each side,

\[r = \sqrt[3]{\frac{3V}{4\pi}}.
\]

### EXERCISE 19

Change the following formulae to the required forms.

1. \( C = \frac{2}{3} (F - 32); \) **subject** \( F \).
2. \( l = \frac{PRT}{100}; \) **subject** \( R \).
3. \( l = a + (n - 1)d; \) **subject** \( d \).
4. \( V = \frac{1}{3} \pi r^2 h; \) **subject** \( h \).
5. \( f = \frac{6a}{10 + u}; \) **subject** \( u \). Evaluate \( u \) if \( f = 2 \).
6. \( N + \frac{p}{100} = P; \) **subject** \( N \).
7. \( \frac{l - x}{x - m} = a; \) **subject** \( x \).
8. \( (n + t)a = 4ta + (n - t)d; \) **subject** \( n \). Simplify the result if \( a = 3d \).

### QUADRATIC EQUATIONS AND PROBLEMS

**Solution by Factors**

**Example 1.** Solve \( 6x^2 + 7x - 3 = 0 \).

\[
6x^2 + 7x - 3 = 0, \quad \therefore \quad (2x + 3)(3x - 1) = 0
\]

\[\therefore \quad either \quad 2x + 3 = 0 \text{ or } 3x - 1 = 0,
\]

\[\therefore \quad \begin{cases} 2x + 3 = 0 & \text{or} & 3x - 1 = 0, \\ 2x = -3 & \text{or} & 3x = 1, \\ x = -\frac{3}{2} & \text{or} & x = \frac{1}{3}. \end{cases}
\]

**Solution by Completing the Square**

Any quadratic equation can be expressed in the form \( ax^2 + bx + c = 0 \).

If there are no simple factors of \( ax^2 + bx + c \), the method of "completing the square" can be used.
Example 2. Solve \(3x^2 + 7x - 5 = 0\), giving the roots to 2 places of decimals.

\[ 3x^2 + 7x - 5 = 0, \quad \therefore \quad x = \frac{-7 \pm \sqrt{49 - 4 \cdot 3 \cdot (-5)}}{6}. \]

If \(\left(\frac{7}{2}\right)^2\) is added to each side, the left side becomes a perfect square,

\[ x^2 + \frac{7}{3}x + \left(\frac{7}{6}\right)^2 = \frac{49}{6} + \frac{49}{6}, \]

\[ \therefore \quad \left(\frac{x + \frac{7}{6}}{6}\right)^2 = \frac{60 + 49}{36} = \frac{109}{36}. \]

Take the square root of each side,

\[ x + \frac{7}{6} = \pm \sqrt{\frac{109}{36}}, \]

\[ \therefore \quad x = -\frac{7}{6} \pm \frac{\sqrt{109}}{6}. \]

From the tables, \(\sqrt{109} = 10.44\) to 2 decimal places,

\[ x = -\frac{7}{6} + \frac{10.44}{6} \quad \text{or} \quad x = -\frac{7}{6} - \frac{10.44}{6}, \]

or \(x = -\frac{7}{6} + 1.74 = -2.91\), to 2 decimal places.

Solution by Formula

The method of Example 2 can be used to prove that the roots of the equation

\[ ax^2 + bx + c = 0 \]

are given by

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}. \]

If \(b^2 - 4ac\) is negative, there are no roots.

If \(b^2 - 4ac = 0\), the equation is said to have equal roots or repeated roots.

Example 3. Solve \(3x^2 + 7x - 5 = 0\) by use of the formula.

The equation \(3x^2 + 7x - 5 = 0\) is equivalent to \(ax^2 + bx + c = 0\) if \(a = 3, b = 7, c = -5\),

\[ x = \frac{-7 \pm \sqrt{49 - 4 \cdot 3 \cdot (-5)}}{6}, \]

\[ \therefore \quad \text{the roots are} \quad x = \frac{-7 \pm \sqrt{49 + 60}}{6}, \]

that is,

\[ x = \frac{-7 \pm \sqrt{109}}{6}. \]

QUADRATIC EQUATIONS AND PROBLEMS

Sum and Product of Roots

The equation, whose roots are \(\alpha\) and \(\beta\), is

\[ (x - \alpha)(x - \beta) = 0 \]

that is,

\[ x^2 - \alpha x - \beta x + \alpha \beta = 0 \]

or

\[ x^2 - (\alpha + \beta)x + \alpha \beta = 0. \]

Hence, if \(\alpha\) and \(\beta\) are the roots of the equation

\[ ax^2 + bx + c = 0 \]

that is,

\[ x^2 + \frac{b}{a}x + \frac{c}{a} = 0, \]

\[ \alpha + \beta = -\frac{b}{a} \quad \text{and} \quad \alpha \beta = \frac{c}{a}. \]

Example 4. Find the sum of the squares of the roots of the equation, \(3x^2 + 7x - 5 = 0\).

Write the equation in the form \(x^2 + \gamma x - \delta = 0\) and denote its roots by \(\alpha\) and \(\beta\); then

\[ \alpha + \beta = -\frac{b}{a} \quad \text{and} \quad \alpha \beta = \frac{c}{a}. \]

It is required to find the value of \(\alpha^2 + \beta^2\).

By squaring, we have

\[ (\alpha + \beta)^2 = (\frac{\gamma}{a})^2, \]

\[ \therefore \quad \alpha^2 + 2\alpha \beta + \beta^2 = \frac{\gamma^2}{a^2}, \]

also

\[ 2\alpha \beta = \frac{-2b}{a}, \]

hence by subtraction

\[ \alpha^2 + \beta^2 = \frac{\gamma^2}{a^2} - \frac{2b}{a}. \]

EXERCISE 20

Solve the following equations:

1. \(x^2 + 7x - 8 = 0\).  [2] \(4x^2 + 3x - 1 = 0\).  [3] \(3x^2 + 5x - 2\).
[4] \(5x^2 - 3x = 0\).  [5] \(9x^2 - 12x - 12 = 0\).  [6] \(3(x^2 + 1) = 10x\).
[7] \(x + \frac{1}{4x} = 1\).  [8] \((3x - 2)^2 = 12\frac{1}{2}\).  [9] \((x - 1)(3x + 1) = 15\).

Find correct to 2 places the roots of the following equations:

10. \(x^2 - 6x = 9\).  [11] \(x^2 - 5x + 1 = 0\).  [12] \(x^2 - x = 5\).
[13] \(2x^2 + 3x = 11\).  [14] \(3x^2 - 5x = 4\).  [15] \(x - \frac{2}{5} = \frac{2}{5}\).
[16] Find in their simplest forms the equations whose roots are (i) \(3, -4\); (ii) \(-5, 0\); (iii) \(\frac{1}{2}, \frac{1}{4}\).
17. Find in their simplest forms the equations whose roots are (i) \( \sqrt{6} \), \( -\sqrt{6} \); (ii) \( 5 + \sqrt{3} \), \( 5 - \sqrt{3} \); (iii) \( -1 + \sqrt{2} \), \( -1 - \sqrt{2} \).

18. If \( s = u + \frac{1}{2} at^2 \), find the positive value of \( t \) for which \( s = -6 \) when \( u = 20 \), \( a = -32 \).

19. If \( y = (x - 2)(x - 6) \), find the values of \( x \) for which \( y = -3 \).

20. If \( y = (2x + 1)^2 - 9 \), find the values of \( x \) for which \( y = 1 \).

21. If \( x^2 - 5x - 8 \) is equal to \( (x - \alpha)(x - \beta) \) for all values of \( x \), find the values of (i) \( \alpha + \beta \), (ii) \( \alpha \beta \).

22. If \( \alpha \) and \( \beta \) are the roots of \( x^2 + 3x - 6 = 0 \), find the values of (i) \( \alpha + \beta \), (ii) \( \alpha \beta \), (iii) \( \alpha^2 + \beta^2 \).

23. If \( x^2 + bx - cx = 0 \), where the values of \( b \), \( c \) are given, find \( x \).

24. If \( x^2 - x = a^2 + a \), where \( a \) is given, find \( x \).

Solve the following simultaneous equations:

25. \( 3x - y + 1 = 0 \). \[ 26. \begin{align*} 2x + y &= -1. \\ 3x^2 + 2xy &= -3. \end{align*} \]

27. \( x - y = 1 \). \[ 28. \begin{align*} x - 3y &= 1. \\ 2x^2 - 3xy - 5y &= 11. \end{align*} \]

29. \( 3y - 2x = 1 \). \[ 30. \begin{align*} \frac{1}{2}x + \frac{1}{2}y &= 3. \\ \frac{4}{5}x - \frac{5}{y} &= 1. \end{align*} \]

31. Find three consecutive numbers such that the square of the middle number exceeds the difference of the squares of the other two by 60.

32. The length of a rectangular tray exceeds its breadth by 1 ft. If the length is halved and the breadth is increased by 6 in., the area of the tray is decreased by 180 sq. in. Find the original length of the tray.

33. A rectangular metal plate of uniform thickness is 30 in. long, 20 in. wide. If a strip \( x \) in. wide is cut off all round, the weight is reduced from 8 lb. to 5 lb.; find the value of \( x \).

34. The lengths of the sides of a right-angled triangle are \( 5x + 2 \), \( 5x \), \( 3x - 1 \), inches. Find the length of each side.

35. The diameter \( AB \) of a circle \( APB \) is 3 in. long; the length of \( PA \) exceeds the length of \( PB \) by 1 inch. Find the length of \( AP \) correct to \( \frac{1}{100} \) in.

---

**GRAPHS**

**EXERCISE 21**

1. An anti-freezing mixture is made by dissolving calcium chloride. If the mixture contains \( w \) lb. per gallon, the freezing point is \( F \) degrees Fahrenheit, as follows:

<table>
<thead>
<tr>
<th>( w )</th>
<th>2</th>
<th>2\frac{1}{2}</th>
<th>3\frac{1}{2}</th>
<th>4</th>
<th>4\frac{1}{2}</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F )</td>
<td>13</td>
<td>7</td>
<td>-8</td>
<td>-17</td>
<td>-27</td>
<td>-39</td>
</tr>
</tbody>
</table>

Estimate from a graph (i) the freezing point for a mixture containing 3 lb. per gallon, (ii) the mixture for which the freezing point is -22°F.

2. The annual premium for a whole life assurance of £100 (with profits) depends on the age at entry as follows:

<table>
<thead>
<tr>
<th>Age at entry</th>
<th>30</th>
<th>34</th>
<th>38</th>
<th>42</th>
<th>46</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Premium</td>
<td>50s.</td>
<td>54s. 6d.</td>
<td>60s.</td>
<td>68s.</td>
<td>78s.</td>
<td>90s. 6d.</td>
</tr>
</tbody>
</table>

Estimate from a graph (i) the premium if the age at entry is 48, (ii) the age at entry if the premium is 70s.

Use the scale: 1 inch for 5 years of age, 1 inch for 10s. of premium.
ALGEBRA

3. A cyclist starts from A at 2 p.m. and rides steadily at 10 m.p.h. towards B, 50 miles away. A motorist leaves B for A at 4 p.m. and travels at 40 m.p.h. Find from a graph at what time and at what distance from A they meet.

When they meet, they stop and talk for half an hour and then the motorist returns to B at 30 m.p.h.; at what time does he arrive back at B?

4. A man starts from A at 10 a.m. and walks along a road at 3 m.p.h.; a cyclist starts from A at 11 a.m. and rides along the same road at 9 m.p.h. to a town B 15 miles away; he waits B for three-quarters of an hour and then returns to A at 12 m.p.h. Find from a graph at what times and at what distances from A the cyclist passes the pedestrian on his outward and on his return journey.

Examples for Oral Work.

The following examples refer to the figure on page 51 which shows the graphs of y where

\[ y = \frac{1}{2}(3x^2 - 5x - 10), \]

1. Is \( y = \frac{1}{2}(x - 1) \) or of \(- \frac{1}{2}(3x + 2)\) ? Give reasons.

Solve the equations, Nos. 2-5:

2. \( 3x^2 - 5x - 10 = 0 \).
3. \( 3x^2 - 5x - 10 = 10 \).
4. \( 3x^2 - 5x - 10 = 6 \).
5. \( 3x^2 - 5x = -22 \).

6. Is there a value of \( x \) for which \( 3x^2 - 5x - 10 \) equals \(-15\)?

7. What is the least value of \( 3x^2 - 5x - 10 \) and for what value of \( x \) is the expression least?

8. For what range of values of \( x \) is \( 3x^2 - 5x - 10 < 5 \)?

9. For what range of values of \( x \) is \( 3x^2 - 5x - 10 < -5 \)?

10. For what range of values of \( x \) is \( x - 1 > 3x^2 - 5x - 10 \)?

11. What equation in \( x \) has the values of \( x \) at the points P, Q as roots? Give the equation in its simplest form.

12. What equation in \( x \) has the values of \( x \) at the points H, K as roots? Give the equation in its simplest form.

13. Calculate algebraically the co-ordinates of the points P, Q.

14. Calculate algebraically the co-ordinates of the points H, K.

15. Calculate algebraically the co-ordinates of the point of intersection of the lines HK, PQ.

FIG. 1.

EXERCISE 22

1. Draw the graphs of \( y \) for values of \( x \) from \(-3 \) to \(+5 \) where

(i) \( y = 2x^2 - 5x - 3 \), (ii) \( y = \frac{1}{2}(x + 1) \), (iii) \( y = 2(x + 6) \). Use the graphs for the following:

(i) Solve \( 2x^2 - 5x - 3 = 0 \). (ii) Solve \( 2x^2 - 5x = 11 \).

(iii) Solve \( 2x^2 - 5x - 3 = \frac{1}{2}(x + 1) \).

(iv) Find the least value of \( 2x^2 - 5x - 3 \).

(v) For what range of values of \( x \) is \( 2x^2 - 5x - 3 < 15 \)?

(vi) For what range of values of \( x \) is \( 2x^2 - 5x - 3 < 2(x + 6) \)?
[2] Draw the graphs of \( y \) for values of \( x \) from \( -2 \) to \( +7 \) where

(i) \( y = 5x - x^2 \), (ii) \( y = \frac{1}{2}(6x - 19) \). Use the graphs for the following:

(i) Solve \( 5x - x^2 = 2 \).

(ii) Solve \( x^2 - 5x - 8 = 0 \).

(iii) Solve \( x - \frac{6}{x} = 5 \).

(iv) For what range of values of \( x \) is \( 5x - x^2 > \frac{1}{2}(6x - 19) \)?

(v) By inserting a suitable straight-line graph, solve the equation \( x^2 - 4x - 2 = 0 \).

3. Draw the graph of \( y \) where \( y = x + \frac{6}{x} \) for values of \( x \) from \( 0.5 \) to \( 12 \). Use the graph for the following:

(i) Find two values of \( x \) such that \( x + \frac{6}{x} = 8 \).

(ii) Solve \( x^2 - 6x + 6 = 0 \).

(iii) Find the smallest positive value of \( x + \frac{6}{x} \) and the value of \( x \) for which it takes this value.

[4] Draw the graph of \( y = x^2 + \frac{48}{x} \) for values of \( x \) from \( 1 \) to \( 7 \), taking the side of a large square for 1 unit on the \( x \)-axis and for 5 units on the \( y \)-axis. Use the graph for the following:

(i) For what value of \( x \) is \( x^2 + \frac{48}{x} \) least?

(ii) Find two values of \( x \) for which \( x^2 + \frac{48}{x} = 40 \).

(iii) Find two values of \( x \) for which \( x^2 + \frac{48}{x} \) is equal to 36.

5. After \( x \) years at 5\% per annum compound interest, £100 amounts to £100(1.05)^x. Taking \( \log 1.05 = 0.02119 \), calculate the values of the expression when \( x \) has the values 5, 10, 15, 20 and draw the graph of \( 100(1.05)^x \) for values of \( x \) from 0 to 20. Estimate from the graph the amount at the end of (i) 12 years, (ii) 18 years. Find the greatest number of years that elapses before the amount exceeds £150.

[6] Draw the graph of \( y = 2^x \) for values of \( x \) from \( -1 \) to \( +4 \), taking the scale 1 inch to represent 1 unit on the \( x \)-axis and 1 inch to represent 2 units on the \( y \)-axis. Use the graph (i) to find the value of \( 2^{1/3} \), (ii) to solve \( 2^x = 10 \).

By drawing the graph of \( y = 2x + 1 \) on the same scale and axes, find the positive value of \( x \) for which \( 2^x = 2x + 1 \).

[7] Draw the graphs of \( y = \frac{6}{x+1} \) and \( y = x^2 \) for values of \( x \) from \( 0 \) to \( 5 \). Find from the graphs an approximate value of \( x \) for which \( x^2 = \frac{6}{x+1} \).

Next make a table of values for \( \frac{6}{x+1} \) for the values of \( x \): 1, 1.1, 1.2, 1.4, 1.5, 1.7, 2. Taking the side of a large square for 0.2 units on each axis, draw the graph of \( \frac{6}{x+1} \) for values of \( x \) from 1 to 2.

With the same scale and axes draw the graph of \( x^2 \) for values of \( x \) from 1.4 to 1.7. Hence find as accurately as possible from these graphs a root of \( x^2 = \frac{6}{x+1} \).

[8] Draw the graph of \( \frac{4x + 3}{x^2 + 1} \) for values of \( x \) from -3 to +3.

Use the graph for the following:

(i) What are the maximum and minimum values of \( \frac{4x + 3}{x^2 + 1} \)?

(ii) For what range of values of \( x \) is \( \frac{4x + 3}{x^2 + 1} \) greater than 2?

(iii) For what values of \( x \) is \( \frac{4x + 3}{x^2 + 1} \) equal to \( 3\frac{1}{2} \)?

Check the answer by solving \( \frac{4x + 3}{x^2 + 1} = 3\frac{1}{2} \) algebraically.

[9] Draw the graph of \( y = \frac{3x(x - 3)}{x + 4} \) for values of \( x \) from -1 to +5.

(i) Find from the graph the value of \( x \) for which \( \frac{x(x - 3)}{x + 4} \) is a minimum.

(ii) Draw the line \( y = 1 \frac{1}{2} \) and express in its simplest form the quadratic equation whose roots are given by the points of intersection of this line and the curve. What are the roots?

(iii) Solve \( x(x - 3) = x + 4 \).
ALGEBRA

10. The graph of \( y = ax^2 + bx + c \) and \( a, b, c \) are constants, passes through the points \( (0, -5), (2, 3), (3, 4) \).

  (i) What is the value of \( c \)?
  (ii) Find the values of \( a, b \).
  (iii) Find the co-ordinates of the points where the graph cuts the \( x \)-axis.

Sketch the graph without making a table of values.

11. Draw the graphs of \( y = x^3 \) and \( y = \frac{1}{2}(10x - 7) \) for values of \( x \) from \(-3\) to \( +3\), taking \( 1 \) inch as the unit on the \( x \)-axis and \( \frac{1}{2} \) inch as the unit on the \( y \)-axis.

  (i) Express in its simplest form the equation whose roots are given by the points of intersection of the curve and the line. What are the roots?
  (ii) By drawing a suitable straight-line graph, solve the equation \( x^3 + 10x - 20 = 0 \).

INDICES, LOGARITHMIC NOTATION AND SURDS

**Indices**

\[
\begin{align*}
a^m \times a^n &= a^{m+n} \quad & a^m \div a^n &= a^{m-n} \quad & (a^m)^n &= a^{mn}.
\end{align*}
\]

\[
\begin{align*}
a^\frac{1}{2} &= \sqrt{a} \quad & a^\frac{2}{3} &= \sqrt[3]{a^2} = (\sqrt[3]{a})^2.
\end{align*}
\]

In particular,

\[
\begin{align*}
a^\frac{1}{2} &= \sqrt{a} \quad & a^\frac{2}{3} &= \sqrt[3]{a^2}.
\end{align*}
\]

\[
\begin{align*}
a^{-n} &= \frac{1}{a^n} \quad & a^0 &= 1.
\end{align*}
\]

**Logarithmic Notation**

If \( x = 10^m \), \( m \) is the logarithm of \( x \) to base 10, and we write

\[ m = \log_{10} x. \]

In particular, since \( 1 = 10^0 \), \( \log_{10} 1 = 0 \),

and, since \( 10 = 10^1 \), \( \log_{10} 10 = 1 \),

and, since \( 100 = 10^2 \), \( \log_{10} 100 = 2 \), and so on.

If there is no special reason for emphasising that the base is 10, \( \log_{10} x \) is represented by \( \log x \).

**INDICES, LOGARITHMIC NOTATION AND SURDS**

**Properties of Logarithms**

\[
\begin{align*}
\log (xy) &= \log x + \log y, \\
\log \left( \frac{x}{y} \right) &= \log x - \log y, \\
\log (x^n) &= n \log x.
\end{align*}
\]

**Surds**

\[
\begin{align*}
\sqrt{a \times \sqrt{b}} &= \sqrt{(ab)} \\
\sqrt{a \div \sqrt{b}} &= \sqrt{\left(\frac{a}{b}\right)}.
\end{align*}
\]

Note. \( \sqrt{a} \div \sqrt{b} = \sqrt{\frac{a}{b}} \) is not equal to \( \sqrt{a + \sqrt{b}} \) unless \( a \) or \( b \) is zero.

Fractions with surds in the denominator can often be dealt with more easily if replaced by equivalent fractions with rational denominators. This process is called **rationalising the denominator**.

**Example 1.** Simplify \( \frac{12}{\sqrt{75}} \).

\[
\sqrt{75} = \sqrt{25 \times 3} = 5\sqrt{3}.
\]

\[
\frac{12}{\sqrt{75}} = \frac{12 \times \sqrt{75}}{5\sqrt{3} \times \sqrt{75}} = \frac{12 \times \sqrt{75}}{5 \times 3 \times \sqrt{3}} = \frac{12 \times \sqrt{75}}{15} = \frac{12 \times \sqrt{75}}{5 \times 3} = \frac{12 \times \sqrt{75}}{5}.
\]

**Example 2.** Simplify \( \frac{12}{\sqrt{3\sqrt{2} - 2\sqrt{3}}} \).

Since \( (a - b)(a + b) = a^2 - b^2 \),

\[
(3\sqrt{2} - 2\sqrt{3})(3\sqrt{2} + 2\sqrt{3}) = (3\sqrt{2})^2 - (2\sqrt{3})^2 = 9 \times 2 - 4 \times 3 = 18 - 12 = 6.
\]

\[
\frac{12}{3\sqrt{2} - 2\sqrt{3}} = \frac{12(3\sqrt{2} + 2\sqrt{3})}{(3\sqrt{2} - 2\sqrt{3})(3\sqrt{2} + 2\sqrt{3})} = \frac{12(3\sqrt{2} + 2\sqrt{3})}{6} = 2(3\sqrt{2} + 2\sqrt{3}).
\]

**EXERCISE 23**

[In this exercise, \( \log x \) means \( \log_{10} x \). Tables are not to be used.]

Write down the values of the following:

1. \( 3^2 \)
2. \( 6^2 \)
3. \( (-2)^2 \)
4. \( 8^2 \)
5. \( 16^{1/8} \).
[6] $\left(\frac{1}{2}\right)^{-1}$. [7] $27^{\frac{1}{3}}$.  [8] $(0.01)^{-1}$.  [9] $(\frac{1}{3})^{-2}$.  [10] $(\frac{1}{5})^{-\frac{1}{4}}$.

11. Simplify (i) $2^4 \times 10^{-3}$; (ii) $(16 \times 10^{-1}) \div 2$; (iii) $16^a \times 9^b = 1 \times 6^a$.

12. Find the value of $a$ if (i) $5^{2a} = 125$; (ii) $10^a = 0.01$.

13. If $x = a + a^{-1}$, $y = a - a^{-1}$, simplify $x^2 - y^2$.

14. Simplify (i) $a^{\frac{1}{2}} \div a^2$; (ii) $a^{\frac{1}{2}} b^{\frac{1}{2}} \times \sqrt{a^2 b}$.

15. Expand $(x^\frac{1}{2} + 1)(x^\frac{1}{2} - x^\frac{1}{4} + 1)$.

16. Find the value of $\log 4 + \log 25$.

17. Express in terms of $\log a$ and $\log b$,

(i) $\log \sqrt[3]{a^2 b}$; (ii) $\log \sqrt[3]{\frac{a}{b^2}}$.

18. Given $\log 2 = 0.301030$, $\log 3 = 0.477122$, find to 5 places of decimals the values of $\log 5$, $\log 6$, $\log 15$, $\log 1.2$.

19. If $y = 1000 \times 5^x$, express $\log y$ in terms of $\log x$.

20. If $\log y = 2 - \frac{1}{2} \log x$, express $y$ in terms of $x$.

21. If $\log x + 3 \log 2 = 2$, find the value of $x$.

22. If $\log x - \log (x - 1) = 1$, find the value of $x$.

23. If $\log (ab) = x$ and $\log (ac) = y$, express $\log \frac{b}{c}$ in terms of $x$ and $y$.

24. Write down the values of (i) $(\sqrt{2})^4$; (ii) $\sqrt{12} \div \sqrt{3}$.

25. Simplify (i) $\sqrt{8} + \sqrt{18} - \sqrt{50}$; (ii) $\sqrt{54} + \sqrt{96} - \sqrt{216}$.

26. Express in a form free of indices, using root signs,

(i) $\frac{3^2}{2}$; (ii) $10^{-\frac{1}{2}}$; (iii) $(\sqrt{2})^{-1}$; (iv) $a^{\frac{1}{2}} b^{\frac{1}{2}} \times \sqrt{a^2 b^2}$.

27. If $x = \sqrt{5} + 1$, evaluate $x^2 - 2x$.

28. If $a = \sqrt{7} + \sqrt{3}$, $b = \sqrt{7} - \sqrt{3}$, evaluate (i) $ab$, (ii) $a^2 + b^2$.

29. Simplify $\frac{1}{\sqrt{5} + 2} - \frac{1}{\sqrt{5} + 2}$.

30. Express with rational denominators the following fractions:

(i) $\frac{1}{\sqrt{2}}$; (ii) $\frac{\sqrt{12}}{\sqrt{5} + \sqrt{3}}$; (iii) $\frac{3\sqrt{2} - 2\sqrt{3}}{\sqrt{3} + \sqrt{2}}$.

31. Given $\sqrt{3} = 1.7321$, find as shortly as possible the value of $x$ which satisfies the equation $x\sqrt{3} - x = 4$, correct to 3 places of decimals.

32. If $a = \sqrt[3]{b^x - 2}$, find $x$ in the form $a^m b^n$.

VARIATION

The statement, $y$ varies directly as $x^n$, means

$$y = cx^n$$

that is,

where the value of $c$ is the same for all values of $x$. The statement is also written in the form, $y \propto x^n$. In particular, if $y$ varies directly as the square of $x$,

$$y = cx^2$$

and if $y$ varies directly as the square root of $x$,

$$y = c\sqrt{x}$$

where as before the value of $c$ is the same for all values of $x$.

The statement, $y$ varies inversely as $x^n$, means

$$y = \frac{c}{x^n}$$

that is,

In particular, if $y$ varies inversely as $x$,

$$y = \frac{c}{x}$$

and if $y$ varies inversely as the square root of $x$,

$$y = \frac{c}{\sqrt{x}}$$

Joint Variation. The weight $W$ lb. of a cylindrical metal bolt, radius $r$ in., length $l$ in., is given by

$$W = \pi r^2 l$$

where $\rho$ lb. per cu. in. is the density of the metal. $W$ is said to vary directly as $\rho$ and as $l$ and as the square of $r$; this is called joint variation.

If a function of a variable consists of two or more terms, it does not vary as any power of the variable, although the separate terms may do so.

If

$$y = a + bx^n$$

where $a$, $b$ are the same for all values of $x$, we say that $y$ is partly constant and partly varies directly as $x^2$.

If

$$y = ax + \frac{b}{x}$$

we say that $y$ partly varies directly as $x$ and partly varies inversely as $x$. 
EXERCISE 24

1. If \( pv \) is constant and if \( p = 120 \) when \( v = 3 \), find the value of \( p \) when \( v = 4.5 \).
   Find the percentage change in \( p \) if \( v \) decreases by 20%.

2. A solid cylindrical metal bar, diameter 3 cm., length 40 cm., weighs 2.7 kg. Find the weight of a solid cylindrical bar of the same metal, diameter 2 cm., length 50 cm.

3. If \( V \) varies as \( d^2 \) and if \( V = 24 \) when \( d = 2 \), find \( V \) when \( d = \frac{1}{2} \).

4. The horse-power transmitted by the engines of a steamer varies as the square of the speed. The speed is 8 knots when the transmitted horse-power is 1000. What horse-power is transmitted when the speed is 12 knots?

5. If \( x \) varies as the square-root of \( A \) and if \( x = 4 \) when \( A = 25 \), find \( x \) when \( A = 4 \).
   Also find \( x \) in terms of \( A \).

6. If \( x \) varies inversely as the square-root of \( h \) and if \( x = 10 \) when \( h = 9 \), find \( x \) when \( h = 64 \).
   Also find \( x \) in terms of \( h \).

7. If a stone takes \( t \) sec. to fall \( s \) feet, \( t \propto \sqrt{s} \). If a stone falls 25 feet in \( 1\frac{1}{2} \) sec., find how long it takes to fall 64 feet.
   [Air-resistance is neglected.]
   Also find \( x \) in terms of \( t \).

8. If \( y = 3 \) when \( x = 8 \), write down, without simplifying, the value of \( y \) when \( x = 11 \),
   (i) if \( y \) varies as \( x \);
   (ii) if \( y \) varies as \( x^2 \);
   (iii) if \( y \) varies as \( \sqrt{x} \);
   (iv) if \( y \) varies as \( \frac{1}{x} \);
   (v) if \( y \) varies as \( x^4 \);
   (vi) if \( y \) varies as \( \frac{1}{x^4} \).

9. Complete the following statements:
   (i) If \( A \propto d^2 \), \( d \propto \cdots \);
   (ii) if \( V \propto d^2 \), \( d \propto \cdots \);
   (iii) if \( t \propto \sqrt{s} \), \( s \propto \cdots \);
   (iv) if \( s \propto vt \), \( t \propto \cdots \);
   (v) if \( H \propto \frac{W}{x^2} \), \( x \propto \cdots \);
   (vi) if \( n \propto \frac{1}{\sqrt{l}} \), \( l \propto \cdots \).

10. If \( S \propto d^2 \) and \( V \propto d^3 \), find how \( S \) varies with \( V \). Find the percentage increase in \( S \), if \( V \) is doubled.

11. The number of equal spherical shot that can be made from a lump of lead varies directly as the weight of the lump and inversely as the cube of the diameter of a shot. From 4 lb. of lead it is possible to cast 1190 shot, each of diameter \( \frac{1}{2} \) inch. Find to 3 figures the number of shot, each of diameter \( \frac{1}{10} \) inch, that can be cast from 10 lb. of lead.

12. The length of a pendulum varies as the square of the time of swing. What percentage change must be made in the length (i) in order to increase the time of swing by 20 per cent., (ii) in order to decrease the time of swing by 20 per cent.?

13. The effort \( P \) lb. required by a machine to raise a load \( W \) lb. is partly constant and partly varies as the load. For loads 30 lb., 60 lb., the necessary efforts are 10 lb., 15 lb. Find \( P \) in terms of \( W \).

14. Part of the total cost of providing dinners for school children is constant and another part varies as the number of children taking dinner. For 180 children, the cost is 9d. per child and for 270 children the cost is 8½d. per child. How many children must take dinner in order that the cost may be only 8d. per child?

15. The cost of manufacturing a motor-car is assumed to consist of a fixed sum together with an additional sum which varies inversely as the number of cars produced per day. The cost of each is £170 when the daily output is 40 cars and is £160 when the daily output is 50 cars. Find the daily output in order to bring the cost down to £145 per car.

16. The expense of a ship's voyage between two places is the sum of two parts, one of which varies directly and the other of which varies inversely as the number of days the voyage lasts. The expense is £9200 if the voyage lasts 10 days and is £10,000 if it lasts 14 days. What is the cost if the voyage lasts 12 days?

17. The illumination of a small object by a lamp varies directly as the candle-power and inversely as the square of its distance from the lamp. If an electric lamp of 32 candle-power, fixed 6 feet above a table, is replaced by a lamp of 18 candle-power, how much must the new lamp be lowered to give the same illumination at the point of the table directly below the lamp?
18. The heat, $h$ calories, developed by an electric current in a wire varies directly as the time, $t$ seconds, and as the square of the voltage, $V$ volts, and inversely as the resistance, $R$ ohms.

48 calories are developed in 1 second when the resistance is 50 ohms and the voltage is 100 volts. Find how many calories are developed in 1 minute by a current in a wire of resistance 2 ohms, if the voltage is 2 volts.

Also find $h$ in terms of $t$, $V$, $R$ and state how $V$ varies with $h$, $t$, $R$.

19. The force, $P$ tons, necessary to stop a train, weight $W$ tons, varies directly as the weight of the train and the square of its velocity, $v$ m.p.h., and inversely as the distance, $s$ feet, within which it is stopped. A train weighing 300 tons, travelling at 40 m.p.h., can be brought to rest in 800 feet by a force of 20 tons. Find the force necessary to stop a train weighing 360 tons, travelling at 45 m.p.h., in a distance of 400 yards.

Also find $P$ in terms of $W$, $v$, $s$ and state how $v$ varies with $P$, $W$, $s$.

20. The horse-power $H$ of a pump necessary to discharge water from a pipe varies directly as the cube of the weight $W$ lb. of the water delivered per second and inversely as the square of the area $A$ sq. in. of the cross-section of the pipe.

(i) A 3-horse-power engine can just deliver 20 lb. of water per second from a pipe, cross-section 2 sq. in. Find to 3 figures what weight of water per second can be delivered from a pipe, cross-section $1\frac{1}{2}$ sq. in., by a 6-horse-power engine.

(ii) Find a formula for $W$ in terms of $A$, $h$.

(iii) If $A$ is increased by 50% and $H$ is decreased by 20%, find the percentage change in $W$ to 2 figures.

21. The weight that can be carried by a cylindrical iron column varies directly as the fourth power of the diameter and inversely as the square of its length.

(i) If the length is doubled, how must the diameter be altered, if the same weight is to be carried?

(ii) If a load $W$ tons is to be carried by a cylindrical column, $l$ feet long and of volume $V$ cu. ft., find how $W$ varies with $l$ and $V$.

**PROGRESSIONS**

Arithmetical progression. The set of numbers

$$a, a + d, a + 2d, a + 3d, \ldots$$

is called a series in arithmetical progression or for short an A.P., having $a$ as first term and $d$ as common difference.

The $n$th term $= a + (n - 1)d$.

If there are $n$ terms and if the last term is $l$,

$$l = a + (n - 1)d,$$

and the sum $s$ of the $n$ terms is given by

$$s = \frac{1}{2}n(a + l) \quad \text{and} \quad s = \frac{1}{2}n(2a + (n - 1)d).$$

Arithmetic mean. If $x, y, z$ are in A.P., the common difference is $y - x$ and is also $z - y$,

$$\therefore \quad y - x = z - y, \quad \therefore \quad 2y = x + z$$

$$\therefore \quad y = \frac{1}{2}(x + z).$$

$\frac{1}{2}(x + z)$ is called the arithmetic mean of $x$ and $z$.

**Example 1.** Find five numbers between 21 and 36 which form with 21 and 36 an A.P.

If the numbers 21, $a$, $b$, $c$, $d$, 36 are in A.P., the first term is 21 and the 7th term is 36; denote the common difference by $d$.

Since the 7th term is 21 + 6$d$.

$$21 + 6d = 36, \quad \therefore \quad 6d = 36 - 21 = 15, \quad \therefore \quad d = \frac{15}{6} = 2\frac{1}{2}$$

$$\therefore \quad \text{the A.P. is} \quad 21, 23\frac{1}{2}, 26, 28\frac{1}{2}, 31, 33\frac{1}{2}, 36.$$ 

This process is called inserting 5 arithmetic means between 21 and 36.

Geometrical progression. The set of numbers

$$a, ar, ar^2, ar^3, \ldots$$

is called a series in geometrical progression or for short a G.P., having $a$ as first term and $r$ as common ratio.

The $n$th term $= ar^{n-1}$.

If there are $n$ terms, the sum $s$ is given by

$$s = \frac{a(r^n - 1)}{r - 1} = \frac{a(1 - r^n)}{1 - r}.$$
Geometric mean. If \( x, y, z \) are in G.P., the common ratio is \( \frac{y}{x} \) and is also \( \frac{z}{y} \).

\[
\therefore \frac{y}{x} = \frac{z}{y} \quad \therefore y^2 = xz,
\]

\[
\therefore y = \pm \sqrt{xyz}
\]

+ \( \sqrt{(xz)} \) is called the geometric mean of \( x \) and \( z \).

Example 2. The 3rd term of a G.P. is 360 and the 6th term is 1215. Find the first term, the common ratio and the sum of the first 6 terms.

Denote the first term by \( a \) and the common ratio by \( r \), then

\[
ar^2 = 360 \quad \text{and} \quad ar^3 = 1215,
\]

\[
\therefore \frac{ar^3}{ar^2} = \frac{1215}{360} \quad \therefore \quad r^3 = \frac{37}{8}
\]

\[
\therefore r = \sqrt[3]{\frac{37}{8}} = \frac{37}{8}
\]

Substitute for \( r \).

\[
a(\frac{37}{8})^2 = 360
\]

\[
\therefore a = \frac{360 \times 8}{37} = 160.
\]

The sum of the first 6 terms:

\[
\frac{a(r^6 - 1)}{r - 1} = \frac{160 \left( \frac{37}{8} \right)^6 - 1}{\frac{37}{8} - 1} = 160 \left( \frac{3729}{64} - 1 \right) + \frac{1}{4}
\]

\[
= \frac{320 \times 665}{64} = 5 \times 665 = 3325.
\]

EXERCISE 25

1. Write down the first 4 terms of a series whose \( n \)th term is 
   (i) \( 3n - 5 \), (ii) \( 15 - 5n \), (iii) \( 3 \times 2^{n-1} \), (iv) \( 2 \cdot (-3)^{n-1} \).

Find the \( n \)th terms of the following arithmetical progressions:

2. 4, 5, 6, 7, . . .

3. 5, 7, 9, 11, . . .

4. 7, 4, 1, -2, . . .

5. 2, 2\frac{1}{2}, 3\frac{1}{2}, 4\frac{1}{2}, . . .

6. 5, 10, 20, . . .

7. 16, -32, 64, -128, . . .

8. 6, -3, 1\frac{1}{2}, -\frac{3}{2}, . . .

9. 18, 12, 8, 5\frac{1}{2}, . . .

10. A man's salary increases each year by £20. For the first year it is £450. Write down an expression for his salary in the \( n \)th year.

11. The second and third terms of an A.P. are 7 and 11; find the first and fifth terms.

12. The first and sixth terms of an A.P. are 7 and 3 respectively; find the second term and the \( n \)th term.

13. Sum to 10 terms the A.P., 5, 8, 11, 14, . . .

14. Sum to 12 terms the A.P., 9, 5, 1, -3, . . .

15. How many terms are there in the A.P., 6, 13, 20, . . ., 90? Find their sum.

16. The first and last terms of an A.P. are 10 and 52, and their sum is 465. Find the number of terms and the 4th term.

17. The first and last terms of an A.P. are 3.5 and 8, and there are 16 terms. Find their sum. How many more terms must be taken to give a term greater than 100 and what is this term?

18. (i) Find the sum of all whole numbers from 100 to 200, both included.

(ii) Find the sum of all whole numbers between 100 and 200 which are divisible by 3.

19. How many terms of the series 1, 2, 3, 4, . . . must be taken to give a sum 210?

20. The first term of an A.P. is 24 and the third term is 4. How many terms must be taken to give a sum 20? What is the last term taken?

21. Find the values of \( x, y \) if 1.5, 2, \( x, y \) are in G.P. Write down an expression for the 10th term and, using logarithms, find its value to 2 figures.

22. The second and fifth terms of a G.P. are 8 and 27. Find the first term and write down the 12th term in index form; evaluate it correct to 2 figures.

23. If 1.2, \( x, 7 \cdot 5 \) are in G.P., find the possible values of \( x \) and calculate the 4th term of the G.P. in each case.

24. State in index form the last term of the following series in G.P.
   (i) 0.5, 2, 8, . . ., to 9 terms; (ii) 3, -1, \( \frac{1}{3} \), . . ., to 10 terms.
ALGEBRA

[25] Find the sum of the G.P., $18 + 12 + 8 + \ldots + \frac{12^8}{3^8}$.
[26] Find the sum of the G.P., $8 - 4 + 2 - 1 + \ldots - \frac{2^{10}}{16}$.

[27] How many terms of the G.P., 6, 12, 24, $\ldots$ are less than 1000?

[28] Evaluate $1 + 1.03 + 1.03^2 + \ldots + 1.03^{20}$, correct to 3 figures.
[log $1.03 = 0.01284$.]

[29] Evaluate $\frac{1}{1.04} + \frac{1}{1.04^2} + \ldots + \frac{1}{1.04^{18}}$, correct to 3 figures.
[log $1.04 = 0.017033$.]

[30] How many terms of the G.P., 2, 3, 4, $\ldots$ must be taken to obtain a sum exceeding 10,000?

GEOMETRY

Geometrical facts are best remembered by drawing or visualising diagrams as in the following summaries. As far as possible, the data are shown by marking the figures. Arrows indicate that lines are given parallel.

ANGLES

1.

2.

3.

4.

5.

6.

7.

8.

9.

Fig. 2.

Fig. 3.

Fig. 4.

Fig. 5.

Fig. 6.

Fig. 7.

Fig. 8.

Fig. 9.

\[a + b = 2 \text{ rt. } \angle s.\]

If \(c + d = 2 \text{ rt. } \angle s\), \(a = c\), vert. opp. \(\angle s\).

POQ is st. line.

\(b = d.\)

\(a = b\), alt. \(\angle s.\)

\(p = q\), corr. \(\angle s.\)

\(p + r = 2 \text{ rt. } \angle s\), int. \(\angle s.\)

The converses of Nos. 4–6 give tests for lines to be parallel.

(i) \(p = a + b\), ext. \(\angle.\)

(ii) \(a + b + c = 2 \text{ rt. } \angle s.\)

(i) \(n\) sides; angle-sum = \(2n - 4\) rt. \(\angle s.\)

(ii) sum of ext. \(\angle s = 4 \text{ rt. } \angle s.\)
EXERCISE 26 (Numerical)

1. In \( \triangle ABC \), \( \angle C = 80^\circ \), \( \angle A = 34^\circ \); \( BC \) is produced to \( D \) so that \( BD = BA \). Calculate \( \angle ABC \), \( \angle CAD \).

2. In \( \triangle ABC \), \( AB = AC \) and \( \angle BAC \) is obtuse; \( CA \) is produced to \( D \) so that \( BD = BA \); \( \angle DBC = 108^\circ \). Calculate \( \angle BAC \).

3. In \( \triangle ABC \), \( AB = AC \) and \( BE \) is the perpendicular from \( B \) to \( AC \); \( \angle EBC = 28^\circ \). Calculate \( \angle BAC \).

4. The angles of a triangle are \( 3x - 4 \), \( 6x + 2 \), \( 7x - 10 \), degrees. Find \( x \) and prove the triangle is isosceles.

5. In fig. 12, \( ABPQ \) is a straight line; \( PQ = PR \) and \( CD \) is parallel to \( PR \). Calculate \( \angle ABC \).

6. In \( \triangle ABC \), \( \angle B = 56^\circ \), \( \angle C = 48^\circ \); the perpendiculars \( BE, CF \) from \( B, C \) to \( AC \), \( AB \) meet at \( H \). (i) Calculate \( \angle BHC \). (ii) Prove that the lines bisecting \( \angle BAC \), \( \angle BHC \) are parallel.

7. In \( \triangle ABC \), \( AB = AC \) and \( \angle A = 48^\circ \). The line which bisects \( \angle ABC \) meets the external bisector of \( \angle BAC \) at \( D \). Calculate the angles of \( \angle ADB \) and prove that \( AD = AC \).

8. \( ABCD \) is a quadrilateral inscribed in a circle, centre \( O \); \( \angle AOB = 110^\circ \), \( \angle COD = 50^\circ \), \( \angle ABC = 80^\circ \). Calculate \( \angle BOC \), \( \angle BCD \), \( \angle CDA \).

9. Four of the angles of a pentagon are \( 72^\circ \), \( 114^\circ \), \( 125^\circ \), \( 150^\circ \). Find the remaining angle.

EXERCISE 27 (General)

1. In \( \triangle ABC \), \( CA = CB \) and \( \angle ACB \) is obtuse; \( AC \) is produced to \( K \) so that \( BK = BC \). Prove \( \angle BKA = 2 \angle BAK \).

2. \( M \) is the mid-point of the side \( AB \) of \( \triangle ABC \). If \( MC = MA \), prove \( \angle ACB \) is a right angle.

3. \( K \) is a point inside \( \triangle ABC \). Prove that \( \angle BKC - \angle BAC = \angle ABK + \angle ACK \).

4. \( ABCDE \) is a regular pentagon; \( AB \) is produced to \( K \). Prove that \( BC \) bisects \( \angle EBK \).

5. \( ABCD \) is a quadrilateral. If \( \angle A = \angle C \) and if \( AB \) is parallel to \( DC \), prove that \( AD \) is parallel to \( BC \).

6. \( ABCD \) is a square; \( N \) is a point on \( AC \) such that \( AN = AB \); \( DN \) is produced to meet \( BC \) at \( K \). (i) Prove that \( CK = CN \). (ii) Calculate \( \angle CDK \) and prove that \( DK \) bisects \( \angle CDB \).

7. If the altitudes \( BE, CF \) of the triangle \( ABC \) meet at \( H \), prove that the bisectors of \( \angle BAC \), \( \angle BHC \) are parallel.

8. \( M \) is the mid-point of the side \( AB \) of a parallelogram \( ABCD \). If \( MC \) bisects \( \angle BCD \), prove that \( MD \) bisects \( \angle ADC \).

9. If \( O \) is a point inside a triangle \( ABC \), prove that \( \angle BOC > \angle BAC \).

10. \( CN \) is the perpendicular from \( C \) to a straight line \( ANPB \). Prove that \( CN < CP \).
CONGRUENT TRIANGLES

There are three general tests for congruence:
(i) SAS or 2 sides, included angle;
(ii) ASA, AAS or 2 angles, corresponding side;
(iii) SSS or 3 sides.

Two triangles, which agree as regards the lengths of two pairs of sides and a not-included angle, need not be congruent (ASS), see fig. 13.

If the not-included angle is a right angle, the triangles are congruent:
(iv) RHS or rt. \(\angle\), hypotenuse, side.

In naming two congruent triangles, write the vertices in orders which correspond:

The statement, \(\triangle ABC \cong \triangle QKN\)
\[\angle A = \angle Q, \; \angle B = \angle K, \; \angle C = \angle N\]
and
\[AB = QK, \; BC = KN, \; CA = NQ.\]

EXERCISE 28

1. Two circles, each with centre O, cut the lines OAH, OBK at A, B and H, K; AK cuts BH at E, see fig. 14. Prove
   (i) \(\triangle OAK \cong \triangle OBH\); (ii) \(AH = BK\);
   (iii) \(\triangle AEH \cong \triangle BEK\);  
   (iv) OE bisects \(\angle AOB\).

2. \(\triangle ABC\) is any triangle; \(\triangle ABX, \triangle ACY\) are equilateral triangles outside \(\triangle ABC\). Prove
   \(BY = CX\).

[3] \(H, K\) are points on the perpendicular bisector of a line \(AB\).
Prove that the triangles \(HAK, HBK\) are congruent.

4. \(ABCD\) is a square; \(X, Y, Z\) are points on \(AB, BC, CD\) such that \(BX = CY\) and \(CZ = BY\). Prove
   (i) \(XY = YZ\); (ii) \(\angle XYZ = 90^\circ\);
   (iii) \(\angle YXZ = 45^\circ\).

5. In fig. 15, \(P, Q, R\) are points on the sides of \(\triangle ABC\) such that \(BR = PC\) and \(QC = PB\); also \(AB = AC\). Prove
   (i) \(PQ = PR\), (ii) \(\angle RPQ = \angle B\).

6. In fig. 16, the bisector of \(\angle BAC\) meets at \(P\) the line which
   bisects \(BC\) at right angles; \(PX, PY\) are the perpendiculars from \(P\) to
   \(AB, AC\). Find in the figure three pairs of congruent triangles and
   prove the congruence. Prove also that \(AX = AY = \frac{1}{2}(AB + AC)\).

7. \(ABCD\) is a parallelogram; \(BCHK, CDMN\) are squares outside the
   parallelogram. Prove that (i) \(\angle HCN = \angle ABC\), (ii) \(HN = AC\).

8. The triangle \(ABC\) is right-angled at \(A; X\) is the point in \(BC\)
   such that \(BX = AC\). The line from \(X\) drawn outside \(\triangle ABC\)
   perpendicular to \(BC\) cuts the circle, centre \(B\), radius \(BC\), at \(Y\) and
   \(BY\) is joined. Prove that (i) the two triangles in the figure are
   congruent; (ii) \(BY\) is parallel to \(AC\).

9. In \(\triangle ABC\), \(\angle BAC = 90^\circ\), \(\angle BCA = 60^\circ\); the bisector of \(\angle ACB\)
   cuts \(AB\) at \(X\); \(XY\) is the perpendicular from \(X\) to \(BC\). Prove
   (i) \(XY = YC\); (ii) \(BX = 2XA\).

[10] In \(\triangle ABC\), \(AB = AC\) and \(D\) is the mid-point of \(AB\). \(AE\) is
drawn parallel to \(CB\), and the line through \(D\) perpendicular to \(BC\)
meets \(BC, CE, CA\) produced at \(H, N, K\) respectively. Prove (i) the
triangles \(\triangle AND, \triangle ANK\) are congruent; (ii) \(KD = 2DH\).

11. \(M\) is the mid-point of the side \(BC\) of a triangle \(ABC\). Prove
    that \(AM < \frac{1}{2}(AB + AC)\). [Produce \(AM\) to \(D\) so that \(AM = MD\);
    join \(BD\).]
PARALLELOGRAMS AND INTERCEPTS

1. Fig. 17.
(i) \( AB = CD \), \( AD = BC \).
(ii) \( \angle A = \angle C \), \( \angle B = \angle D \).

2. Fig. 18.
(i) \( AK = KC \), \( BK = KD \).
(ii) \( AC \) bisects \( ABCD \).

3. Test for a parallelogram.
If \( AB = CD \) and \( AB \parallel DC \), \( ABCD \) is a parallelogram.

4. Fig. 19.

5. Fig. 20.
**Rectangle**
\( AC = BD \).

6. Fig. 21.
**Rhombus**
\( \angle AKB = 1 \text{ rt. } \angle \).

7. Fig. 22.
**Square**
\( AC = BD \),
\( \angle AKB = 1 \text{ rt. } \angle \).

8. Fig. 23.
(i) \( HK \) is parallel to \( BC \).
(ii) \( HK = \frac{1}{2} BC \).

9. Fig. 24.
(i) \( AK = KC \).
(ii) \( HK = \frac{1}{2} BC \).

10. Fig. 25.
If \( BC = CD \) and \( BE \), \( CR, DS \) are parallel, then \( QR = RS \).

11. Fig. 26.
(i) \( AG \) produced bisects \( BC \).
(ii) \( DG = \frac{1}{3} DA \).

EXERCISE 29

1. State, without proof, the name of a quadrilateral in which
   (i) the diagonals bisect each other,
   (ii) the diagonals bisect each other at right angles,
   (iii) the diagonals are equal and bisect each other.

2. \( ABCD \) is a quadrilateral. If \( AB = DC \) and \( AD = BC \), prove that \( ABCD \) is a parallelogram.

3. \( ACB \) and \( COD \) are diameters of a circle. Prove that \( ACBD \) is a rectangle.

4. \( ABCD \) is a parallelogram; \( AP \), \( AQ \) are the perpendiculars from \( A \) to \( BC, CD \). If \( AP = AQ \), prove that \( ABCD \) is a rhombus.

5. In fig. 27, \( ABCD \) is a parallelogram. Parallel lines \( AMN \), \( CPQ \) cut the sides of \( ABCD \), produced where necessary, at \( M, N \) and \( P, Q \). Prove that \( MN = PQ \).

6. In fig. 28, \( AP \) is equal and parallel to \( BC \), and \( BQ \) is equal and parallel to \( AD \). Prove that \( PQ \) and \( CD \) bisect each other.

7. \( AB, DC \) are the parallel sides of a trapezium \( ABCD \), and \( M \) is the mid-point of \( AC \); \( BM \) and \( CD \) meet, when produced, at \( E \). Prove that \( AE = BC \).
GEOMETRY

[8] ABCD is a square; AB is produced to E. The bisector of ∠DBE meets AC produced at P. Prove that CP = CB.

9. ABCD is a parallelogram; AD is produced to E. Prove that the bisector of ∠EDC is perpendicular to the bisector of ∠ABC.

[10] D is the mid-point of the side BC of ∆ABC; parallel lines BH, CK meet AD, produced where necessary, at H, K. Prove that BK = CH.

11. H, K are the mid-points of the sides AB, AC of a triangle ABC; CP, drawn parallel to BA, meets HK produced at P. Prove that (i) ∆CPK = ∆AKH, (ii) PHBK is a parallelogram, (iii) HK = EB.

12. In fig. 29, △BGE and △CGF are straight lines and E, F, G are the mid-points of AC, AB, AH. Prove that (i) BGEH is a parallelogram, (ii) AH bisects BC, (iii) EG = 1/2 GB and EG = 1/2 EB.

[13] P, Q are the mid-points of the sides AB, DC respectively of the parallelogram ABCD; PD, BQ cut AC at H, K respectively. (i) Explain why PBQD is a parallelogram. (ii) Prove that AH = HK = KC.

14. P, Q, R, S are the mid-points of the sides AB, BC, CD, DA of a quadrilateral ABCD. Prove that PQRS is a parallelogram. What can you say about PQRS if AC = BD?

15. The side AB of a triangle ABC is produced to D so that AB = BD; E is a point on CA such that CE = 1/3 CA; M is the mid-point of DE. Prove that BM = EC. Hence prove that DE bisects BC.

USE OF INSTRUMENTS

EXERCISE 30

1. Draw the triangle ABC given AB = 2.5 in., BC = 2.5 in., CA = 1.6 in. Using ruler and compasses only, construct the perpendicular from A to BC and measure its length.

2. Draw the triangle ABC given AB = 6 cm., BC = 4 cm., CA = 5 cm. Construct a point P in AB which is equidistant from B and C, and a point Q in AC which is equidistant from AB and BC. Construct also a point O which is equidistant from A, B, C and draw the circle which passes through A, B, C.

[8] Draw the triangle ABC given AB = 8 cm., BC = 5.6 cm., ∠ABC = 40°. Construct a point P inside the triangle at distance 4 cm. from A and 1.6 cm. from BC.

4. Draw as many non-congruent triangles as possible in which the lengths of two of the sides are 3 in., 2.2 in., and one angle is 42°. Measure the remaining side in each case.

[8] Construct a triangle ABC in which AB = 3 in., ∠C = 66°, and the length of the perpendicular from A to BC is 2 in. Measure BC.

6. Construct in one figure two triangles ABC having AB = 6 cm., BC = 4 cm., ∠ABC = 30°. Construct the perpendicular from B to AC and, making any measurements, calculate the difference between the areas of the two triangles.

7. Using ruler and compasses only, construct a parallelogram whose diagonals are of lengths 12 cm., 8 cm. and include an angle 45°. Measure the lengths of the sides.

8. Construct a parallelogram ABCD having AB = 2 in., AC = 2.6 in., BD = 3.6 in. Measure AD.

9. Construct a rhombus ABCD, having AC = 5 cm., BD = 8 cm. Measure AB.

10. Using ruler and compasses only, inscribe a regular hexagon in a circle of radius 4 cm. Measure the distance between two opposite sides and find the area of the hexagon.

11. Construct a plan, scale 1 in. to 100 feet, of a field ABCD from the following data: The bearings from A of B, C, D are north, north-west, west respectively and B bears north-east from D; AD = 250 feet, BC = 280 feet. Find in feet the length of the side CD. Mark on the plan the position of a pump P which is equidistant from B, C, D.

12. Construct a trapezium ABCD in which AB is parallel to DC and AB = 6.8 cm., BC = 2.8 cm., CD = 3.6 cm., DA = 2.4 cm. Measure BD.

13. ABCD is the base of a rectangular block; AP, BQ, CR, DS are the edges of the block; AB = 6 cm., AD = 5 cm., AP = 4 cm. Find by drawing and measurement the length of the diagonal AR and the angle which AR makes with the base ABCD.
GEOMETRY

14. The base of a right pyramid, vertex V, is the equilateral triangle ABC; \( AB = 6 \text{ cm.}, VA = 8 \text{ cm.}; \) VN is the perpendicular from V to the base ABC. Find by drawing and measurement the lengths of AN and VN.

15. V is the vertex of a pyramid standing on a square base ABCD, side 6 cm.; the four sloping faces are equilateral triangles. P, Q are points on VC, VD such that VP = VQ = 2 cm. Find by drawing and measurement the lengths of BP, PQ, QA and then make an accurate drawing of the cross-section APQB and hence find the distance of P from AB.

AREAS

1. Area of \( \square ABCD \\)
   \[ = AB \times PN \]
   \[ = \text{base} \times \text{height}. \]

2. Area of \( \triangle ABC \)
   \[ = \frac{1}{2} BC \times AD \]
   \[ = \frac{1}{2} \text{base} \times \text{height}. \]

3. If, in fig. 32, AD is parallel to BC, \( \triangle ABC = \triangle DBC. \)

4. If, in fig. 32, \( \triangle ABC = \triangle DBC, \) AD is parallel to BC.

5. If in fig. 33, AQP is parallel to BC,
   \( \triangle ABC = \frac{1}{2} \square PQBC. \)

EXERCISE 31

1. Calculate the area of a trapezium, given that the lengths of its parallel sides are 6 in., 1.5 in., and that the distance between them is 4 in.

2. The dimensions of fig. 34 are shown in cm. (see Exercise 32, No. 5):
   (i) Find the areas of \( \square ABCD, \triangle PBQ, \) trapezium ADPQ.
   (ii) Find the lengths of the perpendiculars from B to AD, from Q to BP, from C to BP.

3. In fig. 35, if AD is parallel to BC, name a triangle equal to \( \triangle ABC \) and prove that \( \triangle AKB = \triangle DKC. \)

4. In fig. 35, prove that \( \frac{\triangle AKD}{\triangle KDC} = \frac{AK}{KC}. \)

5. If fig. 35, if AK = KC, prove that \( \triangle ABD = \triangle GBD. \)

6. In fig. 36, ABP is a straight line and CP is parallel to DB. Prove that quad. ABCD = \( \triangle APD. \)

7. The area of \( \triangle ABC \) is 4.8 sq. in.; D is the mid-point of BC and N is a point on AC such that AN = \( \frac{1}{3}AC. \) Find the areas of \( \triangle ABD \) and \( \triangle DNC. \)

8. Construct a quadrilateral ABCD in which \( AB = BC = CA = 5 \text{ cm., } CD = 4 \text{ cm., } \angle ACD = 90^\circ. \)
   Construct a triangle having AB as one side and equal in area to quad. ABCD. By making any measurements, find the area of ABCD.
[9] P is any point on the median AD of a triangle ABC. Prove that $\triangle ABP = \triangle APC$.

[10] ABCD is a parallelogram; P, Q are points on AB produced, AD produced, respectively. Prove that $\triangle PCD = \triangle QBC$.

11. Three parallel lines AP, BQ, CR cut one line at A, B, C and another line at P, Q, R. Prove that
   (i) $\triangle AQC = \triangle BQR$,
   (ii) $\triangle ARB = \triangle PCQ$.

12. AB, BC are the parallel sides of the trapezium ABCD and $AB = 3CD$; H is the mid-point of AD, and HPK is drawn parallel to AB to meet AC, BC at P, K. Prove that (i) $\triangle ABC = 3\triangle ADK$,
   (ii) $\triangle ADC = 4\triangle AHP$,
   (iii) $\triangle PKC = \text{quad. HPICD}$.

RIGHT-ANGLES, CHORDS AND TANGENTS

1. Pythagoras' theorem.
   If $\angle BAC = 1 \text{ rt. } \angle$, $BC^2 = BA^2 + AC^2$.

2. If O is the centre of the circle in fig. 37, $\angle OMA = 1 \text{ rt. } \angle$.

3. If O is the centre of the circle in fig. 38, CN = ND.

![Fig. 37.](image1)
![Fig. 38.](image2)
![Fig. 39.](image3)

4. If, in fig. 39, $AB = CD$ and O is the centre, OH = OK.

5. If BAC touches a circle, centre O, at A, $\angle OAB = 1 \text{ rt. } \angle$.

![Fig. 40.](image4)
![Fig. 41.](image5)

6. If TP, TQ touch a circle, centre O, at P, Q,
   (i) $TP = TQ$;
   (ii) OT bisects $\angle PTQ$.

RIGHT-ANGLES, CHORDS AND TANGENTS

EXERCISE 32

1. ABCD is a square, side 3 in.; $AB$ is produced to $P$ so that $BP = 1$ in. Find (i) the area of $\triangle PCD$, (ii) the length of the perpendicular from C to PD.

2. AD, BC are the parallel sides of a trapezium ABCD; $AB = 5$ cm, $BC = 9$ cm, CD = 4 cm, $\angle BCD = 1 \text{ rt. } \angle$. Find the length of AD and the area of ABCD. (Two cases.)

3. The lengths of the diagonals of a rhombus are 9 in., 12 in.; find the length of a side of the rhombus.

4. In $\triangle ABC, AB = 6$ cm, $AC = 8$ cm, $\angle BAC = 1 \text{ rt. } \angle$, and AD is the perpendicular from A to BC. Calculate the lengths of $BD, DC, AD$.

   If BCPQ is the square on BC outside $\triangle ABC$, calculate the length of the perpendicular from Q to AC.

5. In fig. 34, p. 75, show that it can be proved that the length of OP is 7 units, given the other dimensions in the diagram.

6. In the triangle ABC, AB > AC and AD is the perpendicular from A to BC; M is the mid-point of BC. Prove that $AB^2 = AC^2 - BD^2 = BC^2 - 2BC \cdot MD$.

7. F, G, H are points on the sides AB, BC, CD of a square ABCD, such that $AF = BG = CH$. Prove that $FH^2 = 2BG^2 + 2GB^2$.

8. The triangle ABC is right-angled at A; BCPQ, CANM, ABHK are squares drawn outside the triangle; E is any point on PQ. Prove that (i) $\angle HAM$ is a straight line, (ii) the quadrilaterals $BCHM, ABEC$ are equal in area.

9. The radius of a circle, centre O, is 10 cm. Find (i) the distance from O of a chord of length 12 cm, (ii) the length of a chord whose distance from O is 6 cm.

10. ABC is a triangle such that $AB = AC = 5$ cm, $BC = 6$ cm. Construct the circle which passes through A, B, C and measure its radius. Find also the radius of the circle by calculation.

11. ABC is a given triangle. State the locus of the centres of the circles (i) passing through B and C, (ii) touching AB, AC, produced each way, (iii) touching AB at A, (iv) of radius 1 inch and passing through A, (v) of radius 1 inch and touching BC.
12. AB is a chord of length 6 cm. of a circle, centre O, radius 5 cm., AB is produced to T so that BT = 2 cm. Calculate (i) the distance of O from AB, (ii) the length of OT, (iii) the length of the tangent from T to the circle.

Draw the figure accurately and construct the tangents from T to the circle and measure their lengths.

[13] PQ is a variable chord of a given circle, centre O, radius 10 cm. If PQ = 12 cm., find the locus of the mid-point of PQ.

14. Two circles, centres A, B, cut at C, D; the line HCK parallel to AB cuts the circles again at H, K. Prove that HK = 2AB.

15. OA, OB are perpendicular radii of a circle and AP, BQ are parallel chords. If ON is the perpendicular from O to AP, prove that BQ = 2ON.

16. The base of a tetrahedron is an equilateral triangle ABC of side 6 in., see fig. 42. The edges VA, VB, VC are each 4 in. long; VO is the perpendicular from V to the base ABC. Calculate the lengths of OA and VO.

17. ABCD is a quadrilateral whose sides touch a circle, centre O, at P, Q, R, S and enclose it. Prove that

(i) $AB + CD = AD + BC$,
(ii) $\angle AOB + \angle COD = 2\text{ rt. } \angle s$.

[18] The circle inscribed in a triangle ABC touches BC at X. If $\angle BAC$ is a right angle, prove that (i) $AB - BX$ equals the radius of the circle, (ii) $AB + AC - BC$ equals the diameter.

**ANGLE PROPERTIES OF A CIRCLE**

1. If, in fig. 43, O is the centre of the circle APQB, $\angle x = 2\angle p$ and $\angle x = 2\angle q$;  (ii) $\angle p = \angle q$.

[Fig. 43]

2. If, in fig. 44, CD is a diameter of the circle CPD,

$\angle CPD = 1\text{ rt. } \angle$.

[Fig. 44]

3. If, in fig. 45, ABCD is a cyclic quadrilateral,

$\angle b + \angle d = 2\text{ rt. } \angle s$.

[Fig. 45]

4. If, in fig. 46, the side AD of a cyclic quadrilateral is produced to E.

$\angle p = \angle b$.

[Fig. 46]

**EXERCISE 33**

[1] ABCD is a cyclic quadrilateral. If $\angle ABD = 32^\circ$, $\angle DBC = 50^\circ$, $\angle BAC = 44^\circ$, calculate the angles of ABCD.

2. ABCD is a quadrilateral inscribed in a circle, centre O; the sides AB, BC are produced to meet E; $\angle BCE = 80^\circ$, $\angle CEB = 25^\circ$, $\angle AOB = 84^\circ$. Calculate $\angle COD$.

3. In fig. 47, O is the centre of the circle; $\angle ABC = 100^\circ$. Calculate $\angle AOC$.

[Fig. 47]

4. In fig. 48, AD = DN = NC and $\angle ABD = 35^\circ$. Calculate $\angle ADB$.

[Fig. 48]

5. In fig. 49, AE is a diameter of the circle ABCDE; AB = BC and AD is parallel to BC; $\angle AEC = 40^\circ$. Calculate the angles of the triangle CDE.

[Fig. 49]
6. If, in fig. 50, \( \angle AED = 36^\circ \) and \( \angle AFB = 28^\circ \); calculate \( \angle BCE \).

7. If, in fig. 50, the points B, D, F, E lie on a circle, prove that AC is a diameter of the circle ABCD.

8. APB is a minor arc of a circle, centre O. Prove that \( \angle APB = \angle OAB = 1 \) rt. \( \angle \).

9. The side CD of a cyclic quadrilateral ABCD is produced to E. Prove that the triangle ABC is isosceles either if AD bisects \( \angle BDE \) or if BD bisects \( \angle ADC \).

10. In fig. 51, the centre O of the circle ABC lies on AD, and CD is parallel to AB. Prove that \( \angle AOB + \angle ADC = 1 \) rt. \( \angle \).

11. In fig. 52, O is the centre of the circle ABDC and AD cuts BC at N. Prove that \( \angle AOB + \angle COD = 2\angle ANB \).

12. In fig. 53, the lines AHK, BPQ are drawn to cut the circles AHPB, AKQB.
   (i) Prove that HP is parallel to KQ.
   (ii) If AP, AQ, BH, BK are joined, prove that \( \angle PAQ = \angle HBK \).

**TANGENTS**

1. If, in fig. 54, BAC touches the circle APDQ,
   \( \angle DAC = \angle APD \) and \( \angle DAB = \angle AQD \)

2. If two circles, centres A, B, touch at P, A, B, P lie on a straight line.

   ![External contact Internal contact](Fig. 55)

   If the contact is external, \( AB = \text{sum of radii} \); if the contact is internal, \( AB = \text{difference of radii} \).

**EXERCISE 34**

1. P is a point on the major arc BC of a circle; the tangents at B, C meet at T. If \( \angle BPC = 65^\circ \), find \( \angle BTC \).

2. The tangent AT at a point A on a circle makes an angle 36° with a chord AB; D is a point on the minor arc AB such that DA = DB. Calculate \( \angle ABD \).

3. In fig. 56, AB is a diameter of the circle APQB; the tangent at P meets AQ produced at T. Calculate \( \angle APQ \) and \( \angle ATP \).

![Fig. 56](Fig. 56)

4. In fig. 57, TPK touches the circle PQR and TSQ is a straight line; PQ = QR. Calculate \( \angle PQR \) and \( \angle QRS \).

5. AB is a diameter of the circle APQB; AQ cuts BP at N, and the tangent at P cuts BA produced at T. If \( \angle ANP = 48^\circ \), calculate \( \angle TPQ \).

6. The tangents at B, C to the circle ABC are parallel to AC, AB respectively. Prove that the triangle ABC is equilateral.

7. The sides PQ, SR of a cyclic quadrilateral PQRS meet when produced at T. Prove that the tangent at T to the circle TQR is parallel to PS.

![Fig. 57](Fig. 57)
8. HK is a common tangent to the circles ABH, ABK. Prove that $\angle HAK + \angle HBK = 2 \text{ rt.} \angle s$.

9. Two circles, radii 4.5 cm., 8 cm., touch externally. Calculate the length of a direct common tangent.

10. In fig. 58, AB, AC are diameters of the circles ABH, ACK which touch at A. The tangent at A meets the common tangent HK at T. Prove that (i) TH = TK, (ii) the circle HAK touches BC at A.

![Fig. 58.](image)

![Fig. 59.](image)

11. AOB is a diameter of a circle, centre O, and N is the midpoint of OA, see fig. 59. A circle is drawn to touch the given circle internally and to touch OA at N. Prove that the radius of this circle is $\frac{ON}{2}$.

12. OA is a radius of a circle, centre O, radius 3 cm.; B is a point such that $\angle OAB = 150^{\circ}$ and AB = 4 cm. Construct the circle which touches the given circle at A and passes through B and measure its radius.

**TESTS FOR CONCYCLIC POINTS AND TANGENCY**

1. If, in fig. 60, $\angle x = \angle y$, the points A, P, Q, B lie on a circle.

![Fig. 60.](image)

![Fig. 61.](image)

2. If, in fig. 60, $\angle x + \angle z = 2 \text{ rt.} \angle s$, the points A, P, B, R lie on a circle.

3. If, in fig. 61, $\angle p = \angle x$, AT touches the circle APB.

4. If the triangle ABC is right-angled at A, the circle on BC as diameter passes through A.

**EXERCISE 35**

1. The altitudes AD, BE of the triangle ABC intersect at H. Prove that (i) A, B, D, E are concyclic, (ii) C, D, H, E are concyclic.

2. In fig. 62, ABCD is a parallelogram; any circle through A, B cuts AD, BC at H, K and cuts BD, AC at P, Q. Prove that (i) C, D, H, K are concyclic, (ii) C, D, P, Q are concyclic.

![Fig. 62.](image)

![Fig. 63.](image)

3. In fig. 63, PT, QT are tangents to the circles PAB, QAB, and PAQ is a straight line. Prove that B, P, T, Q are concyclic.

4. ABCD is a quadrilateral such that AB = 7 cm., BC = 24 cm., CD = 15 cm., DA = 20 cm., and $\angle ABC = 90^{\circ}$. Calculate the length of AC and prove that ABCD is a cyclic quadrilateral.

5. ABC is a triangle inscribed in a circle. A line parallel to BC cuts AB at H and cuts the tangent CK at K. Prove that the points A, C, H, K are concyclic.

6. In fig. 64, AP, AQ are tangents to the circles ABQ, ABP; AB is produced to C. Prove that (i) BC bisects $\angle PBQ$, (ii) the centre of the circle APQ lies on the circle PBQ.

![Fig. 64.](image)

![Fig. 65.](image)

7. In fig. 65, the circles BHD, CKD touch at a point D on the base BC of a triangle ABC and cut AB, AC at H, K. Prove that the points A, H, D, K are concyclic.
[8] AT is the tangent at A to the circle ABC; H, K are points on the chord BC such that $\angle BAH = \angle CAK$. Prove that AT touches the circle AHK.

9. The tangent at A to a circle ABC is parallel to BC; a chord AP cuts BC or BC produced at Q. Prove that in either case AB touches the circle BQ.

**EQUAL RATIOS, PROPORTION AND SIMILAR TRIANGLES**

1. If $\frac{x}{a} = \frac{y}{b}$, then $\frac{x}{y} = \frac{a}{b}$. Each relation is equivalent to $bx = ay$.

2. Two numbers x, y in the given ratio $a : b$ can be expressed in the form $ak, bk$.

3. If $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$, the numbers x, y, z are said to be proportional to a, b, c. If each of these equal ratios is denoted by k, then $x = ak, y = bk, z = ck$.

**Example 1.** If $\frac{a}{b} = \frac{c}{d}$, prove $\frac{a}{a+b} = \frac{c}{c+d}$.

Let $\frac{a}{b} = \frac{c}{d} = k$, then $a = bk, c = dk$.

\[
\frac{a}{a+b} = \frac{bk}{bk+b} = \frac{bk}{b(k+1)} = \frac{k}{k+1},
\]

and

\[
\frac{c}{c+d} = \frac{dk}{dk+d} = \frac{dk}{d(k+1)} = \frac{k}{k+1}.
\]

\[
\therefore \frac{a}{a+b} = \frac{c}{c+d}.
\]

**Example 2.** If, in fig. 66, $\frac{AP}{PB} = \frac{AQ}{QC}$, prove that $\frac{AP}{AB} = \frac{AQ}{AC}$.

Let $AP = a, PB = b, AQ = c, QC = d$; then it is given that $\frac{a}{b} = \frac{c}{d}$, and so, by Example 1, $\frac{a}{a+b} = \frac{c}{c+d}$, that is, $\frac{AP}{AP} = \frac{AQ}{AC}$.

**EQUAL RATIOS AND SIMILAR TRIANGLES**

4. If, in fig. 67, a line parallel to BC cuts AB, AC, produced if necessary, at H, K,

\[
\frac{AH}{HB} = \frac{AK}{KC}.
\]

If $\frac{AH}{HB} = \frac{AK}{KC}$, K is called the fourth proportional to AH, HB, AK.

5. The statement that $\triangle ABC \sim \triangle XYZ$ means that $\angle A = \angle X, \angle B = \angle Y, \angle C = \angle Z$

and that $\frac{BC}{CA} = \frac{AB}{XY}$.

There are three general tests for similarity:

- $\triangle ABC \sim \triangle XYZ$ are similar
- if (i) $\angle B = \angle Y$ and $\angle C = \angle Z$, equiangular triangles;
- or if (ii) $\frac{BC}{CA} = \frac{AB}{XY}$, sides proportional;
- or if (iii) $\angle A = \angle X$ and $\frac{AB}{AC} = \frac{AC}{XY}$, ratio of 2 sides, inc. $\angle$.

6. If $\triangle ABC \sim \triangle XYZ$ are similar,

\[
\frac{\triangle ABC}{\triangle XYZ} = \frac{BC}{YZ}.
\]
1. If \( \frac{a}{b} = \frac{c}{d} \), prove that \( \frac{a+c}{b+d} = \frac{a-c}{b-d} \).

2. If \( \frac{a}{b} = \frac{c}{f} \), prove that \( \frac{pa + qc + re}{pb + qd + rf} = \frac{a}{b} \).

3. If \( \frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k \), prove that:
   (i) \( c = dk, b = dk^3, a = dk^5 \);
   (ii) \( \frac{a-b}{b-c} = \frac{b-c}{c-d} \).

4. If \( \frac{a}{b} = \frac{c}{d} \), state ratios equal to
   (i) \( b : a \);
   (ii) \( c : a \);
   (iii) \( a + b : b \);
   (iv) \( d : (b - d) \).

5. In fig. 69, arrows indicate that lines are given parallel; calculate the values of \( a \), \( b \), \( c \), \( d \).

6. In fig. 70, arrows indicate that lines are given parallel; calculate the values of \( i \), \( j \), \( k \), \( l \).

7. In fig. 71, \( PQBC \) is a parallelogram and \( RBA \) cuts \( PQ \), \( PR \) produced at \( R \), \( A \); \( BC = 5 \text{ cm.}, \ CA = 6 \text{ cm.}, AB = 4 \text{ cm.}, PR = 8 \text{ cm.} \). Calculate (i) the lengths of \( BR, BQ \), (ii) the ratios of the areas of \( \triangle ABC, \triangle BRQ \), |gram \( PQBC \).
   Prove that \( CQ \) is not parallel to \( AR \).

8. In fig. 72, \( ABCD \) is a parallelogram; \( BHK \) cuts \( AC \), \( CD \) produced at \( H, K \); \( DH \) is produced to cut \( AB \) at \( N \); \( CD = 2DK \). (i) Find the value of the ratio \( \frac{BH}{HK} \).
   (ii) If \( DK = x \text{ in.} \), find the length of \( BN \) and the value of the ratio \( \frac{BN}{NA} \).

9. \( ABCD \) is a cyclic quadrilateral; \( AC \) cuts \( BD \) at \( E \); \( AB \) and \( DC \) meet, when produced at \( F \).
   (i) Name with reasons a triangle similar to \( \triangle ACF \) and a triangle similar to \( \triangle AEB \).
   (ii) Write down two ratios equal to \( \frac{FB}{FC} \) and two ratios equal to \( \frac{AB}{CD} \).

10. Draw a triangle \( ABC \) in which \( AC = 4 \text{ cm.}, CB = 6 \text{ cm.}, \angle ABC = 1 \text{ rt.} \angle \). Construct points \( P, Q \) which divide \( AB \) internally and externally in the ratio \( 5 : 2 \). Construct also the fourth proportional to the lines \( AC, CB, BA \), and measure its length.

11. Draw an equilateral triangle \( ABC \), side \( 4 \text{ cm.} \), and construct a point \( D \) on \( BC \) so that \( BD : DC = 1 : 2 \). Construct also a line \( PDQ \) cutting \( AC \) at \( P \) and \( AB \) produced at \( Q \) so that \( PD : DQ = 2 : 3 \).

12. \( AB, DC \) are the parallel sides of a trapezium \( ABCD \); \( AC \) cuts \( BD \) at \( E \); \( AB = 12 \text{ in.}, CD = 8 \text{ in.} \).
   (i) Calculate the values of the ratios \( \frac{AE}{BE} \), \( \frac{EC}{BD} \).
   (ii) If the area of \( \triangle ABC \) is \( 50 \text{ sq. in.} \), find the areas of \( \triangle ABC, \triangle AEB, \triangle DEC, \triangle AED \).

13. \( AB \) is a diameter of the circle \( APQB \); \( PN \) is the perpendicular from \( P \) to \( AQ \). Prove that \( \frac{AP}{AB} = \frac{PN}{BC} \).

14. \( ABC \) is a triangle right-angled at \( A \); \( AD \) is the perpendicular from \( A \) to \( BC \). Prove that (i) the triangles \( ADC, BAC \) are similar,
   (ii) \( CA^2 = CD \cdot CB \) and \( BA^2 = BD \cdot BC \), (iii) \( AD^2 = BD \cdot DC \).

15. \( AB, DC \) are the parallel sides of the trapezium \( ABCD \). If \( BD^2 = AB \cdot DC \), prove that the triangles \( ABD, BDC \) are similar.
   Hence show that \( \frac{AD^2}{AB} = \frac{BC^2}{DC} \).

16. \( ABC \) is a triangle inscribed in a circle; the line bisecting \( \angle ABC \) meets \( BC \) at \( D \) and the circle at \( E \). Prove that (i) the triangles \( ABD, AEC \) are similar,
   (ii) \( AD \cdot AE = AB \cdot AC \). Complete the relation, \( AB \cdot EC = \ldots \).
RECTANGLE PROPERTIES OF INTERSECTING CHORDS AND TANGENTS

1. If, in fig. 73, the chords AB, CD of a circle intersect at X,
   \[XA \cdot XB = XC \cdot XD.\]

2. If, in fig. 74, the lines AB, CD intersect at a point X such that
   \[XA \cdot XB = XC \cdot XD,
   \]
   the points A, B, C, D are concyclic.

3. If, in fig. 75, the chord AB of a circle ABT meets at X the
tangent at T,
   \[XA \cdot XB = XT^2.\]

4. If, in fig. 76, XAB and XT are lines such that
   \[XA \cdot XB = XT^2,
   \]
   the circle ABT touches XT at T.

5. If APG is a semicircle and if PB is the perpendicular from P to AC,
   \[BP^2 = AB \cdot BC
   \]
   and
   \[AP^2 = AB \cdot AC.\]

   BP is called the mean proportional between AB and BC,
   AP is called the mean proportional between AB and AC.

   The square whose side is BP equals in area the rectangle whose
   adjacent sides are equal to AB and BC; and the square whose side
   is AP equals in area the rectangle whose adjacent sides are equal to
   AB and AC.

EXERCISE 37

1. In fig. 78, TX touches the circle PQRXS and TQP, TRS are
   straight lines; dimensions are shown in inches. Find the lengths
   of TX, TR, QR.

   Fig. 78.

2. In fig. 79, AB is a diameter of the circle APBQ; BQR, PAR
   are straight lines; dimensions are shown in cm. Find the lengths
   of AP, PB, AQ.

   [3] ABC is a straight line; AB = 9 cm., BC = 16 cm.; AT is the
tangent to a circle passing through B and C. Find the length of AT.

   [4] AB is a diameter of the circle ABP; BP produced meets the
tangent AT at T. If AB = 6 cm. and AT = 8 cm., find the length
   of BP.

   5. In \(\triangle ABC\), AB = 2.5 cm., BC = 4.5 cm., \(\angle ABC = 1\) rt. \(\angle\).
   A circle is drawn to touch BC at C and pass through A.
   If the circle cuts BA produced at P, calculate the length of BP
   and the radius of the circle.

   [6] In \(\triangle ABC\), AB = AC = 10 cm., BC = 16 cm.; the perpendicular
   AD from A to BC is produced to meet the circle ABC at E. Find
   the length of DE and the radius of the circle.

   7. Construct an equilateral triangle of side 8 cm. and then
   construct a square of equal area. Measure the side of the square.

   8. In fig. 80, prove that the common chord
   AB, when produced, bisects the common
   tangent HK of the two circles.

   If TP, TQ are the tangents from any point
   T on AB produced, prove that TP = TQ.

   [8] If two chords CD, EF of the circles
   ABP, ABQ intersect on AB, prove that the
   points C, D, E, F are concyclic.

   Fig. 80
GEOMETRY

10. In fig. 81, \(XAB, XDC\) are lines cutting the circle \(ABCD\); \(XY\) is drawn parallel to \(DB\) to cut \(CA\) produced at \(Y\). Prove that \(XY^2 = YA \cdot YC\).

![Fig. 81.](image)

11. In fig. 82, \(HK\) is a common tangent of the circles \(HAC, KADE\), which touch \(XA\) at \(A\); \(XCH, XDK, CDE\) are straight lines. Prove that (i) \(H, K, C, D\) are concyclic, (ii) \(EK\) is parallel to \(GH\).

![Fig. 82.](image)

ANGLE-BISECTORS OF A TRIANGLE

1. (i) If \(AD\) bisects \(\angle BAC\) internally and cuts \(BC\) at \(D\),
\[
\frac{BD}{DC} = \frac{BA}{AC}.
\]

(ii) If \(AD\) bisects \(\angle BAC\) externally and cuts \(BC\) produced at \(D\),
\[
\frac{BD}{DC} = \frac{BA}{AC}.
\]

![Fig. 83.](image)

2. (i) If \(D\) is a point on \(BC\) such that \(\frac{BD}{DC} = \frac{BA}{AC}\), \(AD\) is the internal bisector of \(\angle BAC\).

(ii) If \(D\) is a point on \(BC\) produced such that \(\frac{BD}{DC} = \frac{BA}{AC}\), \(AD\) is the external bisector of \(\angle BAC\).

![Fig. 84.](image)

ANGLE-BISECTORS OF A TRIANGLE

EXERCISE 38

1. \(ABC\) is a triangle in which \(BC = 8\) cm, \(CA = 3\) cm, \(AB = 7\) cm. If the bisectors of \(\angle BAC\) cut \(BC\) and \(BC\) produced at \(P, Q\), calculate the lengths of \(BP\) and \(PQ\).

If \(D\) is a point on \(AC\) such that \(AD = 14\) cm, prove that \(BD\) bisects \(\angle ABC\) internally.

2. In \(\triangle ABC\), \(AB = 7\) cm, and \(AC = 5\) cm. The internal and external bisectors of \(\angle ACB\) meet \(AB\) and \(AB\) produced at \(D\) and \(E\). Find the lengths of \(DB\) and \(BE\).

Prove that, if \(CD = 2.4\) cm, \(EC = EB\). [Use Pythagoras.]

3. In \(\triangle ABC\), \(AB = 5\) in., \(AC = 7\) in., and the bisector of \(\angle BAC\) cuts \(BC\) at \(D\). If the area of \(\triangle ABC\) is 6 sq. in., find the area of \(\triangle ABD\).

4. \(ABCD\) is a parallelogram. The line bisecting \(\angle BAD\) cuts \(DB\) at \(P\) and \(DC\) at \(Q\). Prove that \(AP : PQ = DC : DA\).

5. In fig. 84, \(AB = AD\) and the bisectors of \(\angle CAB, \angle CAD\) meet \(CB, CD\) at \(H, K\). Prove that \(HK\) is parallel to \(BD\).

![Fig. 84.](image)

6. In fig. 85, the circles, centres \(A, B, C\), touch at \(O\); \(PQR\) is parallel to \(AB\); \(QB, RB\) meet \(PA\), when produced, at \(M, N\). Prove that \(MO\) bisects \(\angle AMB\) and \(NO\) bisects \(\angle ANB\).

7. \(ABCD\) is a quadrilateral such that the lines which bisect \(\angle BAD\) and \(\angle BCD\) meet at a point on \(BD\). Prove that \(AB : CD = AD : BC\).

8. \(A, B\) are fixed points and \(P\) is a variable point such that \(AP : BP = 5 : 3\). If the internal and external bisectors of \(\angle APB\) cut \(AB\) and \(AB\) produced at \(H\) and \(K\), and if \(AB = 4\) cm, find the lengths of \(HB\) and \(BK\). Prove that \(\angle HPK\) is a right angle, and state precisely the locus of \(P\).

9. In \(\triangle ABC\), \(\angle ACB\) is a right angle and \(ON\) is an altitude. The bisector of \(\angle CAN\) cuts \(ON\) at \(H\) and the bisector of \(\angle BCO\) cuts \(BN\) at \(K\). Prove that \(HK\) is parallel to \(BC\).
TRIGONOMETRY

RIGHT-ANGLED TRIANGLES

Sine, cosine and tangent of an acute angle.

With the data of fig. 86,

\[
\sin \theta = \frac{\text{opp.}}{\text{hyp.}} = \frac{x}{z},
\]

\[
\cos \theta = \frac{\text{adj.}}{\text{hyp.}} = \frac{y}{z},
\]

\[
\tan \theta = \frac{\text{opp.}}{\text{adj.}} = \frac{x}{y}.
\]

Hence \(x = z \sin \theta, \ y = z \cos \theta, \ x = y \tan \theta.\)

Also

\[
\sin \theta = \frac{x}{z} = \frac{z \sin \theta}{z} = \tan \theta,
\]

\[
\cos \theta = \frac{y}{z} = \frac{z \cos \theta}{z} = \sin \theta.
\]

and

\[
\sin^2 \theta + \cos^2 \theta = \frac{x^2}{z^2} + \frac{y^2}{z^2} = \frac{x^2 + y^2}{z^2} = 1.
\]

The third angle of the triangle in fig. 86 is 90° - \(\theta;\)
hence

\[
\sin (90° - \theta) = \frac{y}{z} = \cos \theta,
\]

\[
\cos (90° - \theta) = \frac{x}{z} = \sin \theta,
\]

\[
\tan (90° - \theta) = \frac{y}{x} = \frac{1}{\tan \theta}.
\]

Angles 45°, 60°, 30°.

Fig. 87 shows an isosceles right-angled triangle.

\[
\sin 45° = \frac{1}{\sqrt{2}}, \ \cos 45° = \frac{1}{\sqrt{2}}, \ \tan 45° = 1.
\]

Fig. 87.

Fig. 88.

Fig. 89.

Fig. 90.

Fig. 91.

RIGHT-ANGLED TRIANGLES

Fig. 88 (i) shows half the equilateral triangle in fig. 88 (ii).

\[
\sin 60° = \frac{\sqrt{3}}{2}, \ \cos 60° = \frac{1}{2}, \ \tan 60° = \sqrt{3}.
\]

\[
\sin 30° = \frac{1}{2}, \ \cos 30° = \frac{\sqrt{3}}{2}, \ \tan 30° = \frac{1}{\sqrt{3}}.
\]

EXERCISE 39

1. In fig. 89, BCD is perpendicular to AB, find \(\angle BAC\) and \(\angle CAD.\)

2. In fig. 90, PR is perpendicular to QRS, find the lengths of QR, PR, RS.

3. In fig. 91, EH, NG are perpendicular to ENF, find the lengths of EF, EH.

4. In \(\triangle ABC, AB = AC = 4\) in. (i) Find BC if \(\angle BAC = 110°;\)
   (ii) find \(\angle BAC\) if BC = 6 in.

5. In fig. 92, TP, TQ are the tangents to the arc PRQ of a circle.
   (i) If the radius is 6 cm., find \(\angle PTQ\) and length of PQ.
   (ii) If \(\angle PRQ = 148°,\) find the radius of the circle and the shortest distance of T from the circumference.

6. In fig. 93, BT touches the semicircle APB. Find the lengths of AP, PT.
[7] In fig. 94, BN is the perpendicular bisector of the chord AC. Find the angle which the arc ABC subtends at the centre of the circle.

[8] ABCD is a rectangle in a vertical plane and A is its lowest vertex; AB = 5 cm., BC = 3 cm., and AB makes an angle 32° with the horizontal. Find the heights of B and C above the level of A and the angle which AC makes with the horizontal.

9. If $\theta$ is an acute angle such that $\cos \theta = \frac{3}{5}$, find without using tables the values of $\sin \theta$, $\tan \theta$, $\sin (90^\circ - \theta)$.

10. If $\theta$ is an acute angle such that $\tan \theta = \frac{5}{12}$, find without using tables the values of $\sin \theta$, $\cos \theta$, $\tan (90^\circ - \theta)$.

[11] In $\triangle ABC$, AB = 6 cm., AC = 5 cm., $\angle BAC = 126^\circ$. Find (i) the length of the perpendicular from C to BA produced, (ii) the area of $\triangle ABC$.

12. In fig. 95, BD is a diameter of the circle ABC; BC = 10 cm., $\angle ABC = 68^\circ$, $\angle ACB = 54^\circ$. What is the size of $\angle BDC$? Hence find the radius of the circle. Find without using tables the radius of the circle which circumscribes an equilateral triangle, side 12 cm.

13. Fig. 96 shows a quarter of the face of a clock, the hour-hand pointing along OA, OB, OC, OD at 12, 1, 2, 3 o'clock. Find the lengths of AB and CD and the sizes of $\angle BCD$, $\angle BCO$.

14. In fig. 97, AP, CQ are perpendicular to ABC; $\angle QBC = 78^\circ$. Find $\angle ABP$ and the length of PQ.

15. A regular polygon of 10 sides is inscribed in a circle of radius 5 in. Find to one significant figure the difference between (i) the perimeter of the polygon and the circumference of the circle, (ii) the area of the polygon and the area of the circle. [Take $\pi = 3.142$.]

---

SOLID GEOMETRY

16. AB, DC are the parallel sides of the trapezium ABCD; AB = 7.5 cm., BC = 8 cm., $\angle ABC = 75^\circ$, $\angle BAD = 62^\circ$. Find the lengths of CD and AD and the area of ABCD.

17. In fig. 98, ABCD is a straight line. Find $\angle BCP$ and the length of AP and the radius of the circle passing through A, D, P.

18. With the data of fig. 99, find the lengths of AE and the perpendicular from C to AE.

19. Two circles, radii 7 cm., 3 cm., touch each other externally. Calculate the angle between their direct common tangents.

20. A triangle ABC circumscribes a circle, radius 2 in.; $\angle CAB = 70^\circ$, $\angle CBA = 50^\circ$; calculate the lengths of AB, BC, CA.

SOLID GEOMETRY

The following examples are intended for oral discussion. Each right-angled triangle used in the argument should be drawn as a separate figure with the lettering of the perspective diagram and any known dimensions shown in the figure.

Example 1. Fig. 100 represents a cuboid.

(i) Explain why the angle which SB makes with the base ABCD is equal to $\angle DBS$ and find its size.

(ii) Explain why the angle between the planes BDS, ADSB is equal to $\angle BDA$ and find its size.

(iii) Name an angle equal to the angle which $SB$ makes with the plane SDAP and find its size.
Example 2. Fig. 101 represents a plane hill-side ABEF sloping at $32^\circ$ to the horizontal plane ABCD; AF, BE are lines of greatest slope and FD, EC are the perpendiculars to ABCD; AE is a track which makes $56^\circ$ with AF.

(i) Name an angle equal to the angle which AE makes with the horizontal.

(ii) If $AE=x$ yards, write down in terms of $x$ the lengths of BE and EC; hence find $\angle EAC$.

(iii) Write down in terms of $x$ the lengths of AB and BC; hence find $\angle CAB$.

(iv) If the bearing of AF is due north, find the bearing of AE.

Example 3. Fig. 102 represents a pyramid whose base is an equilateral triangle ABC, side $6\, \text{cm.}$, and whose vertex is V; VN is the perpendicular from V to the base ABC; $VA=VB=VC=4\, \text{cm.}$

(i) Explain why $NA=NB=NC$ and prove that $NC=2\sqrt{3}\, \text{cm.}$

(ii) Find the lengths of VN and ND.

(iii) Name an angle equal to the angle VA makes with the plane ABC and find its size.

(iv) Name an angle equal to the angle between the planes VBC, ABC and find its size.

EXERCISE 40

1. V is the centre of the face PQRS of the cuboid in fig. 100, and E is the mid-point of AB. Find (i) the angle VE makes with the base ABCD, (ii) the angle VA makes with the base ABCD, (iii) $\angle VAE$.

2. Draw a diagram like fig. 101 for the following example:

From a point B on level ground a factory chimney CE bears due north and the angle of elevation of the top E from B is $27^\circ$; A is a point in the horizontal plane ABCD due west of B and the bearing of C from A is N. $41^\circ\ 30^\prime$ E.; $AB=200$ feet. Find the height CE of the chimney.

3. In fig. 103, ABCD is a horizontal rectangle and AP is a vertical pole; $AB=8\, \text{ft.}$, $BC=6\, \text{ft.}$, $AP=9\, \text{ft.}$ Find (i) the angle of elevation of P from B, (ii) the inclination of CP to the horizontal, (iii) the angle which the plane PDC makes with the horizontal.

4. In fig. 104, the distance between the parallel rectangular faces ABCD, PQRS of a truncated right pyramid is 4 ft. Find (i) the angle which the edge QB makes with the base ABCD, (ii) the angle which the base PQBA makes with the base.

5. A bowl is suspended from a point O by 4 equal chains OA, OB, OC, OD, each 11 inches long, see fig. 105, attached to points A, B, C, D on the rim of the bowl, not shown in the figure; ABCD is a square, side 12 inches. Find (i) the height of O above the plane ABCD, (ii) the angle which OA makes with the vertical, (iii) the angle which the plane OAB makes with the horizontal.

6. Fig. 106 shows the rectangular sloping face PQRS of a desk whose base is the horizontal rectangle ABCD and whose vertical face is the rectangle ABQD. Find (i) the angle which the face PQRS makes with the horizontal, (ii) the angle which the diagonal QS makes with the horizontal.

7. A hoarding is a rectangle 20 ft. long, 12 ft. high and stands vertically with its lower edge on level ground and faces south. Find the area of the shadow cast by the hoarding on the ground when the sun is 20° west of south at an angle 42° above the horizon.
TRIGONOMETRY

[8] The length of each edge of a regular tetrahedron $ABCD$ is 6 cm. Find (i) the lengths of the perpendiculars from $A$ and from $D$ to $BC$, (ii) the length of the perpendicular from $A$ to the plane $BCD$, (iii) the angle between the faces $ABC$, $DBC$, (iv) the angle which $AD$ makes with the face $BDC$.

[9] The base of a pyramid, vertex $V$, is an equilateral triangle $ABC$, side 10 in.; $VA=VB=VC=13$ in.; $VN$ is the perpendicular from $V$ to the base $ABC$. Find (i) the lengths of $AN$ and $VN$, (ii) the angle which $VA$ makes with the base $ABC$, (iii) the angle between the planes $VBC$, $ABC$.

[10] The base of a right pyramid, vertex $V$, is a regular polygon of 12 sides. The length of each edge of the base is 4 in. and the length of each sloping edge is 10 in. Find (i) the length of the perpendicular from $V$ to the base, (ii) the angle which a sloping edge makes with the base, (iii) the angle which a sloping face makes with the base.

11. A ship steams due west from $P$ to $Q$ along the parallel of latitude $58^\circ$ N., see fig. 107. Find the distance travelled when the longitude has altered by $20^\circ$, taking the radius of the earth to be 3960 miles.

If $O$ is the centre of the earth, find $\angle POQ$ and the distance measured along the great circle from $P$ to $Q$.

12. Fig. 108 represents a circular cone, vertex $V$, height 6 cm., base-radius 8 cm.; $P, Q$ are points on the rim of the base, centre $N$, such that $\angle PNQ = 80^\circ$. Find $\angle PVQ$ and the angle which the plane $VPQ$ makes with the base.

SINE FORMULA FOR A TRIANGLE

SINE FORMULA FOR A TRIANGLE

Obtuse angles.

\[
\sin (180^\circ - \theta) = \sin \theta; \quad \cos (180^\circ - \theta) = -\cos \theta;
\]
\[
\tan (180^\circ - \theta) = -\tan \theta.
\]

Area.

Area of triangle $ABC = \frac{1}{2}bc \sin A$
\[
= \frac{1}{2}ac \sin B = \frac{1}{2}ab \sin C.
\]

Fig. 109.

Sine formula.

In any triangle $ABC$,

\[
a = \frac{b}{\sin A} \quad \frac{c}{\sin B} \quad \frac{c}{\sin C} = 2R,
\]

where $R$ is the radius of the circle through $A$, $B$, $C$.

Example 1. Solve $\triangle ABC$ given that $B = 42^\circ 32'$, $C = 36^\circ 20'$, $c = 5.824$.

From the sine formula,

\[
\frac{b}{\sin 42^\circ 32'} = \frac{5.824}{\sin 36^\circ 20'}
\]

\[
\therefore \quad b = \frac{5.824 \times \sin 42^\circ 32'}{\sin 36^\circ 20'}
\]

\[
\therefore \quad b = 6.646 = 6.65, \text{ to } 3 \text{ figures.}
\]

Since $B + C = 78^\circ 52'$, $A = 180^\circ - 78^\circ 52' = 101^\circ 8'$.

From the sine formula,

\[
\frac{a}{\sin 101^\circ 8'} = \frac{5.824}{\sin 36^\circ 20'}
\]

But $\sin 101^\circ 8' = \sin (180^\circ - 101^\circ 8') = \sin 78^\circ 52'$,

\[
\therefore \quad a = \frac{5.824 \times \sin 78^\circ 52'}{\sin 36^\circ 20'}
\]

\[
\therefore \quad a = 9.64(5). \quad \text{The reader should work this out for himself.}
\]
Example 2. Solve \( \triangle ABC \) given that
\[
b = 5.5 \text{ cm}, \quad c = 7 \text{ cm}, \quad B = 35^\circ.
\]

It is possible to draw two unequal triangles satisfying the given conditions, namely \( \triangle ABC_1 \) and \( \triangle ABC_2 \) in fig. 13, p. 68.

From the sine formula,
\[
\frac{\sin C}{\sin 35^\circ} = \frac{5.5}{b}
\]
\[
\therefore \sin C = \frac{7 \sin 35^\circ}{5.5}
\]

From the tables,
the logarithm of \( \sin 46^\circ 53' \) is 1.8633;
but \( \sin 46^\circ 53' = \sin (180^\circ - 46^\circ 53') = \sin 133^\circ 7' \)
\[
\therefore \text{the logarithm of } \sin 133^\circ 7' \text{ is also } 1.8633.
\]
\[
\therefore \text{either } C = 46^\circ 53' \text{ or } C = 133^\circ 7'.
\]
these are the angles \( AC_1B, AC_2B \) in fig. 13, p. 68.

(i) Since \( B = 35^\circ \), if \( C = 46^\circ 53' \), then \( A = 98^\circ 7'.
\]
\[
\therefore \frac{a}{\sin 98^\circ 7'} = \frac{5.5}{\sin 35^\circ}
\]
but \( \sin 98^\circ 7' = \sin (180^\circ - 98^\circ 7') = \sin 81^\circ 53'.
\]
\[
\text{hence, as in Example 1, } a = \frac{5.5 \sin 81^\circ 53'}{\sin 35^\circ} \approx 9.49.
\]

(ii) Since \( B = 35^\circ \), if \( C = 133^\circ 7' \), then \( A = 11^\circ 53'.
\]
\[
\therefore \frac{a}{\sin 11^\circ 53'} = \frac{5.5}{\sin 35^\circ}
\]
\[
\text{hence, as in Example 1, } a = \frac{5.5 \sin 11^\circ 53'}{\sin 35^\circ} \approx 1.97.
\]

In fig. 13, p. 68, \( BC_1 = 1.97 \text{ cm} \) and \( BC_2 = 9.49 \text{ cm} \).

EXERCISE 41
1. If \( \sin \theta = \frac{1}{2} \) and if \( \theta \) is obtuse, find \( \cos \theta \) and \( \tan \theta \).
2. If \( \tan \theta = -\frac{1}{2} \) and if \( \theta \) is between \( 90^\circ \) and \( 180^\circ \), find \( \sin \theta \) and \( \cos \theta \).

SINE FORMULA FOR A TRIANGLE

3. (i) If \( AB = 4 \text{ in}, \ AC = 5 \text{ in}, \ \angle BAC = 130^\circ \), find the area of \( \triangle ABC \).

(ii) If \( CA = 9 \text{ cm}, \ CB = 6 \text{ cm} \) and if the area of \( \triangle ABC \) is 21 sq. cm, find \( \angle ACB \). [Two answers.]

4. Find the area of the parallelogram \( ABCD \) if \( AB = 4 \text{ in.}, \ BC = 3 \text{ in}., \ \angle ABC = 149^\circ \).

5. B is 2000 yards due east of A; a tower T bears N. 51° 30' E. from A and N. 33° 12' W. from B. Find the distance of T from A and from the straight road AB.

6. A and B are points due south of a tower HK on level ground; \( AB = 110 \text{ feet} \). From A, the angle of elevation of the top K of the tower is 28° 36'; from B, the angle of elevation of K is 53° 24'. Find the height of the tower.

7. In the triangle \( ABC, AB = 2.85 \text{ in.}, \ AC = 4.2 \text{ in.}, \ \angle ACB = 31^\circ \) and \( \angle ABC \) is obtuse. Find \( \angle ABC \), the length of \( BC \) and the radius of the circle \( ABC \).

8. Fig. 110 shows a crank mechanism, not drawn to scale; \( OA \) rotates about a fixed point \( O \) and \( B \) slides along a fixed arm \( OC \). Calculate \( \angle ABO \) (i) when \( \angle AOB = 52^\circ \), (ii) when \( \angle AOB = 116^\circ \). Find the greatest angle that \( BA \) makes with \( BO \) during the motion.

9. In the framework in fig. 111, find the lengths of \( BD \) and \( BC \).

10. B and C are two buoys 500 yards apart and the bearing of B from C is 050° (50° east of north). From a ship S, the bearing of B is 304° and the bearing of C is 252°. Find the distances of \( S \) from \( B \) and from \( C \).
Fin also the angle of depression of \( B \) from a masthead of \( S \), 120 feet above the sea.
COSINE FORMULA FOR A TRIANGLE

Extensions of Pythagoras’ theorem.

If \(AN\) is an altitude of the triangle \(ABC\),
\[
c^2 = a^2 + b^2 - 2ac \cdot NC \text{ if } \angle ACB \text{ is acute,}
\]
\[
c^2 = a^2 + b^2 + 2ac \cdot CN \text{ if } \angle ACB \text{ is obtuse.}
\]

Hence in any triangle \(ABC\),
\[
since \cos (180^\circ - C) = -\cos C,
\]
\[
c^2 = a^2 + b^2 - 2ab \cos C
\]
both when \(\angle ACB\) is acute and when \(\angle ACB\) is obtuse.

Similarly
\[
a^2 = b^2 + c^2 - 2bc \cos A
\]
and
\[
b^2 = c^2 + a^2 - 2ca \cos B.
\]

Apollonius’ theorem.

If \(AD\) is a median of \(\triangle ABC\),
\[
b^2 + c^2 = 2AD^2 + 2DC^2 - 2AD^2 + \frac{1}{2}a^2.
\]

Use the cosine formula for the triangles \(ADC, ADB\),
\[
b^2 = AD^2 + DC^2 - 2AD \cdot DC \cos \theta,
\]
\[
c^2 = AD^2 + DB^2 + 2AD \cdot DB \cos \theta;
\]
the result follows by addition.

Example. In \(\triangle ABC\), \(a = 4.6\), \(b = 3.2\), \(c = 6.5\), find \(c\).

From the cosine formula,
\[
6.5^2 = 4.6^2 + 3.2^2 - 2 \times 4.6 \times 3.2 \cos C,
\]
\[
\therefore 22.56 = 21.16 + 10.24 - 9.2 \times 3.2 \cos C,
\]
\[
\therefore 9.2 \times 3.2 \cos C = 31.40 - 42.25 = -10.85,
\]
\[
\therefore \cos C = -\frac{10.85}{9.2 \times 3.2} = -0.3231
\]
\[
\therefore C = 108^\circ 15'.
\]

EXERCISE 42

1. Find whether a triangle is obtuse-angled or acute-angled if the lengths of its sides are
   (i) 9 cm, 10 cm, 14 cm; (ii) 5 in, 9 in, 10 in.

2. In \(\triangle ABC\), \(b = 3\) in, \(c = 5\). Find \(a\) if \(A = 56^\circ 38'\).
   (ii) if \(A = 123^\circ 22'\).

3. Find the angles of \(\triangle ABC\) if \(a = 9\), \(b = 6\), \(c = 5\) and the area of the triangle.

4. Find the largest angle of the triangle \(ABC\) if \(a = 10.2\) cm, \(b = 12.4\) cm, \(c = 18.3\) cm.

5. In fig. 114, find the lengths of \(PQ\) and \(RS\) and the area of the quadrilateral \(PQRS\).

6. Fig. 115 represents the plan of an open window \(AQ\) pivoted at \(A\) and held in position by a bar \(CE\) attached to \(AQ\) at \(C\) and passing through a slot \(B\) on the sill \(AP\). Find \(\angle QAP\) (i) if \(CB = 6\) in, (ii) if \(CB = 10\) in.
7. B is 3 miles from A in direction N, 74° E.; C is 2 miles from A in direction S, 38° 24' E. Find the distance and bearing of B from C.

[8] ABC is a triangular field; AB = 120 yd., AC = 140 yd., \( \angle BAC = 125° \). Find the length of BC and the distance of A from BC.

9. A ship steams at 8 knots on a course 048° (48° east of north) and there is a constant current of 3 knots setting in the direction S, 65° E. Find in nautical miles the distance travelled by the ship in half an hour and the bearing of the track made good by the ship.

[10] A motor-boat leaves a port P and is headed due west at 12 knots. After 15 minutes, owing to a current, the boat is 2.5 nautical miles from P in the direction N, 67° W. Find the speed and direction of the current.

11. The base of a pyramid, vertex V, is a horizontal equilateral triangle ABC, side 10 in.; VA = 16 in., VB = VC = 13 in. Find (i) the angle which VA makes with the horizontal, (ii) the angle which the face VBC makes with the base ABC, (iii) the length of the perpendicular from V to the base.

12. The sides of a triangle are 8 in., 9 in., 11 in.; find the lengths of the two shorter medians and the acute angle between them.

[13] ABCD is a parallelogram; AB = 9 cm., AD = 5 cm., AC = 6 cm. Find (i) the length of BD, (ii) the obtuse angle between the diagonals AC, BD.

14. A, B are fixed points such that AB = 14 in.; P is a variable point in a plane through AB such that PA² + PB² is equal to 130 sq. in. Find precisely the locus of P. Find also the least value of \( \angle APB \).

ROUTINE PAPERS, A1—A15

A 1

1. Divide £83 7s. 6d. by 30.

2. How many revolutions per minute are made by the wheel of a bicycle, diameter 28 inches, when the bicycle is travelling at 15 m.p.h.? [Take \( \pi = \frac{22}{7} \).]

3. Evaluate \( a^3 - 2ab \) when \( a = -4 \), \( b = 8 \).

4. Factorise \( 24 - 2z - z^2 \).

5. AB, BC, CD, DE are consecutive sides of a regular polygon of 10 sides. Find the acute angles which BE makes with AD and with AC.

6. AB, BC are equal chords of a circle, centre O; AB is produced to D. Prove \( \angle CBD = \angle AOB \).

7. In fig. 116, BCD is at right angles to AB; calculate \( \angle BAC \) and \( \angle CAD \).

Fig. 116.

A 2

1. Express in prime factors in index form, \( 6^3 \times 4^2 \times 12 \).

2. If 84% of a sum of money is £1470, how much is 58% of this sum?

3. How far does a car travel in 5t minutes at 4n miles an hour?

4. Solve: \( x - 3y = 8 \), \( 2x - 5y = 14 \).

5. In \( \triangle ABC \), \( AB = AC \); BC is produced to P so that CP = AC and BA is produced to Q. Prove that \( \angle PAQ = 3 \angle APB \).

6. In fig. 117, arrows indicate lines are given parallel. Find \( a, b, c, d \).

7. In \( \triangle ABC \), \( AB = AC = 7 \) cm., \( \angle BAC = 105° \), 40'; calculate the lengths of BC and the altitude AD.

Fig. 117.
A 3

1. Divide 0·01972 by 13·6.
2. Find in cwt. the weight of a wooden beam, 16 ft. 4 in. long, 8 in. wide, 4 1/2 in. deep, if the wood weighs 48 lb. per cu. ft.
3. Solve \((2x + 1)^2 = 9\).
4. If \(4(R + r) = kR\), find \(R\) in terms of \(k\) and \(r\).
5. In fig. 118, ABCD and ABEF are parallelograms; prove DE and CF bisect each other.
6. A segment of a circle is cut off by a chord, length 4 in.; the height of the segment is 1 in. Find the radius of the circle.
7. Find an acute angle \(\theta\) such that (i) \(\cos \theta = \frac{1}{2}\); (ii) \(\sin \theta = \frac{1}{2} \cos \theta\).

A 4

1. Express 17s. 11½d. as a decimal of £1 correct to 4 places.
2. Find the simple interest on £138 for 14 months at 3 3/4% p.a.
3. The smallest of 6 consecutive odd numbers is \(2k - 3\). If the sum of the six numbers is \(n\), express \(k\) in terms of \(n\).
4. Factorise \(x(x + z) = y(y + z)\).
5. The side BC of the parallelogram ABCD is produced to any point K; and the lines AC, AK, DK are drawn.
   (i) Name with reasons a triangle equal in area to \(\triangle DCK\).
   (ii) Prove \(\triangle ABK = \text{quad.} AOKD\).
6. In fig. 119, the circle touching AB at A and passing through C cuts BC produced at P. If AB = AC, prove that PA = PB.
7. The base of a right pyramid, vertex V, is a square ABCD, side 8 in., and the height VN of the pyramid is 7 in. Calculate (i) the angle which the face VAB makes with the base, (ii) the angle which the edge VA makes with the base.

A 5

1. Multiply 70·8 by 96·5.
2. A greengrocer buys apples at 45 for 2s. and sells them at 8d. a dozen. Find his gain per cent.
3. Solve \(x - \frac{2x - 3}{5} = 1 + \frac{1}{2}x\).
4. Complete the relation, \(x^2 - 7x + \ldots = (x - \ldots)^2\), and find to 2 places of decimals the roots of \(x^2 - 7x + 5 = 0\).
5. ABCD is a quadrilateral in which \(\angle B = \angle C = 1\) rt. \(\angle\), \(AB = 25\) cm., \(BC = 30\) cm., \(CD = 9\) cm.; the line bisecting \(\angle BAD\) cuts BC at P. (i) Find the length of AD. (ii) Prove BP = PC. [Produce AP, BC to meet at Q.]
6. The chords AB, CD of the circle ACBD intersect at K. Complete the statement, \(AD : AK = BC : \ldots\), giving reasons.
7. ABC is a triangle inscribed in a circle, radius 10 cm.; \(\angle A = 58^\circ\), \(\angle B = 74^\circ\). Calculate the lengths of BC, CA, AB.

A 6

1. Find in prime factors the square root of 5184, and write down the square root of 0·005184.
2. Find in sq. ft. the area of a path 3 ft. 6 in. wide which runs all round a rectangular lawn 20 yd. long, 14 yd. wide.
3. Solve \(2x - y - 3 = 3x - y + 4 = 5x - 8y - 4\).
4. Simplify \(\frac{1}{t-2} - \frac{5}{t^2 + t - 6}\).
5. In fig. 120, ABC is an equilateral triangle; BC, CA are produced to P, Q so that BP = CQ; PA is produced to cut EQ at R.
   (i) Name with reasons an angle equal to \(\angle APC\).
   (ii) Prove \(\angle PRB = 60^\circ\).
8. AC is a diameter of the circle ABCD; \(\angle BAC = 52^\circ\); find \(\angle ADB\).

In \(\triangle ABC\), \(\angle A = 69^\circ\) 36', \(\angle B = 52^\circ\) 48', \(BC = 6\) 28 in. Calculate the lengths of AB, AC and the area of \(\triangle ABC\).
A 7

1. Telegraph poles are set up at intervals of 55 yards; how many poles are there every 12 miles?

2. Find, correct to 3 figures, \(5.862 \times 0.759\), \(0.698 \times 1.072\).

3. Find the values of \(c\) if \(x^2 - 2(c + 1)x + 8c + 1\) is a perfect square for all values of \(x\).

4. If \(z\) varies directly as \(x^2\) and inversely as \(y\), and if \(z = 4\) when \(x = 2\) and \(y = 12\), find \(z\) in terms of \(x\) and \(y\).

5. \(AB, DC\) are the parallel sides of the trapezium \(ABCD\), whose diagonals \(AC, BD\) cut at \(K\). \(\angle ABD = 1\) rt. \(\angle AB = 9\) cm., \(CD = 3\) cm., \(BD = 5\) cm. Find (i) the area of \(ABCD\), (ii) the lengths of \(AC\) and \(AK\).

6. In fig. 121, \(AP\) touches the circle \(ABQ\); \(QB\) produced cuts the circle \(APB\) at \(R\). Prove that \(PR\) is parallel to \(AQ\).

7. The base of a right pyramid, vertex \(V\), is a rectangle \(ABCD\), and \(VN\) is the height of the pyramid; \(AB = 10\) cm., \(BC = 6\) cm., \(VN = 4\) cm. Find the angles which the faces \(VAB, VBC\) make with the base and the angle which \(VA\) makes with the base.

A 8

1. Simplify \((3\frac{1}{2} - 1\frac{1}{4}) + (2\frac{2}{3} - \frac{1}{3})\).

2. If 12\% of a bill is deducted, £154 is left. How much is the bill?

3. If \(x = \frac{3}{4}y\) and \(x + y = 0.1\), find \(x\).

4. Factorise \((2x + 7)(x - 3) - (x + 7)(x - 1)\).

5. \(ABCD\) is a parallelogram; \(ABMN\) and \(ADQR\) are squares outside \(ABCD\). Prove \(NR = AC\).

6. In fig. 122, arrows indicate lines are parallel. Find the values of \(x, y, z, w\).

7. Use the results obtained in No. 6 to calculate (i) \(\angle ACD\), (ii) the length of \(DR\), (iii) the length of \(BC\).

A 9

1. Taking 1 lb. = 0.4536 kg., express 9\(\frac{1}{2}\) oz. in grams to the nearest gram.

2. Customs receipts on beer increased from £263,170,704 in 1944 to £278,876,870 in 1945. Find the increase per cent. to 3 figures.

3. Solve \(0.2(n - 0.1) = 0.3(n - 0.2)\).

4. If \(x = 4 - 2\sqrt{3}\), prove that \(x^2 + 4 = 8x\).

5. Fig. 123 is formed by 7 straight lines. Prove that the sum of the 7 marked angles is 6 right angles.

6. The internal bisector of \(\angle BAC\) meets \(BC\) at \(P\) and the external bisector of \(\angle ABC\) meets \(BA\) produced at \(Q\); \(AB = 8\) in., \(AC = 6\) in., \(BP = 6\) in. Find the lengths of \(CP\) and \(AQ\).

7. In \(\triangle ABC\), \(AB = 6.52\) in., \(AC = 4.65\) in., \(\angle C = 125^\circ 30'\), find \(\angle B\), the length of \(BC\) and the area of \(\triangle ABC\).

A 10

1. Find the square root of 302.76, without using tables.

2. A cylindrical tank, internal diameter 7 ft. 6 in., contains 1,000 gallons of water. Find in inches, to 3 figures, the depth of the water. [1 gallon = 277.3 cu. in.]

3. Factorise \(x^2 + z^2 - z - 1\).

4. Express as powers of 8 the following:

5. In fig. 124, arrows indicate that lines are given parallel. Prove that \(\triangle ABD = \triangle BPC\). [Join \(AC, BD\).]

6. \(ABC\) is an equilateral triangle; \(HBCK\) is a straight line such that \(\angle HAK = 120^\circ\). Prove (i) \(\angle HAB = \angle HKA\), (ii) \(HB \cdot CK = BC^2\).

7. In \(\triangle ABC\), \(BC = 9\) in., \(CA = 6\) in., \(AB = 5\) in.; \(D\) is the midpoint of \(AC\). Calculate the length of \(BD\) and the size of \(\angle ABD\).
A 11

1. Express as a decimal, $59 \div (8 \times 5^4)$.

2. On a map of scale 4 in. to the mile, an estate occupies an area of 9:37 sq. in. Find its actual area to the nearest acre.

3. Solve: $2x - y + 3 = 0$, $x^2 + y^2 + 6x = 0$.

4. $V$ varies directly as $h$ and as $r^3$. Find the percentage change in $h$ when $V$ increases by $20\%$ and $r$ increases by $25\%$.

5. In fig. 125, $AB = AC$, $PQ = PB$, and $PQ$ produced cuts $AC$ at $R$. Prove that $\angle ARP = \angle ABP$.

6. $P, Q, R, S$ are points on the sides $AB, BC, CD, DA$ of a quadrilateral such that $PQ$ and $SR$ are parallel to $AC$, and $QR$ parallel to $BD$. Prove that (i) $AP : PB = AS : SD$; (ii) $PS$ is parallel to $QR$.

7. A man, walking due south along a straight level road, sees a church tower bearing S. 48° W.; 3 miles further on, he sees it on a bearing N. 56° W. Find in yards, to 3 figures, the shortest distance of the church from the road.

A 12

1. Find, to 3 figures, $\sqrt[4]{0.472} \div \sqrt{0.081}$.

2. Find, to 3 figures, the capacity in gallons of a cylindrical tank, diameter 2 ft. 9 in., height 5 ft. 6 in. [1 gall. = 0.1604 cu. ft.]

3. If $\frac{d}{n} = \frac{n - 1}{n}$, find the value of $n$ when $d = 0.7$, $t = 1.2$.

4. Find the value of $\log_{10} 4 + 2 \log_{10} 5$, without using tables.

5. $AB$, $DC$ are the parallel sides of the trapezium $ABCD$; $M$ is the midpoint of $AC$; the line through $M$ parallel to $AB$ cuts $AD, BC$ at $H, K$. Prove that $AB + DC = 2HK$.

6. In fig. 126, the chords $PQ, HK$ of a circle meet when produced at $T$; $HN$ is parallel to $KQ$. Find the values of $a, b$.

A 13

7. $ABCD$ is a square sheet of paper, side 8 in.; the paper is folded about $BD$ so that the two halves of the square make an angle 70° with each other. (i) Find the distance between the corners $A, C$. (ii) Find the angle between the edges $BA, BC$.

A 14

1. Divide £3 into three parts in the ratios 5 : 7 : 8.

2. If $(1 + \pi)^n = 3$, find the value of $n$ to 3 figures.

3. Solve: $x - y + 2z = 1$, $3x + 9y - 3z = 2$, $6x + 30z = 13$.

4. Find the roots of $3x^2 - 4x - 6$ to 2 places of decimals.

5. $AB$, $DC$ are the parallel sides of a trapezium $ABCD$; a line parallel to $AB$ cuts $AD$ at $P$ and cuts $BC$ at $Q$. Prove that $\triangle PBD = \triangle QAC$. 

7. Simplify (i) $(\frac{3}{4})^{-3}$; (ii) $(4^2)^{\frac{1}{2}}$; (iii) $\sqrt{18} + \sqrt{32} - \sqrt{50}$.

5. In fig. 127, $BPQ, CAD$ are straight lines cutting the circles $APBC, AQDB$. Prove that $CP$ is parallel to $QD$.

6. $H, K$ are points on $AB, AC$ respectively such that $AH = HB = 5 : 4$ and $AK = KC = 3 : 7$; $BK$ cuts $CH$ at $N$. Calculate the ratios $BN : NK$ and $CN : NH$. [Draw $KP$ parallel to $CH$ to meet $AB$ at $P$.]

7. (i) $\alpha$ is an obtuse angle such that $\sin \alpha = \frac{3}{4}$, calculate the value of $\cos \alpha$.

(ii) $\beta$ is an acute angle such that $\tan \beta = \frac{1}{4}$, calculate the values of $\sin (180^\circ - \beta)$, $\cos (180^\circ - \beta)$, $\frac{1}{2} \sin \beta + \cos \beta$. 

A 14
6. In fig. 128, BH, BK are tangents to the circles ABP, ABQ; PBQ and PAR are straight lines. Prove $\angle HBK = \angle QAR$.

7. K is the mid-point of the side CD of the parallelogram ABCD; AB = 8 cm., AD = 5 cm., $\angle BAD = 65^\circ$; find the length of AK and the size of $\angle KAB$.

A 15

1. The time of 25 swings of a pendulum is measured to be 184 sec. to the nearest $\frac{1}{2}$ sec. Between what limits does the time of 1 swing lie?

2. At what rate per cent. p.a. simple interest will 18s. amount to £1 in 16 months?

3. Find the values of $a$, $b$ if $x+1$ and $x-2$ are factors of $x^2 + ax^2 + bx - 6$. What is then the remaining factor?

4. If $\alpha$ and $\beta$ are the roots of $3x^2 + x = 5$, find the value of (i) $(\alpha + 1)(\beta + 1)$, (ii) $\alpha^2 + \beta^2$. Find also the equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$, in a form with integral coefficients.

5. BC is the hypotenuse of a right-angled triangle ABC; AD is an altitude of $\triangle ABC$; DP, DQ are the perpendiculars from D to AB, AC. Prove that the points B, C, P, Q are concyclic.

6. AB is a diameter of a circle ACDB, centre O; the tangent at A meets OC produced at T; TB cuts the circle at D; AT = 3 in., CT = 1 in. Find the lengths of OC and BD.

7. V is the vertex of a pyramid whose base is an equilateral triangle ABC, side 6 cm.; VA = VB = VC = 4 cm. Find (i) the height of the pyramid, (ii) the angle which VA makes with the base, (iii) the angle which the face VBC makes with the base.

MISCELLANEOUS REVISION PAPERS, B1—B15

B 1

1. On a journey of 2 miles in London, a taxi averages 20 m.p.h. for $\frac{3}{4}$ of the distance, 30 m.p.h. for $\frac{1}{4}$ of the distance, and 10 m.p.h. for the remainder. In addition the taxi is stationary in traffic blocks for $\frac{1}{2}$ minutes. What is the average speed for the whole journey?

2. The 5s. shares of a rubber company pay a dividend of 1s. 5½d. per share and cost 17s. 6d. per share. The £1 shares of a tin company pay a dividend of 6 per cent. and stand at £4. A man has £1050 to invest. From which of the two companies will he obtain the greater income and by how much? What would be the price of the tin shares if they gave the same income as the rubber shares?

3. Find T and $f$ in terms of $m_1$, $m_2$, $g$ from the equations $T - m_1 g = m_2 f$, $m_2 g - T = m_2 f$.

4. A circular rubber disc of radius 15 inches is of uniform thickness. If the weight is reduced by 36% when a strip $t$ inches wide all round is cut away, find the value of $t$.

5. BE is an altitude of the triangle ABC. If $AB = AC$, prove that $\angle BAC = 2 \angle EBC$.

6. In fig. 129, AB, DC, PQRS are parallel. (i) Name three ratios equal to $PQ : DC$. (ii) Prove that $PQ = RS$.

7. The trapezium ABCD is right-angled at B and C; BC = 5 in., CD = 4 in., AC = 8 in. Calculate (i) $\angle BCA$, (ii) AB, (iii) $\angle BAD$.

B 2

1. A man, walking at 3½ m.p.h., passes a mile-stone at 2 p.m.; at 2.20 p.m., a cyclist, riding the same way at 10½ m.p.h., passes the same mile-stone. At what time will he overtake the pedestrian?
2. A man buys a new car for £720 and enters into a covenant not to sell it for 2 years. Annual tax is £10 and annual insurance is £16. The car averages 24 miles to the gallon of petrol and uses one gallon of oil every 800 miles. The petrol costs 3s. 6d. per gallon and the oil 10s. per gallon. After 2 years, he sells the car at 20% above the price he paid for it. If he has travelled 12,000 miles in the 2 years and spent £22 on upkeep, find in pence the net cost per mile.

3. Given any four consecutive integers, prove that the product of the two greatest exceeds the product of the other two by the sum of the four integers.

4. The number of beats per minute made by a pendulum varies inversely as the square root of the length of the pendulum. If the length is increased by 50%, find the percentage change in the number of beats per minute.

5. With the data of fig. 130, in which the marked angles are right angles, find \( y \) in terms of \( x \) and find the area of the compartment A in terms of \( x \).

If the areas of the compartments A, B are equal, find the value of \( x \) correct to 3 figures.

6. In fig. 131, \( AB = AC \) and \( BPQ \) is a straight line. Prove that \( AP \) bisects \( \angle CPQ \).

7. \( AB, DC \) are the parallel sides of the trapezium \( ABCD \); \( AB = 12 \) cm, \( BC = 6 \) cm, \( CD = 4 \) cm, \( DA = 7 \) cm. Calculate the sizes of \( \angle BAD \) and \( \angle BCD \) and the area of \( ABCD \). [Draw \( DK \) parallel to \( CB \) to meet \( AB \) at \( K \).]

B 3

1. A man buys a house for £1600 and furnishes it at a cost of £300. The house is rated at £64 and rates are 14s. 6d. in the £. What rent per month must be charged to give him 6% on his investment and to cover the cost of the rates for which he is responsible and annual repairs estimated at £25 a year?

B 4

8. A cylinder which holds 35 gallons is 1 ft. 10 in. high internally. Find in inches to 3 figures the internal diameter.

[1 gallon = 277.3 cu. in.]

9. If \( 3x^2 - 4x \) is expressed in the form \( 3(x + 1)^2 + p(x + 1) + q \), where \( p, q \) are constants, find their values.

10. A number of two digits is such that the difference of the squares of the digits is 40. If the order of the digits is reversed, the number is decreased by 36. Find the number.

11. In fig. 132, \( ABCD \) is a parallelogram, and the parallel lines \( AK, CH \) meet \( BD, AB \) respectively at \( K, H \). Prove that \( \triangle GKH = \triangle ADH \).

12. FN is the perpendicular from a point \( P \) to the diameter \( AB \) of a circle \( APB \). Prove that the perpendiculars from \( A \) and \( B \) to the tangent at \( P \) are equal to \( AN \) and \( NB \).

13. The base of a pyramid, vertex \( V \), is a horizontal rectangle \( ABCD \) and the edge \( VA \) is vertical; \( AB = 3 \) in., \( BC = 4 \) in., \( \angle ABV = 52^\circ \). Calculate (i) the height \( AV \), (ii) \( \angle ADV \), (iii) the angle which the plane \( VBD \) makes with the horizontal.
5. X, Y are the mid-points of the sides AB, AD respectively of
the square ABCD; DX cuts CY at N. Prove that (i) \( \angle CND \) is a
right angle, (ii) the area of \( \triangle CND \) is one-fifth of the area of
the square.

6. In fig. 133, the chords CA, CE cut the
chord BD at H, K. If \( AB = BC \) and \( CD = DE \),
prove that \( CH = CK \).

7. A sphere, radius 2 in., rests on the inner
surface of an inverted hollow cone whose axis is
vertical and whose semi-vertical angle is 50°.
The diameter of the base of the cone is 10 in.
Find to 2 figures the distance of the highest
point of the sphere above the plane of the rim of the cone.

B 5

1. The engine of a car makes 21 revolutions for each 4 revolutions
of the road wheel whose diameter is 92-3 cm. Find to 3 figures the
number of revolutions per minute of the engine when the car is
travelling at 55 m.p.h. [1 ft. = 30.48 cm.]

2. A set of solid brass weights are made in the same shape. In
this set, the 40 lb. weight is 6 inches high. What does one which is
3 inches high weigh? What is the height, to 3 figures, of one which
weighs 20 lb.?

3. If the values of \( x \) and \( y \), for which \( x = \frac{2t}{1+t^2},  y = \frac{1-t^2}{1+t^2} \) satisfy
the equation \( 7y - x = 5 \), find the numerical values of \( x \) and \( y \).

4. A pepper-pot, volume \( V \) cu. in., is in the form of a cylinder,
height \( h \) in., base-radius \( r \) in., surmounted by a
cone, and the total height of the pepper-pot is \( h \) in. Express \( x \) in terms of \( \pi, r, h, V \).
[Volume of cone = \( \frac{1}{3} \) base-area \( \times \) height.]

5. In fig. 134, ABCD is a trapezium with right
angles at B and C; the perpendicular bisector
of AD cuts BC at Q. With the data in the
figure, prove that \( 2bx = b^2 + c^2 - a^2 \).

Explain what happens (i) if \( b^2 + c^2 = a^2 \), (ii) if \( a^2 + b^2 = c^2 \).

6. ABCD is a square; the bisectors of \( \angle BAC, \angle ADB \) meet
BC, AC respectively at P, Q. Prove that (i) the triangles APC, DQA
are similar, (ii) the area of \( \triangle APC \) is twice the area of \( \triangle DQA \).

7. P is a place on the equator and Q is on the same meridian of
longitude as P, 1500 miles north of P. Taking the radius of the
earth as 3960 miles, find the latitude of Q. If the longitude of a
ship sailing due west from Q along a parallel of latitude changes
by 12°, how far does the ship travel?

B 6

1. The average monthly rainfall at my home for the 8 driest
months of a year was 2:18 inches and for the remaining 4 months
was 2:93 inches. Find the average rainfall for the whole year.
The average monthly rainfall for the period January to August,
both months inclusive, was 2:76 inches, what was it for the period
September to December?

2. The area of a triangle, whose sides are of lengths \( a, b, c \) in.,
is \( \sqrt{s(s-a)(s-b)(s-c)} \) sq. in. where \( s = \frac{1}{2}(a+b+c) \). Find to
3 figures the length of the side of a square whose area equals the
area of triangle with sides of lengths \( 2:37, 2:06, 7:13 \) in.

3. An aircraft can carry petrol for \( \frac{5}{4} \) hours’ flight. It is to
fly due west and then return to its starting point. Owing to an
east wind, its outward and homeward speeds are 260 m.p.h. and
180 m.p.h. Find the greatest distance it can reach from its base
and return safely.

4. Draw the graph of \( y = 2x^2 + x - 1 \), taking 1 inch as unit on
each axis, for values of \( x \) from \(-2\) to \(+1\). With the same scale
and axes, draw the graph of \( y = \frac{4}{3}(x+1) \). Use the graphs for the following:
(i) Solve \( 2x^2 + x = 2 = 0 \). (ii) Solve \( 4x^2 + 2x - 7 = 0 \).
(iii) For what range of values of \( x \) is \( 2x^2 + x - 1 \) less than \( \frac{1}{2}(x+1) \).

5. AB is a fixed line of length 3 inches; a variable point \( P \) moves
in a plane through \( AB \) and on one side of \( AB \) so that
\( \angle APB = 30° \). State the form of the path
traced out by \( P \) and prove that its length is
5m. inches.

6. In fig. 135, the bisector of \( \angle BAC \) meets
BC at D; N is a point on AD such that \( CN = CD \).
Name with reasons (i) an angle in the figure
equal to \( \angle ACN \), (ii) a triangle similar to \( \triangle ACN \).
Hence prove that \( AB : AC = BD : DC \).

7. D is the mid-point of the side BC of \( \triangle ABC \); \( \angle B = 47°, \angle C = 61°, BC = 8 \) cm. Calculate the lengths of \( AB, AD \) and the
size of \( \angle ADB \).
B 7

1. A man had an earned income of £600 a year and a net income of £80 a year from house property. One-tenth of his earned income and an allowance of £140 was free of tax. On the remainder, he paid tax at 3s. in the £ on the first £50 and tax at 6s. in the £ on the next £75 and tax at 9s. in the £ on the remainder. Find the total tax he paid.

2. A manufacturer makes an article for 6s. 3d. and sells it to a retailer. The retailer sells it to a customer for 9s. If the manufacturer and retailer make the same percentage profit on their costs, what actual profit does each of them make on the article?

3. If \( x^2 - 8x + 15 \) and \( 5x^2 - 18x^2 - kx - 10 \) have a common factor for an integral value of \( k \), find the common factor and the value of \( k \). Find the values of \( a, b \) if \( x^2 - 8x + 15 \) is a factor of \( 5x^3 - 18x^2 + ax + b \).

4. (i) If \( p = (2 + t)q \) and \( r = \left( \frac{t}{2} + 1 \right)s \), express \( q \) in terms of \( p, r, s \).
(ii) If \( x^2 = \pi e^2 = \pi^3 \) and if \( x \) is not zero, find \( a \) in terms of \( b \).

5. In \( \triangle ABC \), \( AB = AC \); a straight line cuts \( BA, BC, AC \) produced, at \( L, M, N \) respectively. Express \( \angle BMN \) in terms of \( \angle BLN \) and \( \angle LNC \).

6. In fig. 136, \( AB \) is a diameter of the circle \( APB \); the tangents at \( A, P \) meet at \( T \); BP, AT are produced to meet at Q. Prove that \( AT = TQ \).

7. In \( \triangle ABC \), \( AB = 3 \text{ in.}, BC = 4 \text{ in.}, \angle ABC = 1 \text{ rt. } \angle \); AP, BQ, CR are drawn perpendicular to the plane of \( \triangle ABC \) and on the same side of it; \( AP = 10 \text{ in.}, BQ = 8 \text{ in.}, CR = 5 \text{ in.} \). Calculate \( \angle PQR \).

B 8

1. On a map, whose scale is 6 inches to the mile, a plot of land is represented by a rectangle 4·5 in. long, 2·4 in. wide. At an auction, the land fetches £12,000. If the owner receives £60 an acre, find as a percentage what commission on the sale price has been deducted by the auctioneer.

2. Formerly 35% of a man's income was taken for tax, 54% of it was used for living expenses, and the remainder was saved. His income has now risen by 45%; his taxes are 80% more, and his living expenses 35% more, than they were before. Find the percentage his present savings are of his former savings.

3. Factorise \( (i) (x + 2y)(x^2 - y^2) + (x + y)(x^3 - 4y^2); \\
(ii) bc + a(b - c - a^2) \).

4. A body is projected vertically upwards so that its height above the ground is \( (80t - 16t^2) \) feet after \( t \) seconds. Find to the second the times at which it is at a height of 48 feet. For what length of time is the body more than 48 feet above the ground?

5. In fig. 137, \( X, Y, Z \) are the mid-points of \( BC, CA, AB \); XP is equal and parallel to BY. Prove that \( P, Y, Z \) are collinear, \( \text{(ii) } AP \text{ is equal and parallel to } ZC \).

6. \( AB \) is a diameter of the circle \( APQB; AN \) is the perpendicular from \( A \) to \( PQ \) produced. Prove that \( \angle BAO = \angle PAB \).

7. In \( \triangle ABC \), \( \angle B = 76^\circ, \angle C = 68^\circ, BC = 10 \text{ cm.} \). Calculate the radius of (i) the circle inscribed in \( \triangle ABC \), (ii) the circle escribed to \( BC \).

B 9

1. An ironmonger sells patent stoves at £5 17s. each at a profit of 30%. When the wholesale price rises by 10s., the sale price is increased so as to give him a profit of 27½%. The number of stoves he sells per year then falls by 14½%. Find the percentage decrease in the profit made annually from sales of the stoves.

2. A closed box whose external measurements are 11 in. by 9 in. by 7½ in. is made of wood ½ in. thick. The wood is cut from a plank 10 ft. long, 9 in. wide, ½ in. thick which weighs 14 lb. 12 oz. Find to the nearest oz. the weight of the box.

3. If a man walks for an hour and cycles for an hour, he goes 16 miles; if he walks a mile and cycles a mile, he takes 20 minutes. Find his rates of walking and cycling.

4. If \( x = c y^{2 + t}, \) express \( z \) in terms of \( c, x, y \). Find also the value of \( z \) when \( c = 8, x = 1000, y = 9 \).
5. A circle is divided into two segments by a chord which subtends a right angle at the centre of the circle. If \( \pi \) is taken to be \( \frac{22}{7} \), prove that the area of the larger segment is 10 times that of the smaller. A closer approximation for \( \pi \) is 3.1416; is the ratio of the areas of the segments actually greater or less than 10?

6. ABCDEF is a regular hexagon; AC cuts BD at N. By using similar triangles, or otherwise, prove that BD = 3BN.

7. On a plane hill-side sloping at 28° to the horizontal, a path is cut making an angle 60° with the line of greatest slope which bears north. Find the angle the path makes with the horizontal plane and the bearing of the path.

If in walking along the path, a man rises 1 foot vertically for every \( n \) feet he moves horizontally, find the value of \( n \) to 3 figures.

B 10

1. A man who has £500 lends £325 to X at 4% p.a. interest and £175 to Y at 6% p.a. interest. The interest is paid by each for the first year, and then X repays the loan in full and Y goes bankrupt and can only pay 16s. in the £. How much better off would the man have been if he had put all the money into the Post Office Savings Bank at 24% p.a. interest?

2. A solid sphere of aluminium weighs twice as much as a solid sphere of silver; silver is 3.96 times as heavy as aluminium. Find the ratio of the diameter of the aluminium sphere to that of the silver sphere in the form \( n : 1 \), giving \( n \) to 3 figures.

3. (i) If \( \frac{x - 1}{x + 1} = y \), express \( \frac{x - 2}{x + 2} \) in terms of \( y \).

(ii) If \( x = \frac{2p + 3q}{2p - 3q} \) and \( y = \frac{2p - 3q}{2p + 3q} \) and \( z = \frac{p}{q} \), express \( \frac{x + y}{x - y} \) in the form \( az + \frac{b}{z} \) where \( a \) and \( b \) are constants.

4. Find the values of \( x + \frac{8}{x} \) for \( x = 0, 1, 2, 3, 4, 5, 6 \). Draw the graph of \( x + \frac{8}{x} \) for this range of values and use the graph for the following: (i) Find the smallest positive value of \( x + \frac{8}{x} \) (ii) Find two values of \( x \) such that \( x + \frac{8}{x} = 7 \). (iii) Solve \( x^2 - 10x + 8 = 0 \).

B 11

1. The addition of a lubricant to the petrol increases the number of miles per gallon of petrol a car runs from 17.6 to 19.2. The petrol costs 3s. 6d. per gallon and the lubricant costs 15s. per 100 gallons of petrol. Find to 2 figures what percentage of the cost is saved by using the lubricant.

2. The length of a metal tube, open at both ends, is 18.3 cm. and its weight is 23.145 grams; the outer diameter is 0.92 cm. and the metal weighs 2.65 gm. per c.c. Find to 2 figures the internal diameter.

3. (i) If \( x = \frac{1}{3} \), find the values of \( x^2 + \frac{1}{x^2} \) and \( x^3 - \frac{1}{x^3} \).

(ii) If \( y = 2 + \sqrt{3} \), find the values of \( y^2 + \frac{1}{y^2} \) and \( y^3 + \frac{1}{y^3} \).

4. The cost of providing school dinners is partly constant and partly varies as the number of pupils. For 200 pupils, the cost is 1s. 6d. each, for 300 pupils it is 1s. 4d. each. Find the number of pupils if the cost is (i) 1s. 3d. each, (ii) 1s. 2d. each. Explain why the cost is never as low as 1s. each, however many pupils there are.

5. Fig. 139 is formed by 5 straight lines. If AB = AC and PQ = PR and TC = TR, prove that (i) B, C, R, Q lie on a circle; (ii) TC and TR touch the circle through C, R, N.
6. In \( \triangle ABC \), \( AB = AC = 2BC \). The line bisecting \( \angle ABC \) meets \( AC \) at \( P \); the circle which touches \( AC \) at \( P \) and passes through \( B \) cuts \( AB, BC \) at \( Q, R \) respectively. (i) Prove that \( CR = \frac{2}{3} CA \).
(ii) Find the ratio \( AQ : QB \). (iii) Prove that \( AB = 2CR \).

7. \( ABCD \) is a quadrilateral inscribed in a circle; \( AB = 4 \text{ cm}, \ BC = 3 \text{ cm}, \ CD = 6 \text{ cm}, \ DA = 7 \text{ cm} \). Calculate \( \angle ABC \) and the length of \( AC \). Find also the radius of the circle.

**B 12**

1. A concrete road-way in a mushroom farm consists of two rectangular sections, running east and south respectively, joined by a curved portion bounded by two concentric quarter circles. The width of the road is 15 feet throughout and the radius of the inner circular quadrant is 12 feet. The lengths of the rectangular sections are 60 yards and 40 yards respectively. Calculate to 3 figures the number of cu. yards of hard core required to form a bed for the road-way to a depth of 9 inches.

2. A man borrows £1200 and is charged interest at 4\% each year on the amount owing at the beginning of that year. He repays £400 at the end of each of the first 2 years. How much, to the nearest penny, must he pay at the end of the third year to clear off the debt?

3. A greengrocer buys \( k \) pine-apples for \( p \) shillings each; he sells \( x \) of them for \( 2p \) shillings each and the remainder for \( (p + 1) \) shillings each. Find his total profit. If \( x = \frac{1}{3} k \), find the value of \( p \) if he makes a profit of 40 per cent.

4. If \( \log_{10} y = \frac{1}{2} \log_{10} x + 2 \), find \( y \) in terms of \( x \) and evaluate \( y \) when \( x = 2y \).

5. In fig. 140, \( AP \) is a side of a regular decagon (10 sides) and \( AQ \) is a side of a regular pentagon (5 sides) inscribed in the circle on \( AB \) as diameter; \( PQ \) cuts \( AB \) at \( R \). Prove that \( QR = \frac{1}{3} AB \).

6. \( ABK, CKD \) are intersecting chords of the circle \( AKB \); \( AK = 2KB \) and \( C \) is the mid-point of the arc \( AB \); the tangent at \( D \) meets \( AB \) produced at \( T \). Prove that (i) \( AD = 2BD \), (ii) \( AT = 4BT \).

7. A sector, whose angle is 120\(^{\circ}\), is cut away from a circle of radius 6 in.; the remainder is rolled into the form of the curved surface of a circular cone. Calculate (i) the radius of the base of the cone, (ii) the semi-vertical angle of the cone.

**B 13**

1. The cost of using an electric cooker consists of a quarterly charge of 15 shillings and a charge of 1½d. per unit of electricity. If the consumption for a quarter is 472 units, find to 3 figures what percentage the fixed charge is of the quarterly bill. For the next quarter the charge per unit is raised to 2½d. and the fixed charge is unaltered. I decide to reduce my consumption so that my quarterly bill does not increase by more than 10 per cent. Find to 2 figures the percentage reduction I must make in my consumption.

2. A tank, 5 ft. long, 4 ft. wide, 6 ft. deep, is half full of water. A solid metal sphere, diameter 2½ feet, is dropped into the tank. Find in inches, to 3 figures, the rise in the water level. [Volume of sphere, radius \( r \), is \( \frac{4}{3} \pi r^3 \).]

3. The purchase tax on an article is increased from \( p \) per cent. of its tax-free price to \( q \) per cent. of it and in consequence the price to the customer is increased by \( k \) shillings. Find the new price the customer pays.

4. Find the value of (i) the square of \( \sqrt{\frac{1}{2}} - \sqrt{\frac{1}{3}} \); (ii) \( \frac{1}{5 + \sqrt{11}} + \frac{1}{5 - \sqrt{11}} \).

5. \( ABCD \) is a parallelogram; \( BCHK \) and \( CDPQ \) are squares outside \( ABCD \). Prove that \( AP \) is equal and perpendicular to \( AK \).

6. In fig. 141, \( AB \) is a diameter of the circle \( APQB; \ PQ = QB; PR \) is drawn parallel to \( QB \) to meet \( AB \) at \( R \). Prove that \( AR = AP \).

7. Two ships \( P, Q \) leave a port at the same time; \( P \) steams at 15 knots in direction N. 25\(^{\circ}\) E. and \( Q \) steams at 18 knots in direction S. 48\(^{\circ}\) E. Find the distance in nautical miles and the bearing of \( P \) from \( Q \) after 20 minutes.
1. A shopkeeper sells in one week 100 pails at a profit of 25 per cent. In the next week he gives a discount of 5 per cent. on the previous week’s price and sells 120 pails. Find his profit per cent. on the fortnight’s sales. Find also the least number of pails he must sell in the second week to obtain at least as much actual profit as in the first week.

2. The section of a hut is a rectangle 18 ft. 8 in. wide, 9 ft. 6 in. high, surmounted by an isosceles triangle; the hut is 15 yards long and the height of the roof-ridge above the ground is 6 ft. 6 in. Find the capacity of the hut in cu. ft. The roofs, sides and ends of the hut are insulated by sheets of fibre-glass. Find in sq. ft. the area of fibre-glass required, assuming that doors at each end, 12 ft. wide, 8 ft. high, are not insulated.

3. Solve correct to 2 places of decimals

\[
\frac{1}{x+1} - \frac{2}{x+2} - \frac{1}{x+4}
\]

4. It is given that an \( n \)-sided polygon has \( \frac{1}{2} n(n-3) \) diagonals. Find expressions for the numbers of diagonals of the polygons which have \((n-1)\) sides and \((2n+1)\) sides.

5. In fig. 142, \( AB > AC \). The bisectors \( BP, CQ \) of \( \angle ABC, \angle ACB \) meet the circle \( ABC \) at \( P, Q \). If \( BP = CQ \), prove that \( \angle BAC = 60^\circ \).

6. In fig. 143, \( BE, CF \) are altitudes of \( \triangle ABC \) and are produced to meet the semicircles, diameters \( AC, AB \), at \( Q, R \). Prove that

(i) \( AE \cdot AC = AF \cdot AB \); (ii) \( AQ = AR \).

7. \( V \) is the vertex of a right circular cone and \( AB \) is a chord of the base subtending an angle of \( 70^\circ \) at the centre of the base. The semi-vertical angle of the cone is \( 50^\circ \). Calculate \( \angle AVB \).

8. A chord \( AB \) of a circle, centre \( O \), radius 2 in., subtends an angle \( 35^\circ \) at \( O \); \( OA \) is produced to \( C \) so that \( OC = 5 \) in. Calculate the radius of the circle which touches the given circle at \( B \) and passes through \( C \).
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**Log Sines**

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**Log Sines**

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Use Interpolation.
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#### Use Interpretation

- 0°: 1.0000
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- 12°: 0.9996
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Where the integer changes, the numbers are interlaced.
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**Natural Tangents**

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|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|

**Use Interpolation:**

1. **For 4°, 5°, 6°, 7°, 8°, 9°, 10°, 11°, 12°, 13°, 14°, 15°, 16°, 17°, 18°, 19°, 20°, 21°, 22°, 23°, 24°, 25°, 26°, 27°, 28°, 29°, 30°, 31°, 32°, 33°, 34°, 35°, 36°, 37°, 38°, 39°, 40°, 41°, 42°, 43°, 44°, 45°, 46°, 47°, 48°, 49°, 50°, 51°, 52°, 53°, 54°:**

2. **For 1°, 2°, 3°, 4°, 5°, 6°, 7°, 8°, 9°, 10°, 11°, 12°, 13°, 14°, 15°, 16°, 17°, 18°, 19°, 20°, 21°, 22°, 23°, 24°, 25°, 26°, 27°, 28°, 29°, 30°, 31°, 32°, 33°, 34°, 35°, 36°, 37°, 38°, 39°, 40°, 41°, 42°, 43°, 44°, 45°, 46°, 47°, 48°, 49°, 50°, 51°, 52°, 53°, 54°:**

3. **For 0°, 1°, 2°, 3°, 4°, 5°, 6°, 7°, 8°, 9°, 10°, 11°, 12°, 13°, 14°, 15°, 16°, 17°, 18°, 19°, 20°, 21°, 22°, 23°, 24°, 25°, 26°, 27°, 28°, 29°, 30°, 31°, 32°, 33°, 34°, 35°, 36°, 37°, 38°, 39°, 40°, 41°, 42°, 43°, 44°, 45°, 46°, 47°, 48°, 49°, 50°, 51°, 52°, 53°, 54°:**

4. **For 0°, 1°, 2°, 3°, 4°, 5°, 6°, 7°, 8°, 9°, 10°, 11°, 12°, 13°, 14°, 15°, 16°, 17°, 18°, 19°, 20°, 21°, 22°, 23°, 24°, 25°, 26°, 27°, 28°, 29°, 30°, 31°, 32°, 33°, 34°, 35°, 36°, 37°, 38°, 39°, 40°, 41°, 42°, 43°, 44°, 45°, 46°, 47°, 48°, 49°, 50°, 51°, 52°, 53°, 54°:**

Where the integer changes, the numbers are italicized.

**Difference for 1°:**

<p>| 1°  | 2°  | 3°  | 4°  | 5°  | 6°  | 7°  | 8°  | 9°  | 10° | 11° | 12° | 13° | 14° | 15° | 16° | 17° | 18° | 19° | 20° | 21° | 22° | 23° | 24° | 25° | 26° | 27° | 28° | 29° | 30° | 31° | 32° | 33° | 34° | 35° | 36° | 37° | 38° | 39° | 40° | 41° | 42° | 43° | 44° | 45° | 46° | 47° | 48° | 49° | 50° | 51° | 52° | 53° | 54° |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|</p>
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</tbody>
</table>

Find the first significant figure and the position of the decimal point by inspection.
ANSWERS

EXERCISE 1

Page 1

1. 22 yd. 2. 20. 3. 52 esc. 4. 4 ton 9 cwt. 32 lb.
5. 14. 6. £12 10s. 6d. 7. 11s. 11½d. 8. 725 tons.
9. 20-5 m.p.h. 10. 2 ft. 2½ in. 11. 2 ft. 0 in.
12. 11 cm. 7 mm. 13. 1s. 3½d. 14. £56 14s.
15. £43 4s. 16. 2³⁄₅ mi. 17. 48 hr.
18. 15·9 m. per sec. 19. 8s. 3¼d. 20. £7 13s. 1d.
21. 10s. 1½d. 22. 31-7 m.p.h. 23. 12.
24. 2½d. per lb. 25. 35 bags; 1 lb. 14½ oz.
26. 65 articles; 2½d.
28. £562 10s.; 3·2d. in the £.
30. 19·5 kr. to the £.

Page 3

EXERCISE 2

1. 72 mi. 2. 12 days. 3. 33 ft. per sec.
4. 460 m.p.h. 5. £2 0s. 10d. 6. £2 10s. 7. 5 hr.
8. 18s. 4d. 9. 1 ton. 10. 36 m.p.h. 11. 45 m.
12. £8 2s. 13. 784 revs. 14. 29·7 gall. 15. £33 15s.
16. £1145 16s. 8d. 17. £96,180; £3 3s.; 6½d. in the £.
18. 19d. in the £; 4s. 7d. 19. 48. 20. 5809 tons.
21. 39 m.p.h. 22. 36½ m.p.h. 23. £6 2s. 6d.
24. 1 cwt. 7 lb. 25. 125 men. 26. 25·2 mi., 3·48 p.m.
27. 113 gm., 170 gm., 198 gm. 28. 30 cm. 5 mm.: 71 cm. 1 mm.
29. 78·0 cu. in. 30. 1006 m. 31. £17 16s. 3d.
32. £298 1s. 8d. 33. £4 10s. 1d. 34. 6 yd. 4 in.
35. £6 3s. 6d. 36. £22 5s. 11½d.

Page 7

EXERCISE 3

1. 2 : 3. 2. 242. 3. 18s.
4. 1s. 6d. 5. 135 mi. 6. £25.
7. 9 : 16. 27 : 64. 8. 3½ lb. 9. 2 sq. mi.
10. 1¼ sq. in. 11. 225 sq. mi.; 4½ in. 12. 96 sq. ft.
13. 650 ft. 14. £31 5s. per ac. 15. 2·025 kg.
16. 13s. 6d. 17. 1200 sq. yd., 1500 sq. yd.
## ANSWERS

### Exercise 3 (continued)

<table>
<thead>
<tr>
<th>Page 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>18. £20, £12, £8.</td>
</tr>
<tr>
<td>19. 6, 10, 5, 15, cm.</td>
</tr>
<tr>
<td>20. 5s, 10d, 2s, 4d.</td>
</tr>
<tr>
<td>21. £5 5s.</td>
</tr>
<tr>
<td>22. 3 s, 14d.</td>
</tr>
<tr>
<td>23. £48, £42, £30.</td>
</tr>
<tr>
<td>24. £220 10s., £12 10s.</td>
</tr>
</tbody>
</table>

### Exercise 4

<table>
<thead>
<tr>
<th>Page 9</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. £12 10s.</td>
</tr>
<tr>
<td>2. 74%</td>
</tr>
<tr>
<td>3. £7 a week.</td>
</tr>
<tr>
<td>4. 750.</td>
</tr>
<tr>
<td>5. 162%</td>
</tr>
<tr>
<td>6. £25.</td>
</tr>
<tr>
<td>7. 7%</td>
</tr>
<tr>
<td>8. 30%</td>
</tr>
<tr>
<td>9. 673/4% ; 1-1%</td>
</tr>
<tr>
<td>10. 20%</td>
</tr>
<tr>
<td>11. 17%</td>
</tr>
<tr>
<td>12. £85.</td>
</tr>
<tr>
<td>13. 40-8%</td>
</tr>
<tr>
<td>14. £80.</td>
</tr>
<tr>
<td>15. 182%</td>
</tr>
<tr>
<td>16. 34%</td>
</tr>
<tr>
<td>17. £1,310,000,000.</td>
</tr>
<tr>
<td>18. £530.</td>
</tr>
<tr>
<td>19. £190.</td>
</tr>
<tr>
<td>20. £6 18s. 8d.</td>
</tr>
<tr>
<td>21. £31 4s.</td>
</tr>
<tr>
<td>22. £8.</td>
</tr>
<tr>
<td>23. £21 16s. 10d.</td>
</tr>
<tr>
<td>24. 6s. 11d.</td>
</tr>
<tr>
<td>25. £960.</td>
</tr>
<tr>
<td>26. 44%</td>
</tr>
<tr>
<td>27. 40%</td>
</tr>
<tr>
<td>28. 1 1/2 yr.</td>
</tr>
</tbody>
</table>

### Exercise 5

<table>
<thead>
<tr>
<th>Page 12</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 613/4% ; 80, 70, 40 ; 631/4%</td>
</tr>
<tr>
<td>2. £21 18s. 9d. ; 481/4%</td>
</tr>
<tr>
<td>3. 34-9%</td>
</tr>
<tr>
<td>4. 17s. 6d.</td>
</tr>
<tr>
<td>5. 614/4%</td>
</tr>
<tr>
<td>6. 46s. 3d.</td>
</tr>
<tr>
<td>7. 42%</td>
</tr>
<tr>
<td>8. 263/4%</td>
</tr>
<tr>
<td>9. 20%</td>
</tr>
<tr>
<td>10. £163 16s. 6d.</td>
</tr>
<tr>
<td>11. £58 decrease.</td>
</tr>
<tr>
<td>12. £2 10s.</td>
</tr>
<tr>
<td>13. 46s. 3d.</td>
</tr>
<tr>
<td>14. 84%</td>
</tr>
<tr>
<td>15. 17s. 6d. in the £.</td>
</tr>
<tr>
<td>16. 14-8% ; 2500.</td>
</tr>
</tbody>
</table>

### Exercise 6

<table>
<thead>
<tr>
<th>Page 15</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. £302 10s.</td>
</tr>
<tr>
<td>2. 770 yd.</td>
</tr>
<tr>
<td>3. 1134 tiles.</td>
</tr>
<tr>
<td>4. £6 3s. 9d.</td>
</tr>
<tr>
<td>5. £16 17s. 6d.</td>
</tr>
<tr>
<td>6. 690 sq. ft.</td>
</tr>
<tr>
<td>7. 58 sq. ft.</td>
</tr>
<tr>
<td>8. 2304 black, 5184 white.</td>
</tr>
<tr>
<td>9. 20 ac.</td>
</tr>
<tr>
<td>10. 374 in. ; 86½ sq. in.</td>
</tr>
<tr>
<td>11. 6s.</td>
</tr>
<tr>
<td>12. 10 m.p.h.</td>
</tr>
<tr>
<td>13. 288 revs.</td>
</tr>
<tr>
<td>14. 7-9 lb.</td>
</tr>
<tr>
<td>15. 3 3/4 ac.</td>
</tr>
<tr>
<td>16. 170½ sq. yd.</td>
</tr>
<tr>
<td>17. 207 sq. in.</td>
</tr>
<tr>
<td>18. 10-7 sq. ft.</td>
</tr>
<tr>
<td>19. £11 2s.</td>
</tr>
<tr>
<td>20. £38.</td>
</tr>
</tbody>
</table>

### Exercise 7

<table>
<thead>
<tr>
<th>Page 17</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 8½ cu. yd.</td>
</tr>
<tr>
<td>2. 500 gall.</td>
</tr>
<tr>
<td>3. 98 lb.</td>
</tr>
<tr>
<td>4. 0-13 in.</td>
</tr>
<tr>
<td>5. 36-2 sq. ft.; 16-8 cu. ft.</td>
</tr>
<tr>
<td>6. 191 gall.</td>
</tr>
<tr>
<td>7. 7 cm.</td>
</tr>
<tr>
<td>8. 1½ in.</td>
</tr>
<tr>
<td>9. 100 tons.</td>
</tr>
<tr>
<td>10. 33½ min.</td>
</tr>
<tr>
<td>11. 0-5 litre.</td>
</tr>
<tr>
<td>12. 230 cu. in.</td>
</tr>
<tr>
<td>13. 39-6 cu. in.</td>
</tr>
<tr>
<td>14. 3-1 in.</td>
</tr>
<tr>
<td>15. 7 in.; 9-2%.</td>
</tr>
<tr>
<td>16. 197 kg.</td>
</tr>
<tr>
<td>17. 1630 lb.; 159 : 208.</td>
</tr>
<tr>
<td>18. 10¾ cu. ft.; 29½ sq. ft.</td>
</tr>
<tr>
<td>19. 1500 cu. ft.</td>
</tr>
<tr>
<td>20. 96,250 gall.; 12½ hr.; 3-5 ft. per sec.</td>
</tr>
<tr>
<td>21. £337 10s.</td>
</tr>
</tbody>
</table>

### Exercise 8

<table>
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<tr>
<th>Page 19</th>
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</thead>
<tbody>
<tr>
<td>22. 201 cu. ft.; 189, (188-5), sq. ft.</td>
</tr>
<tr>
<td>23. 4 in.; 3170 cu. in.; 1810 sq. in.</td>
</tr>
<tr>
<td>24. 0-47 in., 10-24 in.</td>
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### Exercise 9

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<tbody>
<tr>
<td>21. 4-01; 4-0.</td>
</tr>
<tr>
<td>22. 0-041; 0-0406.</td>
</tr>
<tr>
<td>23. £2 12s. 2½d.</td>
</tr>
<tr>
<td>24. £0.4240.</td>
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<tr>
<td>25. 9 stone 2½ lb.</td>
</tr>
<tr>
<td>26. £153 7s. 4d.</td>
</tr>
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### Exercise 10

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<tbody>
<tr>
<td>21. 20-6 mi.</td>
</tr>
<tr>
<td>2. 15-1 cm.</td>
</tr>
<tr>
<td>3. 103 per sq. km.</td>
</tr>
<tr>
<td>4. 552 lb.</td>
</tr>
<tr>
<td>5. 4580 fr.</td>
</tr>
<tr>
<td>6. 17-5 (5) cm.</td>
</tr>
<tr>
<td>7. 14-5(5) hr., 29-1 hr.</td>
</tr>
<tr>
<td>8. 4-93 in.</td>
</tr>
<tr>
<td>9. 95-3 lb.</td>
</tr>
<tr>
<td>10. 3180.</td>
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<tr>
<td>11. 1440.</td>
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<tr>
<td>12. 0-254 mm.</td>
</tr>
<tr>
<td>13. £397.</td>
</tr>
<tr>
<td>14. £717.</td>
</tr>
<tr>
<td>15. £633 15s. 8d.</td>
</tr>
<tr>
<td>16. £707 10s. 6d.; £87 10s. 6d.</td>
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### Exercise 11

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<th>Page 26</th>
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<tbody>
<tr>
<td>1. £825; £37 10s.</td>
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<tr>
<td>2. £148; £8 8s.</td>
</tr>
<tr>
<td>3. £312 10s. stock; £8 11s. 10½d.</td>
</tr>
<tr>
<td>4. 600 shares; £31 10s.</td>
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<tr>
<td>5. 8s. 6d.</td>
</tr>
<tr>
<td>6. 5½%</td>
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<tr>
<td>7. 4½%</td>
</tr>
<tr>
<td>8. £3 0s. 7d. per cent</td>
</tr>
<tr>
<td>9. 83½</td>
</tr>
<tr>
<td>10. £75.</td>
</tr>
<tr>
<td>11. £3 6s. 8d. more from stock; 136.</td>
</tr>
<tr>
<td>12. 96.</td>
</tr>
<tr>
<td>13. £42.</td>
</tr>
<tr>
<td>14. 3½% stock.</td>
</tr>
<tr>
<td>15. 6 15s.</td>
</tr>
<tr>
<td>16. £6000 stock; £17 10s. less.</td>
</tr>
<tr>
<td>17. £3 17s. 6d.</td>
</tr>
<tr>
<td>18. 6s. 6d.</td>
</tr>
<tr>
<td>19. £4000 stock; £88.</td>
</tr>
<tr>
<td>20. £4000 stock.</td>
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<tr>
<td>21. £102.</td>
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</tbody>
</table>
**ANSWERS**

**Page 28**

**EXERCISE 12**

1. 36\(\frac{2}{3}\) lb.; 2 qr. 2. 70 esc. per day. 3. 231,800. 4. 6.3 dollars; £2 5s. 5. £1 11s. 6. £188 7s. 6d. 7. 59.04%. 8. 24 m.p.h. 9. £7 19s. 11d. 10. £4 11s. 3d.; 78\% 11. 12. 2. : 1. 12. A £10,000; B £30,000; C £5000. 13. £2 10s. 14. 16. £421 5s. 6d.; £422 10s. 15. 17. £954, £1288 13s. 4d., £1176 6s. 8d. 18. 19. 17 6d. in the £; £3 6s.; 15\%\% (15\frac{3}{4})%. 20. 12 8 gall. per min.

**Page 31**

**EXERCISE 13**

1. 16. 2. 11. 3. 3. 5. 800. 6. 1. 7. 8x - y; - 2x - 2y. 8. 5b - 5a. 9. 2a^2. 10. 36a^2. 11. 8c. 12. a - b - c. 13. x^2 + y^2. 14. 6a + 1. 15. be^2 - c^2. 16. 2x^2 + 6x - 1. 17. 2y^2 - y - 3. 18. z^3 - z + 2; - 1. 19. 28^2 - 2ab. 20. 21. 22. x^2 + 2 + \frac{1}{x^2}; 7. 23. 165 lb. 24. 12 - 4x^2.

**Page 32**

**EXERCISE 14**

1. 1\frac{1}{2}. 2. 180. 3. 24\frac{1}{2}. 4. - 6\frac{1}{2}. 5. - 1. 6. 9. 7. - 2\frac{1}{2}. 8. 27. 9. 13. 10. 5. 11. 12. \frac{3}{2}. 13. \frac{1}{2}. 14. - 6. 15. 5. 16. 12. 17. 12. 18. 25; 54\^4, 54\^5, 72\^4. 19. 144 ft. 20. 27. 21. 21 mi. 22. 36 m.p.h. 23. 655 runs. 24. 2 8 mi. 25. 21, 35. 26. 3\frac{1}{2} pm. 27. 2x in place of 5x.

**Page 35**

**EXERCISE 15**

1. - 1, 2. 2. 3\frac{1}{8}, - 1. 3. 21, 20. 3. 4. 5. \frac{3}{2}, 41. 6. 5, - 10\frac{2}{3}. 7. 0.75, 0.5. 8. 11, 3. 9. 8, 6. 10. 2, 3, - 1. 11. - 5, 2, 6. 12. - a, 2a. 13. \alpha, \beta - \gamma. 14. \frac{2m_mog}{m_1 + m_2}, \frac{(m_1 - m_2)(m_1 + m_2)}{m_1 + m_2}. 15. - 2\frac{1}{2}, - 5; - 1\frac{9}{10}, - 2. 16. 3, - 5; 0, 1\frac{1}{2}.

**Page 39**

**EXERCISE 16**

EXERCISE 17 (continued)

22. \( \frac{3}{(x - 2)(x - 3)(2x - 3)} \)
23. \( \frac{6}{x - 1} \)
24. \( \frac{2x - 5}{(x - 1)(x + 2)(x + 3)} \)
25. \( \frac{2y - 7}{(y - 3)(y - 4)(y + 4)} \)
26. \( \frac{-5x - 1}{x - 8} \)

EXERCISE 18

1. \( n \) inches.
2. \( \frac{10x + 9y}{2x + 3y} \) pence per lb.
3. \( \frac{4}{5} \) lb.
4. \( \frac{1}{4} \) sq. ft.
5. \( \frac{m^2 + 2ny}{m + n} \) lb. per cu. ft.
6. 4t years.
7. \$2x \) w.
8. \( \frac{cgn}{dp} \) men.
9. \( (h + 2hb + 2hd) \) sq. ft.
10. 10a + 4.
11. \( \frac{d(12W - w)}{12} \) shillings.
12. \( \frac{3a + b}{2a} \)

EXERCISE 19 (continued)

9. \( t = \frac{d \sqrt{r}}{r_1 - r_2} \) \( ; \) 1.99.
10. \( r = \frac{b + a}{b - a} \)
11. \( v = \frac{uf}{u - f} \)
12. \( r = \pm \sqrt{(a^2 - c^2)} \)
13. \( r = \pm \sqrt{(\frac{3V}{16h})} \)
14. \( t = \frac{g_0^2}{4x^2} \), \( g = \frac{4\pi^2I}{x^2} \)
15. \( u = \pm \sqrt{(y^2 - 2as)} \)
16. \( r = \frac{3V}{4} \) shillings.
17. \( V = \frac{1}{4} \) y(S - 2y^2).
18. \( d = \frac{2c}{\pi + 2} \)
19. \( r = \frac{5l}{8} \)
20. \( \frac{4}{8} \)

EXERCISE 20

1. 1, -8.
2. \( \frac{1}{1} \), -1.
3. \( \frac{1}{3} \), -2.
4. 0, \( \frac{1}{3} \).
5. \( \frac{1}{5} \), 2.
6. \( \frac{1}{8} \), 3.
7. \( \frac{1}{2} \) repeated.
8. \( -\frac{1}{6} \), \( \frac{1}{6} \).
9. \( 2 \frac{1}{2} \), -2.
10. 7.24, -1.24.
11. 4.79, 0.21.
12. 2.79, -1.79.
14. 2.26, -0.59.
15. 1.63, -1.23.
16. \( x^2 + x = 12 \); \( x^2 + 5x = 0 \); 12\( x^2 + x = 6 \).
17. \( x^3 = 6 \); \( x^2 - 10x + 22 = 0 \); \( 2x^3 + 4x + 1 = 0 \).
18. \( \frac{1}{3} \).
19. 3, 5.
20. \( 1 \frac{1}{2} \), -2\( \frac{1}{2} \).
21. 5, -8.
22. -3, -6.
23. 0, c - b.
24. \( a + 1 \), -a.
25. -3, -8 or 24, 9.
26. 3, -5 or -1, 3.
27. 2, 1 or -3, -4.
28. 7, 2 or -11\( \frac{1}{2} \), -4\( \frac{1}{2} \).
29. 7, 5 or -6\( \frac{1}{2} \), -3\( \frac{1}{2} \).
30. 2, 5 or 24, -6.
31. 9, 10, 11 or -5, -6, -7.
32. 30 in.
33. \( 2 \frac{1}{2} \).
34. 17, 15, 8.
35. 2.56 in.
36. 15 m.p.h.
37. 7 boys.
38. 80 men.
39. 24 m.p.h.
40. 20 (or -25).

EXERCISE 21

1. \( 0^0 \) F.; \( 4.25 \) lb. per gall.
2. 83s. 6d.; 43 years.
3. 4.36 p.m., 26 mi.; 5.54 p.m.
4. 1.30 a.m., 4.5 mi.; 1.44 p.m., 11.2 mi.
Page 50

**EXAMPLES FOR ORAL WORK**

1. \( \frac{1}{2}(x - 1) \).  
2. \(-1\cdot17, 2\cdot84\).  
3. \(-1\cdot88, 3\cdot55\).  
4. \(-1\cdot62, 3\cdot29\).  
5. \(-2, 3\cdot67\).  
6. No.  
7. \(-12\cdot08, 0\cdot83\).  
8. \(-1\cdot55 < x < 3\cdot22\).  
9. \(-0\cdot70 < x < 2\cdot37\).  
10. \(-1 < x < 3\).  
11. \(3x^2 - 2x - 8 = 0\).  
12. \(x^2 - 2x - 3 = 0\).  
13. \((-\frac{1}{2}, \frac{3}{2})\).  
14. \((-\frac{1}{4}, -\frac{1}{4})\).

Page 51

**EXERCISE 22**

1. \(-0\cdot5, 3\).  
2. \(-1\cdot41, 3\cdot91\).  
3. \(-0\cdot53, 3\cdot28\).  
4. \(-6\cdot12\).  
5. \(-2 < x < 4\cdot5\).  
6. \(-1\cdot5 < x < 5\).  
7. \(0\cdot4, 4\cdot56\).  
8. \(-1\cdot27, 6\cdot27\).  
9. \(-0\cdot54, 5\cdot54\).  
10. \(-0\cdot5 < x < 4\cdot75\).  
11. \(-0\cdot45, 4\cdot45\).  
12. \(0\cdot84, 7\cdot16\).  
13. \(1\cdot27, 4\cdot73\).  
14. \(4\cdot9, 2\cdot45\).  
15. \(2\cdot88, 1\cdot25\).  
16. \(5\cdot61, 1\cdot79\).  
17. \(4\cdot36\).  
18. \(127\cdot6, 162\cdot9, 207\cdot9, 265\cdot4\).  
19. \(179\cdot6, 240\cdot6\).  
20. \(8\cdot3\) yr.  
21. \(2\cdot83, 3\cdot32\).  
22. \(2\cdot66\).  
23. \(7, 1\cdot5\).  
24. \(1\cdot54\).  
25. \(4, -1\).  
26. \(-0\cdot22 < x < 2\cdot22\).  
27. \(0\cdot14, 1\).  
28. \(3\cdot1\).  
29. \(2x^2 - 7x - 4 = 0\).  
30. \(-0\cdot5, 4\).  
31. \(-0\cdot83, 4\cdot83\).  
32. \(-5, 1, 6\).  
33. \((1, 0), (5, 0)\).  
34. \(11, 2x^3 - 10x + 7 = 0, -2\cdot52, 0\cdot8, 1\cdot72, 1\cdot60\).  

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**EXERCISE 23**

1. \(\frac{1}{2}-\).  
2. \(1\).  
3. \(\frac{1}{2}\).  
4. \(4\).  
5. \(64\).  
6. \(\frac{2}{3}\).  
7. \(\frac{7}{2}\).  
8. \(100\).  
9. \(25\).  
10. \(\frac{27}{2}\).  
11. \(\frac{8}{3} \times 10^{-3}\).  
12. \(\frac{1}{2}\).  
13. \(-2\).  
14. \(\frac{1}{a^2}\).  
15. \(x + 1\).  
16. \(2\).  
17. \(\log a + \frac{1}{2} \log b; \frac{1}{2} \log a - \frac{1}{2} \log b\).  
18. \(0\cdot69897, 0\cdot77815, 1\cdot17609, 0\cdot07918\).  
19. \(\log a = 3 + 1\frac{1}{2} \log a\).  
20. \(y = \frac{100}{\sqrt[3]{x^3}}\).  
21. \(12\frac{1}{2}\).  
22. \(1\frac{1}{3}\).  
23. \(x - y\).  
24. \(4\).  
25. \(6\).  
26. \(\sqrt[3]{27}; \sqrt[3]{0\cdot1}; \frac{1}{3} \sqrt[3]{2}; \frac{\sqrt{a}}{b}\).  
27. \(4\).  
28. \(4\).  
29. \(4\).  
30. \(\frac{\sqrt{2}}{2}; \sqrt[3]{15} - 3; 5 \sqrt[3]{6} - 12\).  
31. \(5\cdot464\).  
32. \(x = \pm \sqrt[3]{8} b^3\).
Page 66
EXERCISE 26
1. 66°, 23°.
2. 132°.
3. 56°.
4. 12°; 32°, 74°, 74°.
5. 54°.
6. 104°.
7. 114°, 33°, 33°.
8. 90°, 110°, 100°.
9. 79°.
10. 36°, 72°.
11. EC.
12. CD, AD, BD.

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EXERCISE 27
6. 22\(\frac{1}{2}\)°.

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EXERCISE 30
1. 1-44 in. 4. 2-01 in.; 4-25 in.; 3-13 in.; 1-33 in.
5. 3-13 in.
6. 7-94 sq. cm.
7. 9-27, 4-25, cm.
8. 2-42 in.
9. 4-72 cm.
10. 6-93 cm; 4-16 sq. cm.
11. 280 ft.
12. 5-89 cm.
13. 8-775 cm.; 27° 8'.
14. 3-46 cm.; 7-21 cm.
15. 5-29 cm., 2 cm., 5-29 cm.; 4-90 cm.

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EXERCISE 31
1. 15 sq. in. 2. 120, 108, 270, sq. cm.; 8, 10-8, 4-2, cm.
7. 2-4, 1-6, sq. in.
8. 20-8 sq. cm.

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EXERCISE 32
1. 4\(\frac{1}{2}\) sq. in.; 1-8 in.
2. 6 cm., 30 sq. cm. or 12 cm., 42 sq. cm.
4. 3-6, 6-4, 4-8, cm.; 14 cm.
7. 8, 16, cm.
10. 3\(\frac{1}{2}\) cm.
12. 4, 6-40, 4, cm.
13. Circle, centre O, radius 8 cm.
11. 3-46, (2\sqrt{3}), in.; 2 in.

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EXERCISE 33
1. 94°, 82°, 86°, 98°. 2. 94°.
3. 160°.
4. 40°.
5. 30°, 130°, 20°.
6. 58°.

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EXERCISE 34
1. 50°.
2. 18°.
3. 130°, 20°.
4. 72°, 84°.
5. 138°.

Page 83
EXERCISE 35
4. 25 cm.

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EXERCISE 36
4. \(d: c; d:b; (c+d):d; c:(a-c)\).
5. \(2\frac{1}{2}; 6\frac{1}{2}, 3\frac{3}{4}, 5\).
6. 6, 6, 7\(\frac{1}{3}\), 3.
7. 2-4, 3-6, cm.; 25 : 9 : 30.
8. 2 : 3; \(\frac{2x}{3}\) in.; 1 : 2.
9. \(\triangle DBF; \triangle DEC; \frac{BD}{DF} = \frac{AE}{BE} = \frac{DE}{GE}\).
10. 10-82 cm.
12. 3 : 2, 3 : 5; 30, 18, 8, 12, sq. in.
16. AE, BD.

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EXERCISE 37
1. 6, 3, 2\(\frac{1}{2}\) in.
2. 3-5, 6, 2-7, cm.
3. 15 cm.
4. 3-6 cm.
5. 8-1, 5-3, cm.
6. 10\(\frac{1}{2}; 8\frac{1}{2}\) cm.

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EXERCISE 38
1. 5-6, 8-4, cm.
2. 1-2, 1-8, cm.
3. 2\(\frac{1}{2}\) sq. in.
8. 1\(\frac{1}{2}\) 6, cm.; circle, radius 3\(\frac{1}{2}\) cm., diameter HK.

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EXERCISE 39
1. 30° 58', 25° 17'.
2. 8-91, 4-54, 5-41.
3. 5-3, 4-26.
4. 6-55 in., 97° 11'.
5. 67° 23', 9-98(5) cm.; 14-40, 2-58, cm.
6. 5-73(5), 2-81, in.
7. 147° 29'.
8. 2-65, 5-19, cm.; 62° 58'.
9. \(\frac{1}{2}; \frac{2}{3}; \frac{3}{4}, \frac{3}{4}\).
10. \(\frac{2}{3}; \frac{1}{2}; \frac{1}{2}\).
11. 4-04(5) cm.; 12-13(5) sq. cm.
12. 58°; 5-90 cm.; 6-93 cm.
13. 6-93, 5-77, in.; 153° 45', 93° 45'.
14. 53° 8', 6-20(5) in.
15. 0-5 in.; 5 sq. in.
16. 1-32, 8-75, cm., 34-1 sq. cm.
17. 70° 32', 24-5 in.; 21-2 in.
18. 10-95, 6-45, in.
19. 47° 10'.
20. 7-14(5), 7-75, 6-32, in.

Page 96
ORAL EXAMPLES
1. 50° 12'; 53° 8'; \(\angle\) ASB, 30° 49'.
2. \(\angle\) EAC; \(x\cos 56°, x\cos 56° \sin 32°, 17° 14'; x\sin 56°, x\cos 56° \cos 32°, 29° 46'; N. 60° 14° E.
3. 2 cm., \(\sqrt{3}\) cm.; \(\angle\) VAN, 30°; \(\angle\) VDN, 49° 6'.

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EXERCISE 40

1. 75° 58'; 67° 23'; 72° 5'.  
2. 115 ft.
3. 48° 22', 41° 59', 56° 19'.  
4. 47° 59'; 63° 26'.
5. 7 in.; 50° 29'; 49° 24'.  
6. 14° 56'; 9° 5'.
7. 250 (5) sq. ft.
8. 5196, 5196; (3√3), cm.; 4899, (2√24), cm.; 70° 32'; 54° 44'.
9. 5774; 11.65, in.; 63° 38'; 76° 5'.
10. 635 in.; 39° 24'; 40° 23'.  
11. 732 (5) mi.; 10° 34', 730 mi.
12. 61° 54'; 44° 24'.

EXERCISE 41

1. $-\frac{3}{4}$; $-\frac{1}{2}$.  
2. $\frac{1}{2}$; $-\frac{1}{2}$.  
3. 7.66 sq. in.; 51° 4', 128° 56'.
4. 618 sq. in.  
5. 1680, 1050, yd.  
6. 104 ft.
7. 130° 38'; 174, 277, in.  
8. 9° 4', 10° 21'; 11° 19'.
9. 588, 104, ft.  
10. 238, 610, yd.; 9° 33'.

EXERCISE 42

1. Obtuse-angled; acute-angled.  
2. 418, 7 11, in.
4. 107° 45'.  
5. 3.32, 3.98, in.; 11.67 sq. in.
6. 46° 34', 108° 13'.
7. 290 mi., N. 34° 26' E.  
8. 231, 59.6, yd.
9. 479 ml., N. 64° 45' E.  
10. 480 knots, N. 35° 35' E.
11. 47° 34', 100° 15'; 11 8 in.  
12. 62, 8 1, in., 74° 31'.
13. 3 cm., 134° 43'.
14. Circle, radius 4 in., centre mid-point of AB; 120° 31'.

A 1. 1. £2 15s. 7d.  
2. 180 revs.  
3. 0.  
4. (6 + z)(4 - z).  
5. 36°, 54°.
7. 38° 30', 12° 40'.
A 2. 1. 2°, 3°.  
2. £10 15s.  
3. $\frac{1}{2}$ mi.  
4. 2, -2.  
6. 6, 2, 2, 3.  
7. 11-16, 4-23, cm.
A 3. 1. 0.00145.  
2. 13 cwt.  
3. 1, -2.  
4. $\frac{4r}{k-4}$.  
6. 2 1, in.
7. 55° 9', 33° 41'.

A 4. 1. £0.890.  
2. £6 0s. 9d.  
3. $\frac{n}{12}$.  
4. $(x-y)(x+y+z)$.  
5.△ACK.  
7. 60° 15', 51° 3'.
A 5. 1. 6832.  
2. 25%.  
3. 14.  
4. $+12\sqrt{2}$, $-3\sqrt{2}$; 6 19, 0 81.
5. 34 cm.
6. CK.  
7. 17.0, 19.2, 14.9, cm.
A 6. 1. 29° 3', 0° 072.  
2. 763 sq. ft.  
3. 5, 7.  
4. $\frac{1}{t+3}$.  
5. $\triangle BQC$.  
6. 38°.
7. 5 66, 5 34, in.; 14-1(5) sq. in.
A 7. 1. 384.  
2. 5.94.  
3. 0 or 6.
4. $z = \frac{6x}{\sqrt{y}}$.  
5. 30 sq. cm.; 73, 75, cm.  
7. 53° 8', 38° 40', 34° 27'.
A 8. 1. $\frac{3}{4}$.  
2. £175.  
3. 0.04.  
4. $(x-7)(x+2)$.  
6. 10, 6, 15, 15.
7. 49° 27', 8 37, 19 04.
A 9. 1. 269 cm.  
2. 5.97%.  
3. 0.4.
6. 45°, 105°, in.  
7. 35° 30', 2 61, in., 4 93(5) sq. in.
A 10. 1. 174.  
2. 43 6, in.  
3. $(2+1)z(z-1)$.  
4. Indices are $\frac{1}{2}$, $\frac{3}{2}$, $-\frac{1}{2}$, $-\frac{1}{2}$.  
7. 5° 63 in.; 25° 14'.
A 11. 1. 0.0118.  
2. 375 ac.  
3. -3, -3; -0.6, 1.8.
4. 23.2% decr.  
7. 3350 yd.
A 12. 1. 2 73(5).  
2. 204 gal.  
3. 2.4.
4. 2.  
6. 4, 2 9.  
7. 6 49 in.; 47° 51'.
A 13. 1. 1 2.  
2. 4 27.  
3. 2, 8, $\frac{1}{3}$.  
4. 27; 16; 2\sqrt{2}.  
6. 8 7; 21 4.  
7. -0.7454; 0.8; -0.6, 1.
A 14. 1. 15, 21, 24, 4.  
2. 2.71.  
3. 1 1, 1, 1, 1.  
4. 223, -0.90.  
7. 7 61 cm.; 36° 33'.
A 15. 1. 0.732, 0.740, sec.  
2. $\frac{8}{5}$.  
3. 2, -5; x + 3.
4. -1, $2\sqrt{2}$; $15x^2 + 31x + 15 = 0$.  
6. 4, 7 49, in.
7. 2 cm.; 30°, 49° 6'.

REVISION PAPERS A 1-A 15 (continued)
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**ANSWERS**

**B 1.**
1. 12 m.p.h.
2. Rubber; £3 10s.; £7 ½s. (14s. 4 8d.)
3. \(\frac{2m_1 m_2 g}{(m_2 - m_1) g}\)
4. 3.
5. \(\frac{m_1 + m_2}{m_1 + m_2}\)
6. 11° 19′; 6° 24(5) in.; 65° 49′.

**B 2.**
1. 2.30 p.m.
2. ½ d. per mile.
3. 18° 3(5)% decn.
4. \(\frac{5x + 8}{3x^2 + 8x}; 18\frac{1}{2}\).
5. 46° 34′, 122° 5′, 40° 7 sq. cm.

**B 3.**
1. £15 9s. per month.
2. 23° 7 in.
3. -10, 7.
4. 73.
5. 83° 15′; £4 8 s.
6. 2. 0° 7071, 2° 4142, 2° 9142.
7. 36 min.
8. 4. 0° 42 in.

**B 5.**
1. 2670.
2. 5 lb.; 4° 76 in.
3. 0° 6, 0° 8; -0° 8, 0° 6.
4. \(3V - \frac{1}{2}h\).
5. (i) Q at B, (ii) Q at C.
6. 21° 43′ N.; 77° 5(5) mi.

**B 8.**
1. 2° 43 in.; 1° 77 in.
2. 2° 98 in.
3. 585 mi.
4. -1° 28, 0° 78; -1° 60, 1° 10; -1° < x < 0° 75.
5. \(\angle ABD; \triangle ABD\).
6. 7° 36, 5° 47(5) cm.; 10° 42′.

**B 7.**
1. £189 15s.
2. 1 s. 3d., 1 s. 6d.
3. \(x = 5, 33; -101, 330\).
4. \(p(e^r - 1); \frac{3}{2r}\).
5. 90° + \frac{1}{4}(\angle BLN + \angle LNC).
6. 100° 27′.

**B 8.**
1. 4%.
2. 82° 44′ 30″.
3. \((x + y)(x + 2y); (2x - 3y); (b - a)(c + a)\).
4. 0° 7, 4° 3 sec.; 3° 6 sec.
5. 3° 62, 6° 87, cm.

**B 9.**
1. 12° 12′.
2. 6 lb. 1 oz.
3. 4 m.p.h., 12 m.p.h.
4. \(x^n y^\frac{3}{2}, \frac{1}{108}\).
5. Greater.
7. 13° 35′; N. 63° E. or N. 63° W.; 4° 14.

**B 10.**
1. £24.
2. 1-99 : 1.
3. \(\frac{3y - 1}{3 + y}; \frac{\frac{1}{3}x + 0.75}{z}\).
4. 5° 66'; 5° 56'; 1° 44'; 9° 12'.
5. 5 : 4.
7. 3° 42', 4° 69', in.

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**ANSWERS**

**B 11.**
1. 44%.
2. 0° 49 cm.
3. 11, 36; 4, 14.
4. 400, 600.
5. 4 : 5.
6. 123° 45′; 6° 19′; 3° 72 cm.

**B 12.**
1. 138 cu. yd.
2. £501 5s. 11d.
3. \(x(p - 1) + k\) shillings.
4. \(100x^\frac{1}{2}; 4 \times 10^4\).
5. 4 in.; 41° 49′.

**B 13.**
1. 20° 3′; 25° 15′.
2. 2° 49 in.
3. \(\frac{k(100 + q)}{q - p}\) shillings.

**B 14.**
1. 21° 3′; 134 pails.
2. 10,920 cu. ft.; 21984 sq. ft.
3. -3.19, -0.31.
4. \(\frac{1}{2}(n - 4); (n + 1)(n - 1); n + 2\).
7. 52° 8′.

**B 15.**
1. £108 17s. 4½d.
2. 11 : 4.
3. \((x + 1)(x + 4)(x - 5)\).
4. 5; 450; 100,000; 25.
5. \(\angle DOF\).
7. 3° 01 in.